Exercises of Mathematical analysis II

In exercises 1. - 8. represent the domain of the function by the inequalities and make a sketch showing the domain in xy-plane.

1. $z = \sqrt{x - \sqrt{y}}$.
2. $z = \arcsin \dfrac{y+2}{x-1} + \ln y$.
3. $z = \sqrt{\sin \pi(x^2 + y^2)}$.
4. $z = \ln x - \ln \sin y$.
5. $z = \sqrt{4 - x^2} + \ln(y^2 - 4)$
6. $z = \sqrt{\arcsin \dfrac{x}{y}}$
7. $z = (y + \sqrt{y})\sqrt{\cos x}$
8. $z = \sqrt{x^2 + y^2 - 1} - 2\ln(9 - x^2 - y^2)$
9. Are the functions

$$z = \sqrt{x \sin y} \quad \text{and} \quad z = \sqrt{x}\sqrt{\sin y}$$

identical? Why?

10. Are the functions

$$z = \ln xy \quad \text{and} \quad z = \ln x + \ln y$$

identical? Why?

Evaluate the limit

11. $\lim\limits_{(x,y)\to(0,0)} \dfrac{3x^2 - y}{2x + y}$
12. $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2 + y^2}{\sqrt{x^2 + y^2 + 1} - 1}$
13. $\lim\limits_{(x,y)\to(0,0)} \dfrac{\sqrt{x^2y^2 + 1} - 1}{x^2 + y^2}$

14. $\lim\limits_{(x,y)\to(0,0)} \dfrac{xy}{x^2+y^2}$

15. $\lim\limits_{(x,y)\to(0,0)} \dfrac{xy(x^2-y^2)}{x^2+y^2}$

16. $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2-y^2}{x^2+y^2}$

17. $\lim\limits_{(x,y)\to(0,0)} \dfrac{\sin(x^3+y^3)}{x^2+y^2}$

In exercises 18. - 28. find partial derivatives with respect to every independent variable.

18. $z = x^2\sqrt[3]{y} + \dfrac{\sqrt{x}}{\sqrt[4]{y}}$

19. $z = \ln\tan\dfrac{x}{y}$

20. $z = e^{-\frac{x}{y}}$

21. $z = \sin xy - \cos\dfrac{y}{x}$

22. $z = \ln(x + \sqrt{x^2+y^2})$

23. $z = \arctan\dfrac{y}{\sqrt{x}}$

24. $z = xy\ln(x+y)$

25. $w = \ln(xy + \ln z)$

26. $w = \tan(x^2+y^3+z^4)$

27. $w = x^{y^z}$

28. $w = e^{2x}\cos(yz)$

29. Evaluate the partial derivatives of $z = \dfrac{x}{\sqrt{x^2+y^2}}$ at $(1;-2)$

30. $z = \dfrac{x\cos y - y\cos x}{1+\sin x+\sin y}$. Evaluate $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$, if $x = y = 0$.

31. $w = \ln(1 + x + y^2 + z^3)$. Evaluate $\frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z}$ at the point $x = y = z = 1$

32. $z = \ln(x^2 - y^2)$; prove that $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} - \frac{2}{x+y} = 0$

33. For the function $z = xy + x \arctan \frac{y}{x}$ prove that et $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = xy + z$.

34. Find the total differential of the function $z = \arcsin \frac{x}{y}$.

35. Find the total differential of the function $z = \sin \frac{x}{y} \cos \frac{y}{x}$.

36. Find the total differential of the function $z = \ln \sin \frac{x}{y}$.

37. Find the total differential of the function $w = x^{yz}$.

38. Find the total differential of the function $z = \frac{xy}{x^2 - y^2}$, if $x = 2$, $y = 1$, $\Delta x = 0,01$ and $\Delta y = 0,03$.

39. Evaluate the increment Δz and total differential dz of the function $z = \frac{x+y}{x-y}$, if $x = -3$, $y = 7$, $\Delta x = -\frac{1}{3}$ and $\Delta y = \frac{1}{4}$.

40. Evaluate the increment Δz and total differential dz of the function $z = xy + \frac{x}{y}$, if x changes from -1 to $-0,8$ and y from 2 to $2,2$.

41. Using total differential, compute the approximate value of $1,96^3 \cdot 2,03^5$.

42. Using total differential, compute the approximate value of $\frac{\sqrt{82}}{\sqrt[3]{28}}$.

43. Using total differential, compute the approximate value of $\arcsin \frac{\sqrt{1,04}}{2,04}$.

44. Using total differential, compute the approximate value of $\ln\left(\sqrt[5]{0,98} + \sqrt[4]{1,04} - 1\right)$.

45. Find $\frac{dy}{dx}$, if $x^2y^2 - x^4 - y^4 = a^2$.

46. Find $\frac{dy}{dx}$, if $2y^3 + 3x^2y + \ln x = 0$ and evaluate it at $x = 1$.

47. Find $\frac{dy}{dx}$, if $y = \sqrt{x}\ln\frac{x}{y}$ and evaluate it at the point $(e^2; e)$.

48. Find $\frac{dy}{dx}$, if $x^y = y^x$ and evaluate it at the point $(1; 1)$.

49. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$, if $x^2 - 2y^2 + z^2 - 4x + 2z - 5 = 0$.

50. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$, if $z = \cos xy - \sin xz$ and evaluate these at the point $\left(\frac{\pi}{2}; 1; 0\right)$.

51. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$, if $xyz = e^z$ and evaluate these at the point $(e^{-1}; -1; -1)$.

52. Prove that $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$, if $z = e^x(\cos y + x \sin y)$.

53. Find $\frac{\partial^2 z}{\partial x^2}$, $\frac{\partial^2 z}{\partial y^2}$ and $\frac{\partial^2 z}{\partial x \partial y}$, if $z = \arcsin(xy)$.

54. Find $\frac{\partial^3 w}{\partial x \partial y \partial z}$, if $w = e^{xyz}$.

55. Evaluate all second order partial derivatives of the function $z = \frac{x}{y^2}$ at the point $(-1; -2)$.

56. Evaluate all second order partial derivatives of the function $z = \arcsin\frac{x}{\sqrt{x^2 + y^2}}$ at the point $(1; -2)$.

57. Evaluate

$$\frac{\partial^2 z}{\partial x^2} \cdot \frac{\partial^2 z}{\partial y^2} - \left(\frac{\partial^2 z}{\partial x \partial y}\right)^2$$

at the point $(0; -2)$, if $z = \frac{\cos x^2}{y}$.

58. For the function $z = \ln(e^x + e^y)$ prove that $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 1$ and $\frac{\partial^2 z}{\partial x^2}\frac{\partial^2 z}{\partial y^2} - \left(\frac{\partial^2 z}{\partial x \partial y}\right)^2 = 0$.

59. Prove that the function $z = \dfrac{x^2y^2}{x+y}$ satisfies the equation

$$x\frac{\partial^2 z}{\partial x^2} + y\frac{\partial^2 z}{\partial x \partial y} = 2\frac{\partial z}{\partial x}$$

60. Find the gradient vector for the scalar field $z = x - 3y + \sqrt{3xy}$ at the point $(3; 4)$.

61. Find the points at which the gradient vector of the scalar field $z = \ln\left(x + \dfrac{1}{y}\right)$ is $\overrightarrow{a} = \left(1; -\dfrac{16}{9}\right)$.

62. Find the gradient vector for the scalar field $w = \arcsin\dfrac{\sqrt{x^2+y^2}}{z}$ at the point $(1; 1; 2)$.

63. Find the directional derivative of the function $z = \arctan\dfrac{y}{x} - \dfrac{4y}{x}$ at the point $(1; \sqrt{3})$ in direction the point $(2; 3\sqrt{3})$.

64. Find the directional derivative of the function $w = xyz$ at the point $A(-2; 1; 3)$ in the direction of $\overrightarrow{s} = (4; 3; 12)$.

65. Find the directional derivative of the function $w = x^2y^2 - z^2 + 2xyz$ at the point $B(1; 1; 0)$ in direction forming with coordinate axes the angles 60°, 45° and 60° respectively.

66. Find the greatest rate of change of the function $z = \ln(x^2 + y^2)$ at the point $C(-3; 4)$

67. Find the greatest value of the derivative of function given by the equation $x^2 + y^3 - z^2 - 1 = 0$ at the point $(3; 2; 4)$.

68. Find the steepest ascent of the surface $z = \arctan\dfrac{y}{x}$ at the point $(1; 1)$.

69. Find the direction of greatest increase of the function $f(x, y, z) = x\sin z - y\cos z$ at the origin.

70. Find the divergence and curl of the vector field $\overrightarrow{F} = \left(\dfrac{x}{y}; \dfrac{y}{z}; \dfrac{z}{x}\right)$.

71. Find the divergence and curl of the vector field $\overrightarrow{F} = (\ln(x^2 - y^2); \arctan(z - y); xyz)$.

72. Find the divergence and curl of the vector field $\overrightarrow{F} = \operatorname{grad} w$, if $w = \ln(x + y - z)$.

73. Find the divergence and curl of the vector field $\overrightarrow{F} = \operatorname{curl} \overrightarrow{G}$, if $\overrightarrow{G} = (x^2y; y^2z; x^2z)$.

74. Find the local extrema of the function $z = 4x^2 - xy + 9y^2 + x - y$ and determine their type.

75. Find the local extremum points of the function $z = x^3y^2(12 - x - y)$, satisfying the conditions $x > 0$ and $y > 0$ and determine their type.

76. Find the local extrema of the function $z = x^2 + xy + y^2 + \frac{1}{x} + \frac{1}{y}$ and determine their type.

77. Find the local extrema of the function $z = e^x(x^2 + y^2)$ and determine their type.

78. Find the local extrema of the function $z = x^3 + y^3 - 3xy$ and determine their type.

79. Evaluate the double integral $\iint\limits_D (x^2 + y^2)dxdy$, if D is the quadrate $0 \leq x \leq 1$ and $1 \leq y \leq 2$.

80. Evaluate the double integral $\iint\limits_D \frac{dxdy}{(x+y)^2}$, if D is the quadrate $1 \leq x \leq 2$ and $3 \leq y \leq 4$.

81. Evaluate the double integral $\int\limits_1^2 dx \int\limits_x^{x\sqrt{3}} xydy$.

82. Evaluate the double integral $\int\limits_0^1 dx \int\limits_{-x}^{x+1} (xy + y)dy$.

83. Evaluate the double integral $\int\limits_0^1 dx \int\limits_{x^2}^{\sqrt{x}} (x^2 + y^2)dy$.

84. Sketch the domain of integration and determine the limits of integration for $\iint\limits_D f(x;y)dxdy$, if D is the region bounded by the line $y = 0$ and the parabola $y = 1 - x^2$.

85. Sketch the domain of integration and determine the limits of integration for $\iint\limits_D f(x;y)dxdy$, if D is the parallelogram bounded by the lines $y = 0$, $y = a$, $y = x$ and $y = x - 2a$.

86. Sketch the domain of integration and determine the limits of integration for $\iint\limits_D f(x;y)dxdy$, if D is the region bounded by $y = \dfrac{2}{1+x^2}$ and $y = x^2$.

87. Sketch the domain of integration and change the order of integration for $\int\limits_0^1 dx \int\limits_{x^3}^{\sqrt{x}} f(x;y)dy$.

88. Sketch the domain of integration and change the order of integration for $\int\limits_{-1}^1 dx \int\limits_{-\sqrt{1-x^2}}^{x+1} f(x;y)dy$.

89. Sketch the domain of integration and change the order of integration for $\int\limits_{-2}^2 dy \int\limits_{y^2-2}^{\frac{y^2}{2}} f(x;y)dx$.

90. Sketch the domain of integration and change the order of integration for $\int\limits_{-1}^1 dx \int\limits_{x^2}^{x^2+2} f(x;y)dy$.

91. Changing the order of integration express the sum $\int\limits_0^1 dy \int\limits_{\sqrt{y}}^1 f(x;y)dx + \int\limits_{-1}^0 dy \int\limits_0^{y+1} f(x;y)dx$ by one double integral.

92. Sketch the domain of integration, determine the limits and evaluate the double integral $\iint\limits_D (x-2y)dxdy$, if D is the region given by inequalities $-1 \leq x \leq 2$ and $0 \leq y \leq x^2+1$.

93. Sketch the domain of integration, determine the limits and evaluate the double integral $\iint\limits_D (x^2+y^2)dxdy$, if D is bounded by the lines $y=x$, $x+y=2a$ and $x=0$.

94. Sketch the domain of integration, determine the limits and evaluate the double integral $\iint\limits_D xydxdy$, if D is the least of segments bounded by the line $x+y=2$ and circle $x^2+y^2=2y$.

95. Sketch the domain of integration, determine the limits and evaluate the double integral $\iint\limits_D e^{x+y}dxdy$, if D is the region bounded by $y=e^x$, $x=0$ and $y=2$.

96. Evaluate the triple integral $\int\limits_0^a dx \int\limits_0^x dy \int\limits_0^{xy} x^3y^2zdz$.

97. Evaluate the triple integral $\iiint\limits_V \frac{dxdydz}{(x+y+z+1)^3}$, if V is the region bounded by planes $x=0$, $y=0$, $z=0$ and $x+y+z=1$.

98. Evaluate the triple integral $\iiint\limits_V xyzdxdydz$, if V is bounded by the surfaces $y=x^2$, $x=y^2$, $z=xy$ and $z=0$.

99. Compute the area bounded by $xy=4$ and $x+y=5$.

100. Compute the area bounded by$y=\frac{8a^3}{x^2+4a^2}$, $x=2y$ and $x=0$ provided a is a positive constant.

101. Compute the volume of solid bounded by the planes $z=0$, $y=0$, $y=x$ and $x=2$ and paraboloid of revolution $z=x^2+y^2$.

102. Compute the volume of solid bounded by the hyperbolic paraboloid (saddle surface) $z=x^2-y^2$ and the planes $z=0$ and $x=3$.

103. Compute the volume of solid bounded by the surfaces $z = x^2 + y^2$, $z = 2(x^2 + y^2)$, $y = x$ and $y^2 = x$.

104. Compute the line integral $\int\limits_L (x^2 + y^2)ds$ where L is the line segment from $A(1;1)$ to $B(4;4)$.

105. Compute the line integral $\int\limits_L y^2 ds$ where L is the arc of cycloid $x = a(t - \sin t)$, $y = a(1 - \cos t)$ between the points $O(0;0)$ and $C(2a\pi;0)$.

106. Compute the line integral $\int\limits_L (x^2 + y^2 + z)ds$ where L is the arc of helix $x = a\cos t$, $y = a\sin t$, $z = bt$ from $t = 0$ to $t = 2\pi$

107. Compute the line integral $\int\limits_L xyzds$ where L is the quarter of circle $x = \dfrac{R}{2}\cos t$, $y = \dfrac{R}{2}\sin t$, $z = \dfrac{R\sqrt{3}}{2}$, which lies in the first octant.

108. Compute the line integral $\int\limits_L \dfrac{ydx + xdy}{x^2 + y^2}$ where L is the segment of the line $y = x$ from $(1;1)$ to $(2;2)$.

109. Compute the line integral $\int\limits_L \arctan\dfrac{y}{x}dy - dx$ where L is the arc of parabola $y = x^2$ from $O(0;0)$ to $A(1;1)$.

110. Compute the line integral $\int\limits_{AB} (x+y)dx + (x-y)dy$ where AB is the arc of ellipse $x = a\cos t$, $y = b\sin t$ from $A(a;0)$ to $B(0;b)$.

111. Compute the line integral $\int\limits_L xdy - ydx$ where L is the arc of astroid $x = a\cos^3 t$, $y = a\sin^3 t$ from $t = 0$ to $t = \dfrac{\pi}{2}$.

112. Compute the line integral $\int\limits_L \dfrac{x}{y}dx + \dfrac{dy}{y-1}$ where L is the arc of cycloid $x = t - \sin t$, $y = 1 - \cos t$ from $t = \dfrac{\pi}{6}$ to $t = \dfrac{\pi}{3}$.

113. Compute the line integral $\int\limits_{AB} \frac{xdx + ydy + zdz}{\sqrt{x^2 + y^2 + z^2 - x - y + 2z}}$ where AB is the line segment from $A(1;1;1)$ to $B(4;4;4)$.

114. Compute the line integral $\int\limits_{L} yzdx + xzdy + xydz$ where L is the arc of helix $x = a\cos t$, $y = a\sin t$, $z = bt$ from $t = 0$ to $t = 2\pi$.

115. Convert the line integral $\oint\limits_{L} (1 - x^3)ydx + x(1 + y^3)dy$ to the double integral over the region D where L is positively oriented, smooth, closed curve and D the region enclosed by L.

116. Convert the line integral $\oint\limits_{L} e^x(1-\cos y)dx+e^x(\sin y+y)dy$ to the double integral over the region D where L is positively oriented, smooth, closed curve and D the region enclosed by L.

117. Use Green's theorem to find $\oint\limits_{L} (x + y^2)dx + (x + y)^2dy$ where L is the contour of triangle ABC with vertices $A(1;0)$, $B(1;1)$ and $C(0;1)$ with positive orientation.

118. Use Green's theorem to find $\oint\limits_{L} (5x - 3y)dx + (x - 4y)dy$ where L is the circle $x^2 + y^2 = 1$ with positive orientation.

119. Use Green's theorem to find $\oint\limits_{L} 2xydx + x^2dy$ where L is the contour of square $|x| + |y| = 1$ with positive orientation. circle $x^2 + y^2 = 5$ with positive orientation.

120. Evaluate $\int\limits_{(0;0)}^{(2;1)} 2xydx + x^2dy$

121. Evaluate $\int\limits_{(-1;2)}^{(2;3)} ydx + xdy$

122. Evaluate $\int\limits_{(1;1)}^{(2;2)} \frac{xdx + ydy}{\sqrt{x^2 + y^2}}$

123. Write the general term of the series $\frac{1}{2} + \left(\frac{2}{5}\right)^3 + \left(\frac{3}{8}\right)^5 + \ldots$.

124. Write the general term of the series $1 - \frac{2}{7} + \frac{3}{13} - \frac{4}{19} + \ldots$.

125. Using the identity $\frac{1}{k(k+1)} = \frac{1}{k} - \frac{1}{k+1}$, find the nth partial sum and the sum of the series $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \ldots + \frac{1}{k(k+1)} + \ldots$.

126. Find the nth partial sum and the sum of the series $\frac{1}{3 \cdot 6} + \frac{1}{6 \cdot 9} + \frac{1}{9 \cdot 12} + \ldots + \frac{1}{3k(3k+3)} + \ldots$.

127. find the nth partial sum and the sum of the series $\sum\limits_{k=1}^{\infty} (\sqrt{k+2} - 2\sqrt{k+1} + \sqrt{k})$.

128. Use the Comparison Test to determine whether the series $\sum\limits_{k=1}^{\infty} \frac{2^k}{5 + 3^k}$ converges or diverges.

129. Use the Comparison Test to determine whether the series $\sum\limits_{k=2}^{\infty} \frac{1}{(k^3 - 1)^{\frac{1}{3}}}$ converges or diverges.

130. Use the d'Alembert Test to determine whether the series $1 + \frac{3}{2!} + \frac{6}{3!} + \frac{12}{4!} + \ldots + \frac{3 \cdot 2^{n-2}}{n!} + \ldots$ converges or diverges.

131. Use the d'Alembert Test to determine whether the series $\sum\limits_{k=1}^{\infty} \frac{k^2}{k!}$ converges or diverges.

132. Use the d'Alembert Test to determine whether the series $\sum\limits_{k=0}^{\infty} \frac{3^k}{(3k+1)!}$ converges or diverges.

133. Use the Cauchy Test to determine whether the series $\sum_{k=1}^{\infty} \arcsin^k \frac{2k-1}{4k+3}$ converges or diverges.

134. Use the Cauchy Test to determine whether the series $\sum_{k=1}^{\infty} \ln^k \frac{2k+3}{k+1}$ converges or diverges.

135. Use the Cauchy Test to determine whether the series $\sum_{k=1}^{\infty} 2^k \left(\frac{k+2}{k+1}\right)^{-k^2}$ converges or diverges.

136. Use the Integral Test to determine whether the series $\frac{1}{4} + \frac{1}{7} + \frac{1}{10} + \ldots + \frac{1}{3k+1} + \ldots$ converges or diverges.

137. Use the Integral Test to determine whether the series $\sum_{k=2}^{\infty} \frac{1}{k(\ln k)^2}$ converges or diverges.

138. Use the Leibnitz's Test to determine whether the series $1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + \frac{1}{\sqrt{5}} - \frac{1}{\sqrt{6}} + \ldots$ converges or diverges.

139. Use the Leibnitz's Test to determine whether the series $\sum_{k=1}^{\infty} \frac{\cos k\pi}{k^3}$ converges or diverges.

140. Does the series

$$1 - \frac{1}{3^2} + \frac{1}{5^2} - \frac{1}{7^2} + \ldots + (-1)^{n+1} \frac{1}{(2n-1)^2} + \ldots$$

converges conditionally or absolutely?

141. Does the series

$$\frac{1}{2} - \frac{1}{2} \cdot \frac{1}{2^2} + \frac{1}{3} \cdot \frac{1}{2^3} - \ldots + (-1)^{n+1} \frac{1}{n} \frac{1}{(2)^n} + \ldots$$

converges conditionally or absolutely?

142. Does the series

$$\frac{1}{\ln 2} - \frac{1}{\ln 3} + \frac{1}{\ln 4} - \frac{1}{\ln 5} + \ldots + (-1)^n \frac{1}{\ln n} + \ldots$$

converges conditionally or absolutely?

In exercises 178. -181. find the radius of convergence and the domain of convergence.

143. $\sum_{k=1}^{\infty} \frac{x^k}{k(k+1)}$.

144. $\sum_{k=0}^{\infty} \frac{x^k}{\sqrt{k}}$.

145. $\sum_{k=0}^{\infty} \frac{k(x-2)^k}{3^k}$.

146. $\sum_{k=0}^{\infty} \frac{2^k(x+3)^k}{k!}$.

In exercises 182. - 187. expand the function in powers of x and determine the domain of convergence.

147. $f(x) = \frac{1}{10+x}$.

148. $f(x) = e^{-x}$.

149. $f(x) = \frac{1}{1+x^2}$.

150. $f(x) = \sinh x$.

151. $f(x) = \cos^2 x$.

152. $f(x) = \arctan x$ (Remark: integrate the result of the exercise 184. in limits from 0 to x).

In exercises 188. - 192. find the Fourier series expansion of the given 2π-periodic function defined on a half-open interval.

153. $f(x) = \begin{cases} -1, & -\pi < x < 0 \\ 1, & 0 \le x \le \pi \end{cases}$

154. $f(x) = x$, if $-\pi < x \le \pi$.

155. $f(x) = x^2$, if $-\pi < x \le \pi$.

156. $f(x) = \sin ax$, if $-\pi < x \le \pi$.

157. $f(x) = \dfrac{\pi - x}{2}$, if $0 < x \le 2\pi$.

In exercises 193. - 196. determine the Fourier transform of function given.

158. Heaviside unit step function $H(x) = \begin{cases} 1, \text{ if } x \ge 0 \\ 0, \text{ if } x < 0 \end{cases}$

159. Rectangular pulse $f(x) = \begin{cases} A, \text{ if } |x| \le 2 \\ 0, \text{ if } |x| > 2 \end{cases}$

160. Two sided exponential pulse

$$f(x) = \begin{cases} e^{ax}, \text{ if } x \le 0 \\ e^{-ax}, \text{ if } x > 0 \end{cases} \quad \text{provided } (a > 0)$$

161. $f(x) = \begin{cases} \sin ax, \text{ if } |x| \le \dfrac{\pi}{a} \\ 0, \quad \text{ if } |x| > \dfrac{\pi}{a} \end{cases}$

Answers

9. No, because the first function is also defined if $x \le 0$ and $\sin y \le 0$. **10.** No, because the first function is also defined if $x < 0$ and $y < 0$. **11.** Does not exist. **12.** 2 **13.** 0. **14.** Does not exist. **15.** 0. **16.** Does not exist. **17.** 0 **18.** $2x\sqrt[3]{y} + \dfrac{1}{2\sqrt{x}\sqrt[4]{y}}$; $\dfrac{x^2}{3\sqrt[3]{y^2}} - \dfrac{\sqrt{x}}{4y\sqrt[4]{y}}$ **19.** $\dfrac{2}{y \sin \frac{2x}{y}}$; $-\dfrac{2x}{y^2 \sin \frac{2x}{y}}$ **20.** $-\dfrac{1}{y} e^{-\frac{x}{y}}$; $\dfrac{x}{y^2} e^{-\frac{x}{y}}$ **21.** $y \cos xy - \dfrac{y}{x^2} \sin \dfrac{y}{x}$; $x \cos xy + \dfrac{1}{x} \sin \dfrac{y}{x}$ **22.** $\dfrac{1}{\sqrt{x^2 + y^2}}$; $\dfrac{y}{(x + \sqrt{x^2 + y^2})\sqrt{x^2 + y^2}}$. **23.** $-\dfrac{y}{2\sqrt{x}(x + y^2)}$; $\dfrac{\sqrt{x}}{x + y^2}$ **24.** $y \ln(x + y) + \dfrac{xy}{x + y}$; $x \ln(x + y) + \dfrac{xy}{x + y}$ **25.** $\dfrac{y}{xy + \ln z}$; $\dfrac{x}{xy + \ln z}$; $\dfrac{1}{z(xy + \ln z)}$ **26.** $\dfrac{2x}{\cos^2(x^2 + y^3 + z^4)}$; $\dfrac{3y^2}{\cos^2(x^2 + y^3 + z^4)}$; $\dfrac{4z^3}{\cos^2(x^2 + y^3 + z^4)}$ **27.** $y^z x^{y^z - 1}$; $x^{y^z} \ln x \cdot z y^{z-1}$; $x^{y^z} \ln x \cdot y^z \ln y$. **28.** $2e^{2x} \cos(yz)$; $-ze^{2x} \sin(yz)$; $-ye^{2x} \sin(yz)$ **29.** $\dfrac{4}{5\sqrt{5}}$; $\dfrac{2}{5\sqrt{5}}$ **30.** 1; -1 **31.** $\dfrac{3}{2}$ **34.** $dz =$

$\dfrac{ydx - xdy}{|y|\sqrt{y^2 - x^2}}$. **35.** $dz = \left(x^2 \cos\dfrac{x}{y}\cos\dfrac{y}{x} + y^2 \sin\dfrac{x}{y}\sin\dfrac{y}{x}\right)\dfrac{ydx - xdy}{x^2y^2}$
36. $dz = \dfrac{ydx - xdy}{y^2 \tan\dfrac{x}{y}}$ **37.** $dw = x^{yz}\left(\dfrac{yzdx}{x} + z\ln xdy + y\ln xdz\right)$ **38.** $\dfrac{1}{36}$ **39.** $\Delta z = \dfrac{19}{635}$; $dz = \dfrac{19}{600}$ **40.** $\Delta z \approx 0,3764$; $dz = 0,35$
41. 259,84. **42.** $2\dfrac{53}{54}$ **43.** $\dfrac{\pi}{6}$. **44.** 0,006. **45.** $\dfrac{x(2x^2 - y^2)}{y(x^2 - 2y^2)}$ **46.** $-\dfrac{1}{3}$ **47.** $\dfrac{3}{4e}$ **48.** 1 **49.** $\dfrac{2 - x}{z + 1}$; $\dfrac{2y}{z + 1}$
50. $-\dfrac{2}{2 + \pi}$; $-\dfrac{\pi}{2 + \pi}$ **53.** $\dfrac{xy^3}{(1 - x^2y^2)\sqrt{1 - x^2y^2}}$; $\dfrac{x^3y}{(1 - x^2y^2)\sqrt{1 - x^2y^2}}$ $\dfrac{1}{(1 - x^2y^2)\sqrt{1 - x^2y^2}}$ **54.** $e^{xyz}(1 + 3xyz + x^2y^2z^2)$ **55.** 0; $\dfrac{1}{4}$; $-\dfrac{3}{8}$
56. $-\dfrac{4}{25}$; $\dfrac{3}{25}$; $\dfrac{4}{25}$ **57.** 0 **60.** $\left(2; -2\dfrac{1}{4}\right)$ **61.** $\left(-\dfrac{1}{3}; \dfrac{3}{4}\right)$; $\left(\dfrac{7}{3}; -\dfrac{3}{4}\right)$ **62.** $\left(\dfrac{1}{2}; \dfrac{1}{2}; -\dfrac{1}{2}\right)$ **63.** $-\dfrac{15}{4}\sqrt{\dfrac{3}{13}}$ **64.** $-\dfrac{30}{13}$ **65.** $2 + \sqrt{2}$ **66.** 0,4; **67.** $\dfrac{3\sqrt{5}}{4}$; **68.** $\dfrac{\sqrt{2}}{2}$ **69.** $(0; -1; 0)$
70. $\operatorname{div}\vec{F} = \dfrac{1}{y} + \dfrac{1}{z} + \dfrac{1}{x}$; $\operatorname{curl}\vec{F} = \left(\dfrac{y}{z^2}; \dfrac{z}{x^2}; \dfrac{x}{y^2}\right)$ **71.** $\operatorname{div}\vec{F} = \dfrac{2x}{x^2 - y^2} - \dfrac{1}{1 + (z - y)^2} + xy$; $\operatorname{curl}\vec{F} = \left(xz - \dfrac{1}{1 + (z - y)^2}; -yz; \dfrac{2y}{x^2 - y^2}\right)$
72. $\operatorname{div}\vec{F} = -\dfrac{3}{(x + y - z)^2}$; $\operatorname{curl}\vec{F} = \vec{\Theta}$ **73.** $\operatorname{div}\vec{F} = 0$; $\operatorname{curl}\vec{F} = (2x; 2x; 2y - 2z)$ **74.** Local minimum at $\left(-\dfrac{17}{143}; \dfrac{7}{143}\right)$ **75.** Local maximum at $(6; 4)$; $z_{max} = 6912$ **76.** Local minimum at $\left(\dfrac{1}{\sqrt[3]{3}}; \dfrac{1}{\sqrt[3]{3}}\right)$; $z_{min} = 3\sqrt[3]{3}$ **77.** There is no local extremum at $(-2; 0)$; local minimum at $(0; 0)$ **78.** There is no local extremum at $(0; 0)$, local minimum at $(1; 1)$
79. $\dfrac{8}{3}$ **80.** $\ln\dfrac{25}{24}$ **81.** $3\dfrac{3}{4}$ **82.** $\dfrac{19}{12}$. **83.** $\dfrac{6}{35}$ **84.** $\displaystyle\int_{-1}^{1} dx \int_{0}^{1-x^2} f(x; y)dy$ **85.** $\displaystyle\int_{0}^{a} dy \int_{y}^{y+2a} f(x; y)dx$ **86.** $\displaystyle\int_{-1}^{1} dx \int_{x^2}^{\frac{2}{1+x^2}} f(x; y)dy$

87. $\int\limits_0^1 dy \int\limits_{y^2}^{\sqrt[3]{y}} f(x;y)dx$ **88.** $\int\limits_{-1}^0 dy \int\limits_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} f(x;y)dx + \int\limits_0^2 dy \int\limits_{y-1}^1 f(x;y)dx$

89. $\int\limits_{-2}^2 dx \int\limits_{-\sqrt{x+2}}^{\sqrt{x+2}} f(x;y)dy - \int\limits_0^2 dx \int\limits_{-\sqrt{2x}}^{\sqrt{2x}} f(x;y)dy$

90. $\int\limits_0^1 dy \int\limits_{-\sqrt{y}}^{\sqrt{y}} f(x;y)dx + \int\limits_1^3 dy \int\limits_{-1}^1 f(x;y)dx - \int\limits_2^3 dy \int\limits_{-\sqrt{y-2}}^{\sqrt{y-2}} f(x;y)dx$ **91.** $\int\limits_0^1 dx \int\limits_{x-1}^{x^2} f(x;y)dy$ **92.** $-10\frac{7}{20}$ **93.** $\frac{4}{3}a^4$ **94** $\frac{1}{4}$ **95.** e

96. $\frac{a^{11}}{110}$ **97.** $\frac{1}{2}\ln 2 - \frac{5}{16}$ **98.** $\frac{1}{96}$ **99.** $\frac{1}{2}(15 - 16\ln 2)$ **100.** $a^2(\pi - 1)$ **101.** $\frac{16}{3}$ **102.** 27 **103.** $\frac{3}{35}$ **104.** $42\sqrt{2}$ **105.** $\frac{256}{15}a^3$ **106.** $2\pi\sqrt{a^2+b^2}(a^2+\pi b)$ **107.** $\frac{R^4\sqrt{3}}{32}$ **108.** $\ln 2$ **109.** $\frac{\pi}{2} - 2$ **110.** $-\frac{a^2+b^2}{2}$ **111.** $\frac{3\pi a^2}{16}$ **112.** $\frac{\pi^2}{24} + \frac{1-\sqrt{3}}{2} - \frac{1}{2}\ln 3$ **113.** $3\sqrt{3}$ **114.** 0 **115.** $\iint\limits_D (x^3+y^3)dxdy$

116. $\iint\limits_D e^x y dxdy$ **117.** $\frac{2}{3}$ **118.** 4π **119.** 0 **120.** 4

121. 8 **122.** $\sqrt{2}$ **123.** $\left(\frac{k}{3k-1}\right)^{2k-1}$ **124.** $(-1)^k\frac{k}{6k-5}$

125. $S_n = 1 - \frac{1}{n+1}$; $S = 1$ **126.** $S_n = \frac{1}{9} - \frac{1}{9(n+1)}$; $S = \frac{1}{9}$ **127**. $1 - \sqrt{2} + \sqrt{n+2} - \sqrt{n+1}$; $1 - \sqrt{2}$ **128.** Convergent. **129.** Divergent. **130.** Convergent. **131.** Convergent. **132.** Convergent. **133.** Convergent. **134.** Convergent. **135.** Convergent. **136.** Divergent. **137.** Convergent. **138.** Convergent. **139.** Convergent. **140.** Absolutely onvergent. **141.**Absolutely onvergent. **142.** Conditionally onvergent. **143.** $R = 1$; $-1 \le x \le 1$ **144.** $R = 1$; $-1 \le x < 1$ **145.** $R = 3$; $-1 < x < 5$ **146.** $-\infty < x < \infty$ **147.** $\frac{1}{10} - \frac{x}{100} + \frac{x^2}{10^3} - \frac{x^3}{10^4} + \ldots,$

converges if $-10 < x < 10$ **148.** $1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \ldots$, converges if $-\infty < x < \infty$ **149.** $1 - x^2 + x^4 - x^6 + \ldots$, converges if $-1 < x < 1$ **150.** $x + \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots$, converges if $-\infty < x < \infty$ **151.** $1 + \frac{1}{2}\sum_{n=1}^{\infty} \frac{(-1)^n (2x)^{2n}}{(2n)!}$, converges if $-\infty < x < \infty$ **152.** $x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots$, converges if $-1 \le x \le 1$ **153.** $\frac{4}{\pi}\sum_{k=1}^{\infty} \frac{\sin(2k+1)x}{2k+1}$ **154.** $2\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} \sin kx$ **155.** $\frac{\pi^2}{3} + 4\sum_{k=1}^{\infty} \frac{(-1)^k}{k^2} \cos kx$ **156.** $\frac{2 \sin a\pi}{\pi} \sum_{k=1}^{\infty} \frac{(-1)^k k}{a^2 - k^2} \sin kx$ **157.** $\sum_{k=1}^{\infty} \frac{\sin kx}{k}$ **158.** $\frac{1}{i\omega}$ **159.** $\frac{2A}{\omega} \sin 2\omega$ **160.** $\frac{2a}{a^2 + \omega^2}$ **161.** $\frac{2ia \sin \frac{\pi\omega}{a}}{\omega^2 - a^2}$