PR4: Robotics

L3 – Basics of Coordinate Transformations

L3 - Basics of Coordinate Transformations

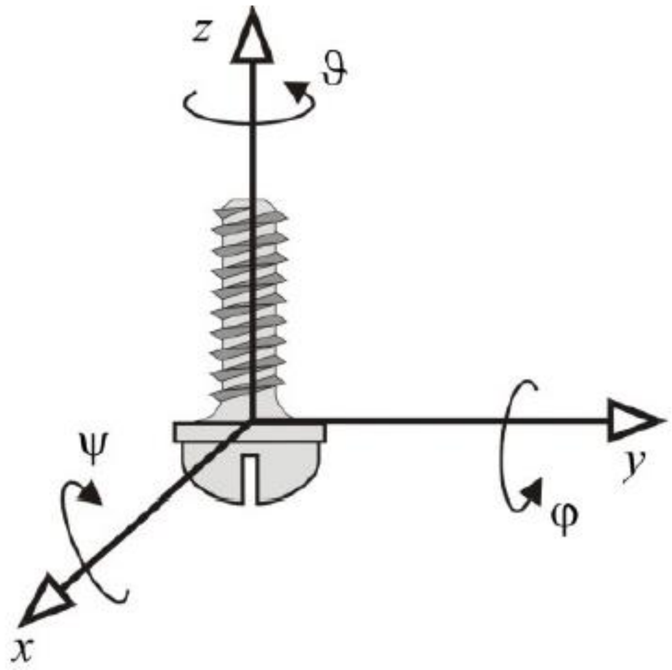
3.1 Coordinate systems (CS)

3.2 Vectors for coordinates

3.3 Simplified representation of the orientation of bodies with Euler angles

3.1 Coordinate systems (CS)

The "right-hand rule" in right-handed Cartesian coordinate systems

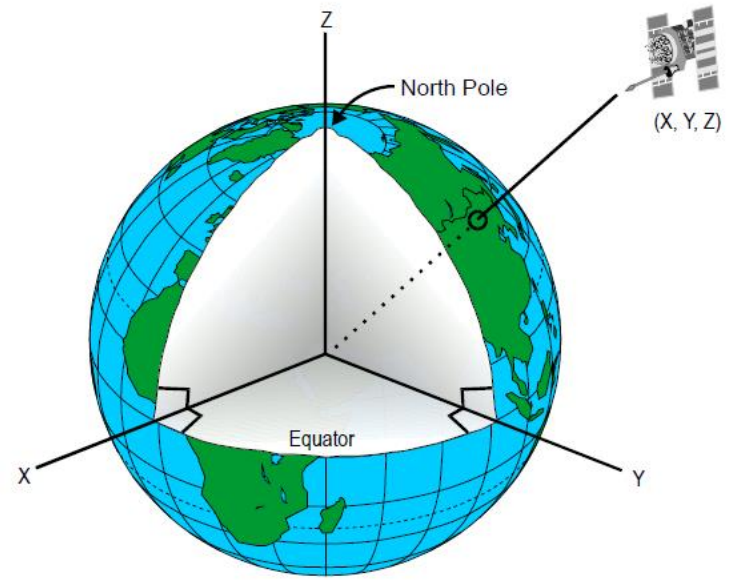


When you rotate the x-axis in the shortest path towards the y-axis, it results in a direction of rotation that would move a right-handed screw towards the positive z-axis.

[Björnsson]

Earth-Centered Inertial (ECI) Coordinate Systems

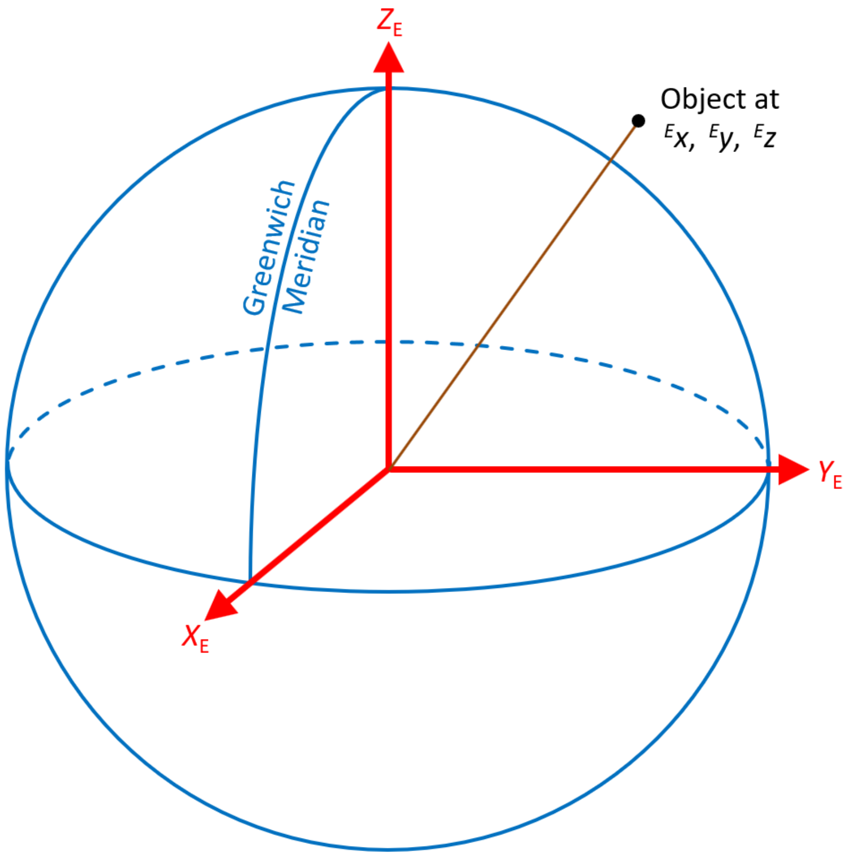
- Cartesian Coordinate System, Origin at the Center of Mass of the Earth.
- The z-axis passes through the North Pole, and the x-axis is fixed to the celestial sphere.
- => The system does not rotate with the Earth.
- Used to describe the motion of objects in Earth's orbit, such as satellites or even aircraft.
- The system is technically not inertial due to its motion with the Earth around the Sun on an elliptical path and the gravitational influence of the Moon.
- Not suitable for objects on the Earth's surface: coordinates change even when at rest



By U.S. Department of Transportation Federal Aviation Administration - Airway Facilities Division - FAA Academy Training Manual - GPS concepts, Course 44221, Public Domain, https://commons.wikimedia.org/w/index.php?curid=16622043

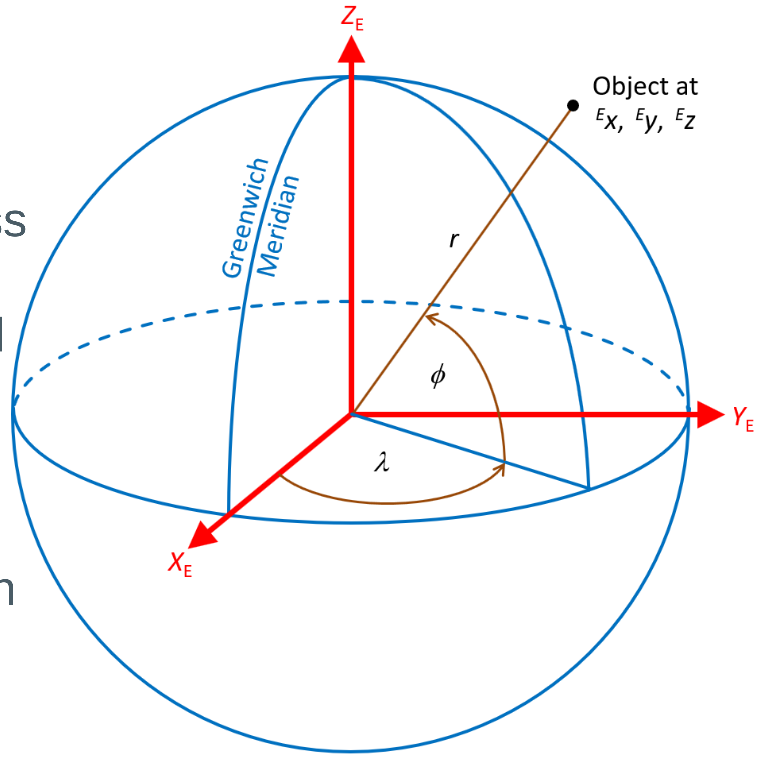
Earth-Centered Earth-Fixed (ECEF) Coordinate Systems

- Cartesian Coordinate System, Origin at the Center of Mass of the Earth.
- The z-axis aligns with the Earth's rotational axis, and the x-axis points to the Greenwich Meridian.
- => The system rotates with the Earth.
- A stationary object on the Earth's surface has constant coordinates.
- The system is non-inertial due to its rotation with the Earth (and for the reasons similar to ECI).
- Drawback: Cartesian coordinates are not very informative about the object's height above the Earth's Surface.

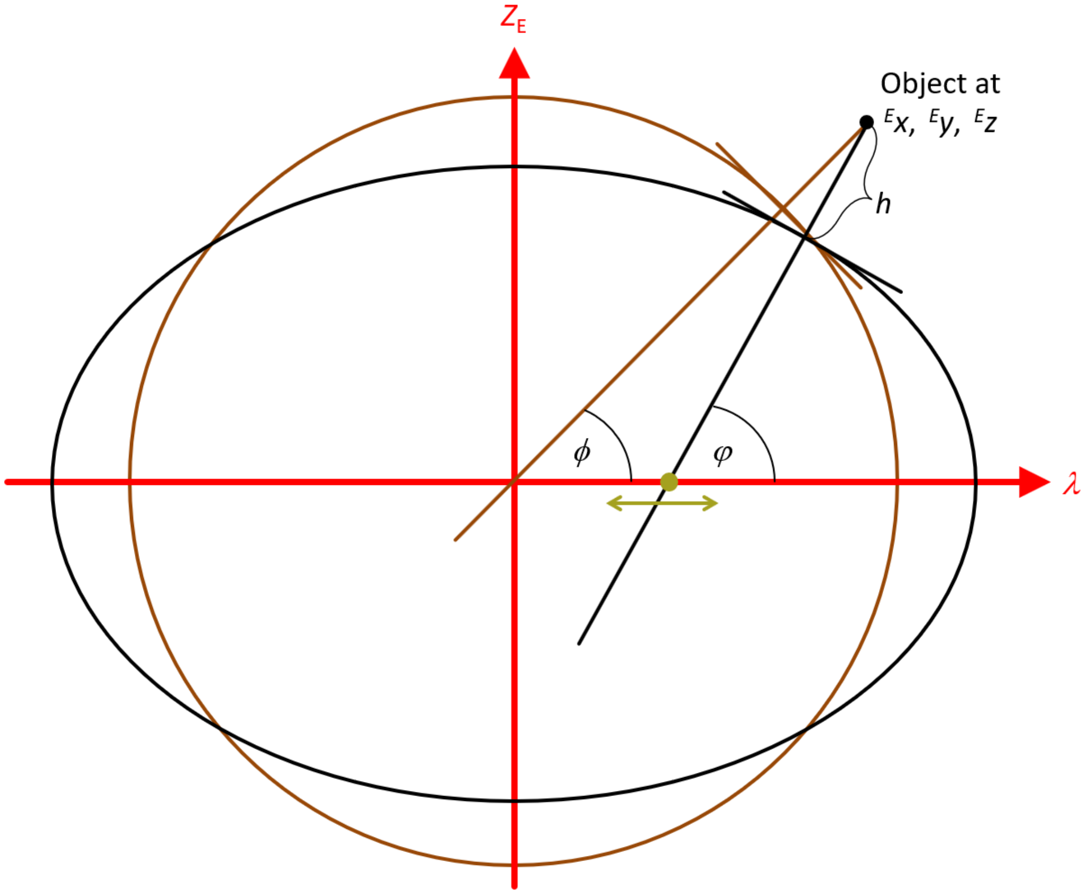


ECEF Geocentric Coordinate System

- Spherical Coordinate System, Origin at the Center of Mass of the Earth.
- Coordinates as distance from the center of mass (r), longitude (λ, eastward from the Greenwich Meridian), and geocentric latitude (ϕ, northward from the equator).
- Distance r minus Earth's radius = height above ground (theoretically…).
- However, the Earth is not a sphere but rather an oblate spheroid...
- This led to the introduction of the geodetic coordinate system.
- In this system, the longitude λ remains the same, but the geodetic (also known as geographic or ellipsoidal) latitude/breadth φ is introduced.



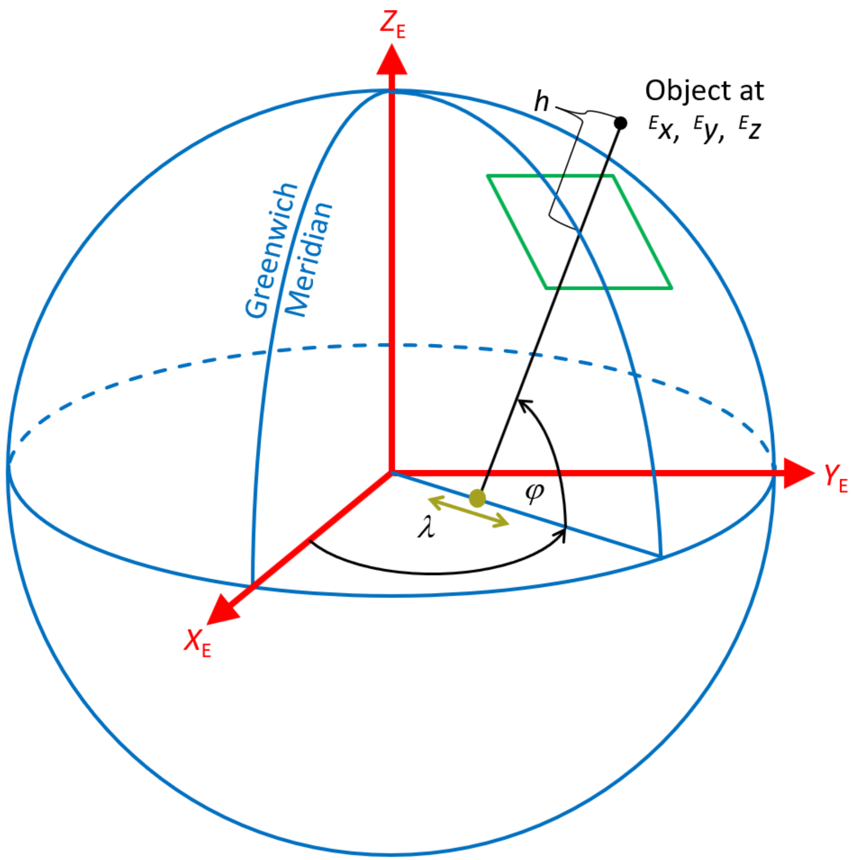
The difference between geocentric and geodetic latitude is as follows



Geocentric latitude ϕ and geodetic latitude φ; in the geodetic coordinate system, the height h above the ellipsoidal surface is introduced as the third coordinate.

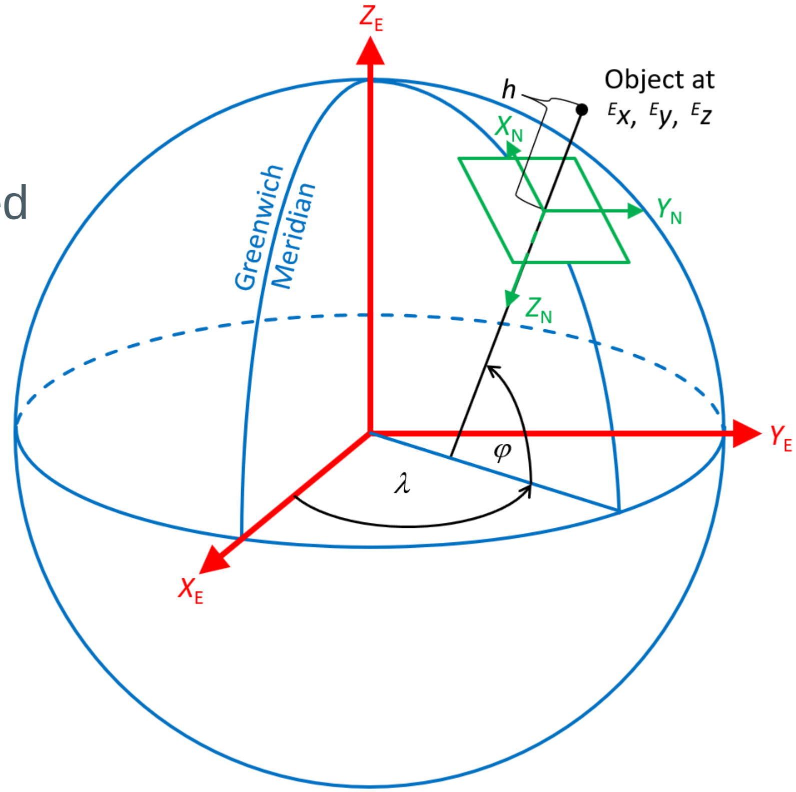
ECEF Geodetic Coordinate System

- Spherical coordinate system, considers the ellipsoidal structure of the Earth, often in the form of the World Geodetic System 1984 (WGS 84).
- Instead of geocentric, the geodetic latitude (φ, northward from the equator) is now introduced: the angle between the equatorial plane and the ellipsoid normal (which is perpendicular to the surface of the ellipsoid).
- Since distance r is no longer a viable coordinate, the height above the ellipsoidal surface (= the height above the plane through a point on the surface and perpendicular to the ellipsoid normal) h is introduced as the third coordinate.



Local 'Inertial System,' e.g., NED (North-East-Down) System

- For local applications, such as an industrial robot, there is a desire to establish a local inertial coordinate system as a base or reference system.
- This is done using the plane described, defined by a point on the Earth's surface according to the WGS 84 model and perpendicular to the ellipsoid normal.
- Here, a Cartesian coordinate system is typically generated again depending on the application, which usually shares only the height coordinate h (often denoted as z or $-z$) with the geodetic system.
- Since it inherits properties from the ECEF system, the local system is not inertial but can be treated as such for practical applications.

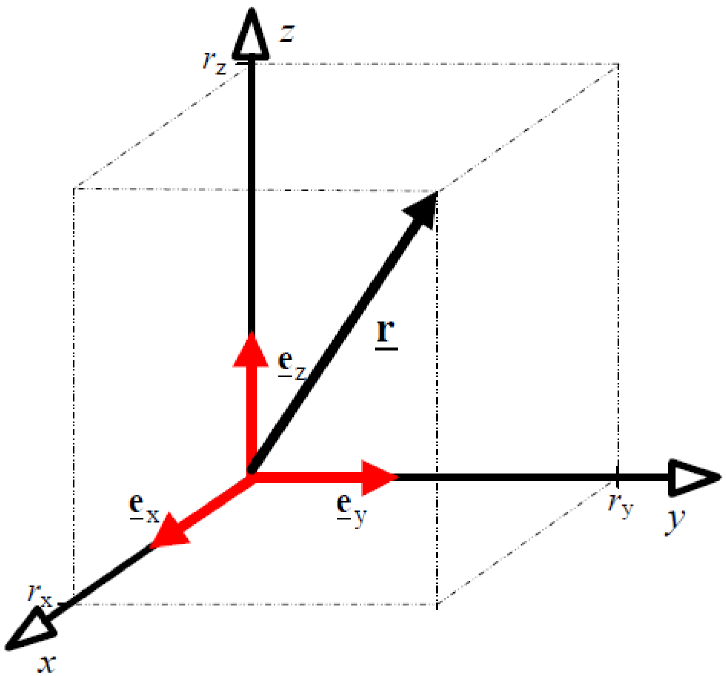


World coordinate system in KUKA robot

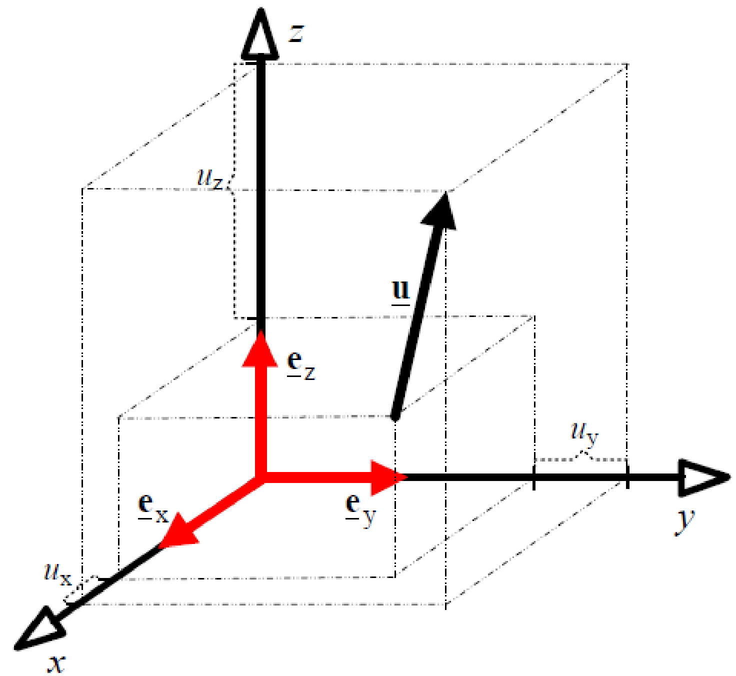
[https://www.youtube.com/watch?v=_wO2lu267e4]

3.2 Vectors for coordinates

Free and Bound Vectors



a) Definition of a position vector in space



b) Definition of any vector in space

[Björnsson]

Scaling / Scalar Multiplication

All elements of the vector are multiplied by the same scalar.

$$\underline{u} = s \cdot \underline{v} = \begin{bmatrix} s \cdot u_x \\ s \cdot u_y \\ s \cdot u_z \end{bmatrix}$$

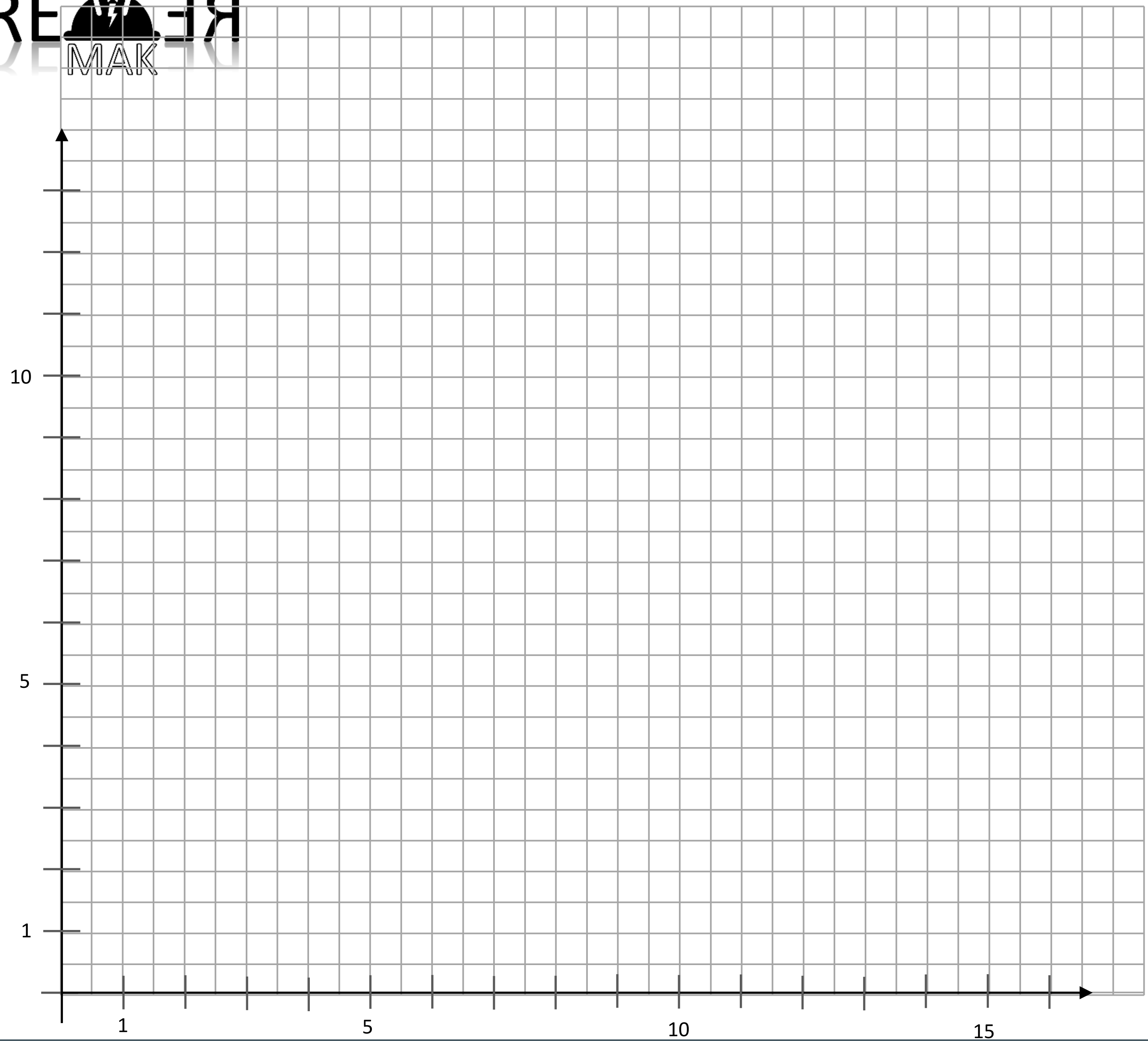
Example: The TCP (Tool Center Point) of a robot is moving with a velocity vector $\underline{v}$. After accelerating to s times the velocity, $\underline{u}$ becomes the new velocity vector.

Addition/ Subtraction

Two vectors of the same dimension are added/subtracted by adding/subtracting their individual components separately.

$$\underline{u} = \underline{v} + \underline{w} = \begin{bmatrix} v_x + w_x \\ v_y + w_y \\ v_z + w_z \end{bmatrix}$$

Example: The TCP (Tool Center Point) of a robot is moving with a velocity vector $\underline{v}$. The entire robot is moving on rails with the velocity vector $\underline{w}$. Then $\underline{u}$ describes the resulting velocity vector of the TCP.



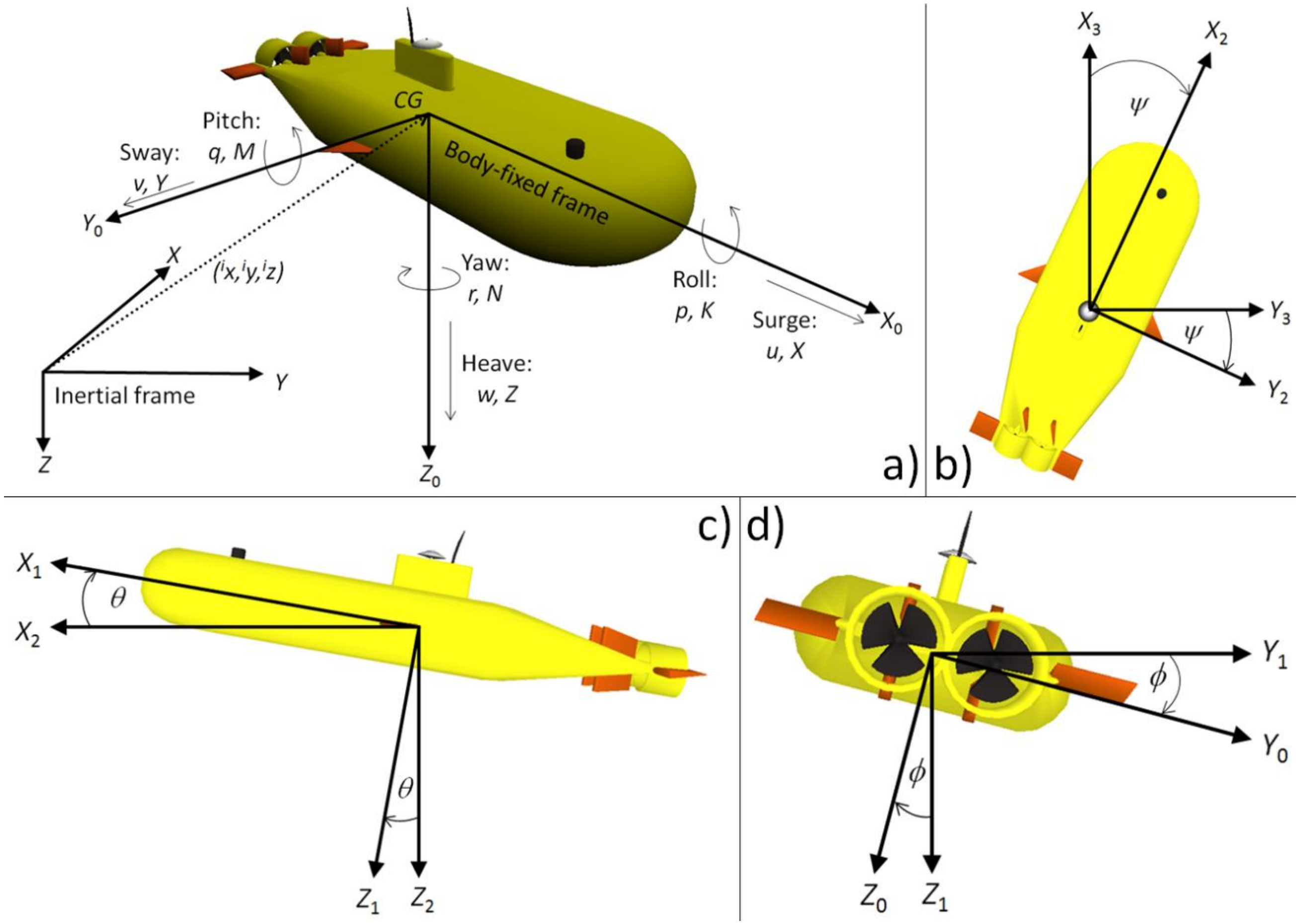
Projection of vectors using the scalar product.

3.3 Simplified representation of the orientation of bodies with Euler angles

Overview of the 24 possible rotations in proper Euler angles and Cardan angles

Proper Euler Angles		Cardan Angles	
intrinsic	extrinsic	intrinsic	extrinsic
$z\text{-}x'\text{-}z''$	$z\text{-}x\text{-}z$	$z\text{-}y'\text{-}x''$	$z\text{-}y\text{-}x$
$z\text{-}y'\text{-}z''$	$z\text{-}y\text{-}z$	$z\text{-}x'\text{-}y''$	$z\text{-}x\text{-}y$
$y\text{-}z'\text{-}y''$	$y\text{-}z\text{-}y$	$y\text{-}z'\text{-}x''$	$y\text{-}z\text{-}x$
$y\text{-}x'\text{-}y''$	$y\text{-}x\text{-}y$	$y\text{-}x'\text{-}z''$	$y\text{-}x\text{-}z$
$x\text{-}y'\text{-}x''$	$x\text{-}y\text{-}x$	$x\text{-}y'\text{-}z''$	$x\text{-}y\text{-}z$
$x\text{-}z'\text{-}x''$	$x\text{-}z\text{-}x$	$x\text{-}z'\text{-}y''$	$x\text{-}z\text{-}y$

Example: Roll-Pitch-Yaw Angles for a Maritime Robot



Thank you very much for your attention!