Functions of several variables

Represent the domain of function by inequalities and sketch in xy-plane.

1. $z = \sqrt{x+y} - \sqrt{x-y}$

2. $z = \sqrt{2x - 3\sqrt{y}}$

3. $z = \dfrac{1}{\sqrt{x^2 - y - 4}}$

4. $z = \arcsin \dfrac{y-1}{x}$

5. $z = \ln(1 - x^2) + \sqrt{1 - y^2}$

6. $z = \ln(x+y) + \dfrac{1}{\ln x}$

7. $z = \ln(y \sin x)$

Are the following functions identical?

8. $f(x,y) = \sqrt{x^2 y}$ and $g(x,y) = x\sqrt{y}$

9. $f(x,y) = \dfrac{xy}{x^2 y^2}$ and $g(x,y) = \dfrac{1}{xy}$

10. $f(x,y) = \sqrt{x}\sqrt{y}$ and $g(x,y) = \sqrt{xy}$

11. $f(x,y) = \ln(x^2 y)$ and $g(x,y) = 2\ln(|x|\sqrt{y})$

Evaluate the limit

12. $\lim\limits_{(x,y)\to(2;2)} \dfrac{x + \sin \pi x}{y}$

13. $\lim\limits_{(x,y)\to(0;3)} \dfrac{\sin(xy)}{x}$

14. $\lim\limits_{(x,y)\to(0+;0+)} \dfrac{x - \sqrt{y}}{x + \ln x}$

15. $\lim\limits_{(x,y)\to(\infty;\infty)} \dfrac{x+y}{x^2 + y^2}$

16. $\lim\limits_{(x,y)\to(0;0)} \dfrac{x^2 + y^2}{x+y}$

17. $\lim\limits_{(x,y)\to(0;0)} (x^2+y^2)\sin\dfrac{3}{xy}$

18. $\lim\limits_{(x,y)\to(0;0)} \dfrac{x^2+y^2}{xy}$

19. $\lim\limits_{(x,y)\to(\infty;\infty)} \left(\dfrac{xy}{x^2+y^2}\right)^{x^2+y^2}$

20. $\lim\limits_{(x,y)\to(\infty;\infty)} \left(1+\dfrac{1}{x^2y^2}\right)^{x^2y^2}$

21. $\lim\limits_{(x,y)\to(3;\infty)} \left(1+\dfrac{x}{y}\right)^{y}$