Power Electronics
Lecture 3

BOOST and BUCK-BOOST DC-DC converters

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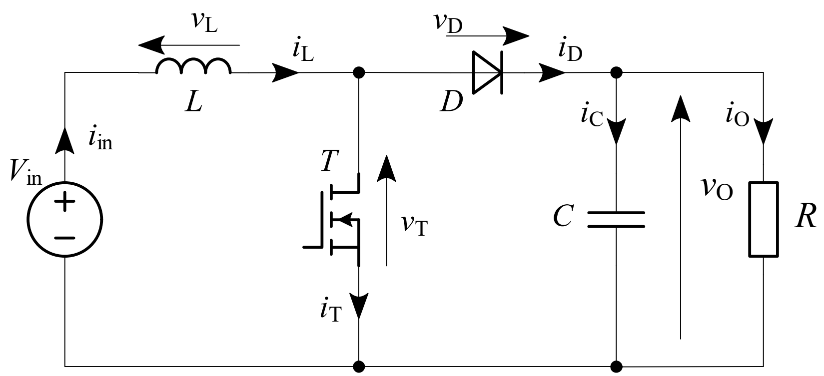
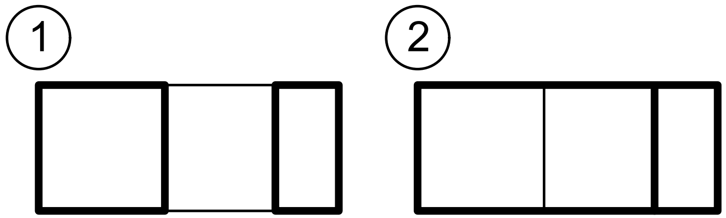
Boost converter

The boost converter (a step-up converter) is composed of a transistor, a diode, a single inductor and capacitor.

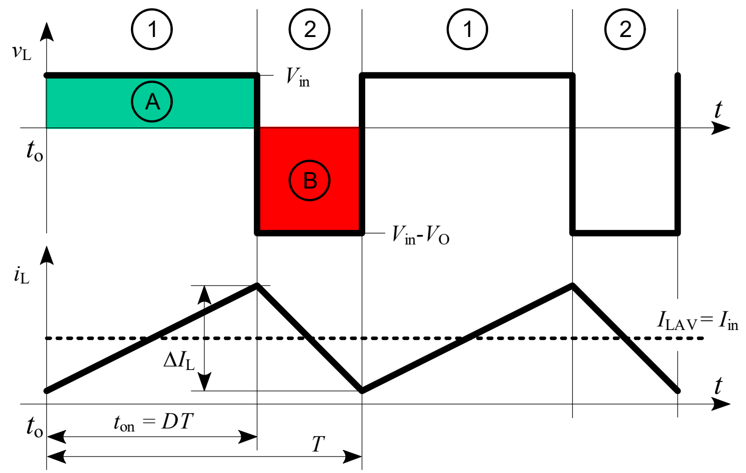


Transistor T is switched between the on- and off-states. For CCM there are two equivalent circuits



Inductor voltage is rectangular and equal to V_{in} when transistor is on, and V_{in}-V_O when transistor is in off-state.

In the steady state operation:



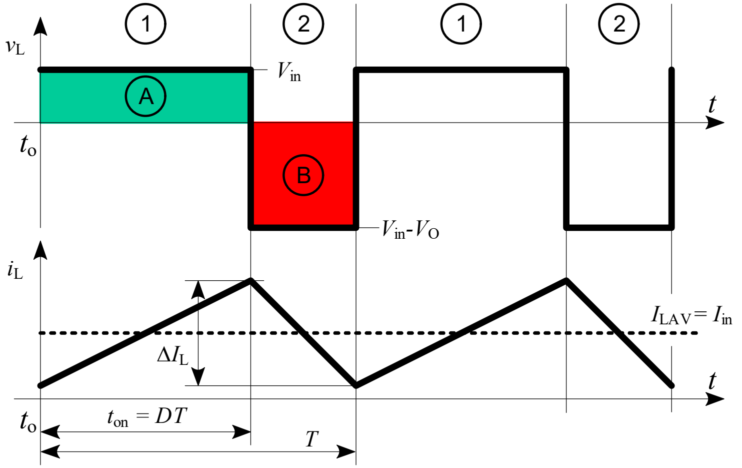
A = B

$$v_L(t) = L\frac{di(t)}{dt}$$

$$\Delta I_L = \frac{1}{L}\int_{t_o}^{t_o+DT} V_{in}\,dt = \frac{V_{in}}{L}DT$$

The output voltage

Due to the fact that in a steady state the average value of the inductor voltage is zero it is possible to derive the output voltage as a function of D and input voltage V_{in}.



$$V_{in}DT + (V_{in} - V_O)(1-D)T = 0$$

$$\underline{V_{in}D} + V_{in} - V_O - \underline{V_{in}D} + V_O D = 0$$

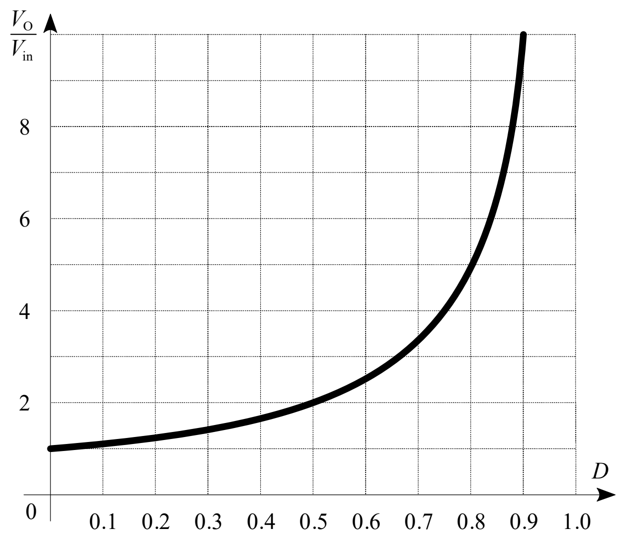
$$V_{in} - V_O(1-D) = 0$$

$$\boxed{V_O = \frac{V_{in}}{1-D}}$$

The characteristic $V_O = V_{in}/(1-D)$ is valid only when the converter works in CCM

The output voltage control characteristic

$$\frac{V_O}{V_{in}} = \frac{1}{1-D}$$



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CCM operation

Inductor current ripple is given as:

$$\Delta I_L = \frac{1}{L}\int_0^{DT} v_L(t)dt = \frac{V_{in}}{L}DT$$

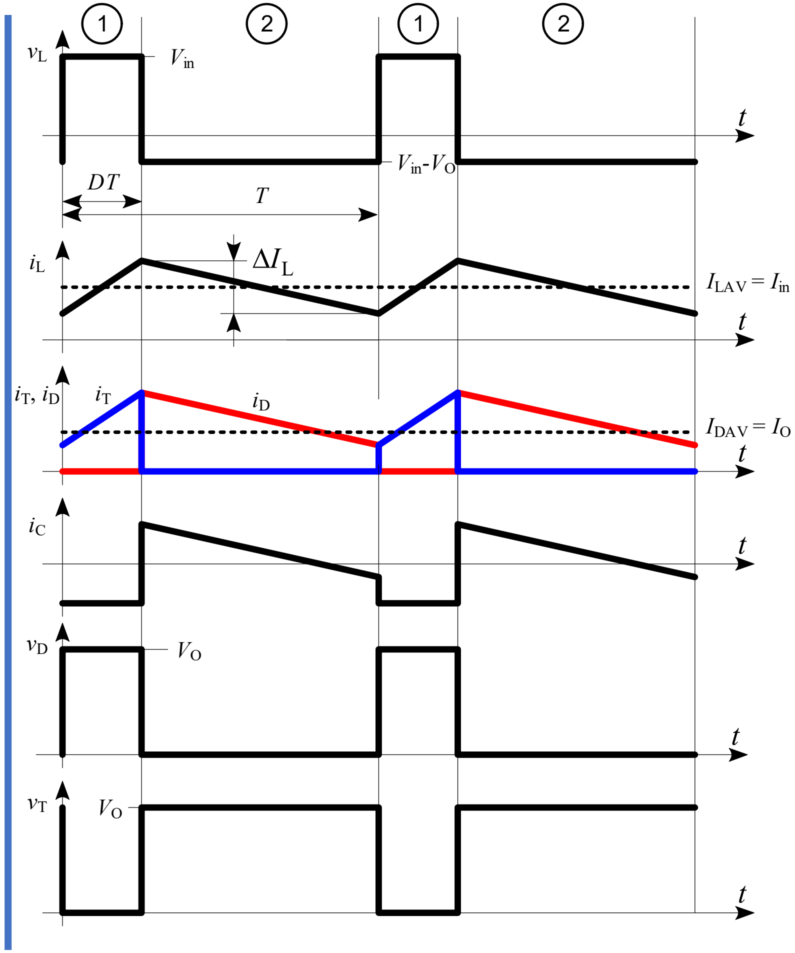
Average value of the inductor current is equal to the average input current:

$$I_{LAV} = I_{in}$$

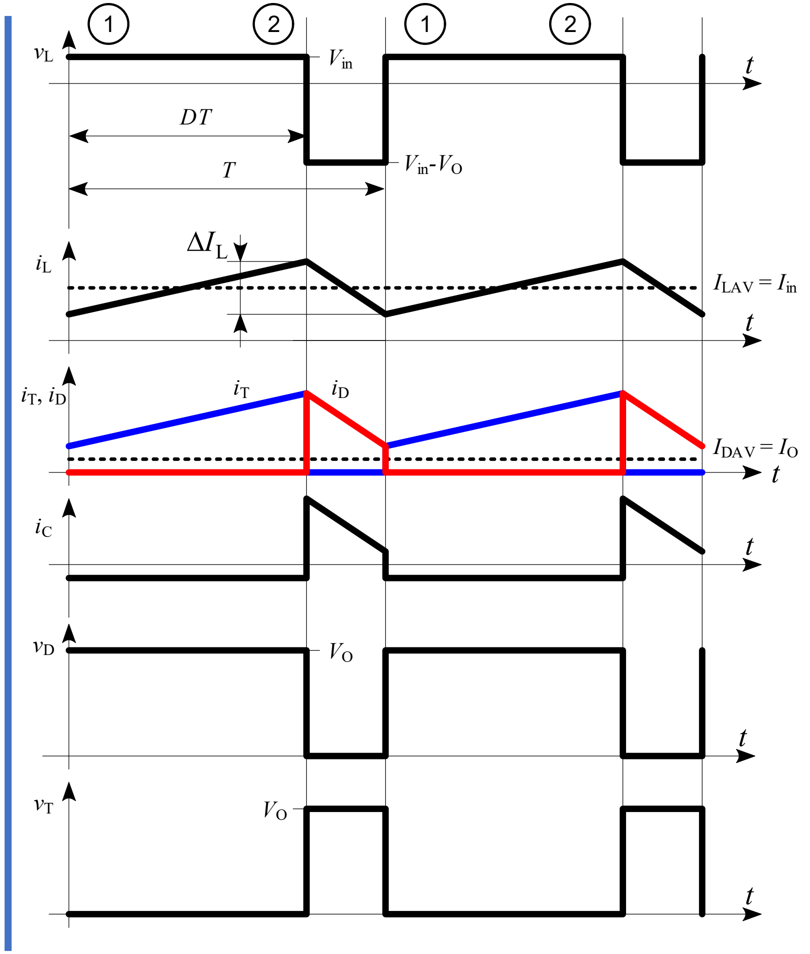
When it is assumed that converter is lossless

$$I_{LAV} = I_{in} \overset{P_{in}=P_O}{=} I_O\frac{V_O}{V_{in}} = I_O\frac{1}{1-D}$$

D = 0.25



D = 0.75



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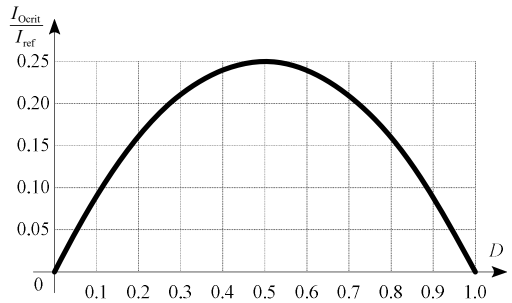
CCM/DCM boundary

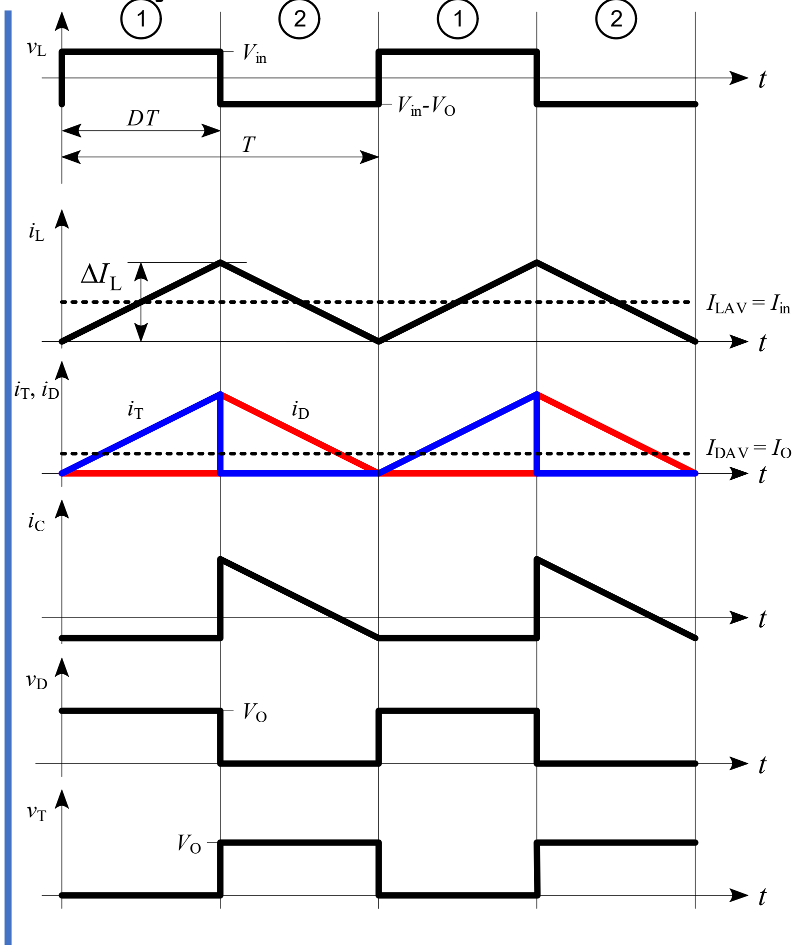
When the output current $I_O = I_{Ocrit} = ½\Delta I_L(1\text{-}D)$ the converter operates at boundary between CCM and DCM

$$I_{Ocrit} = \frac{1}{2}\Delta I_L(1-D) = \frac{V_{in}T}{2L}D(1-D)$$

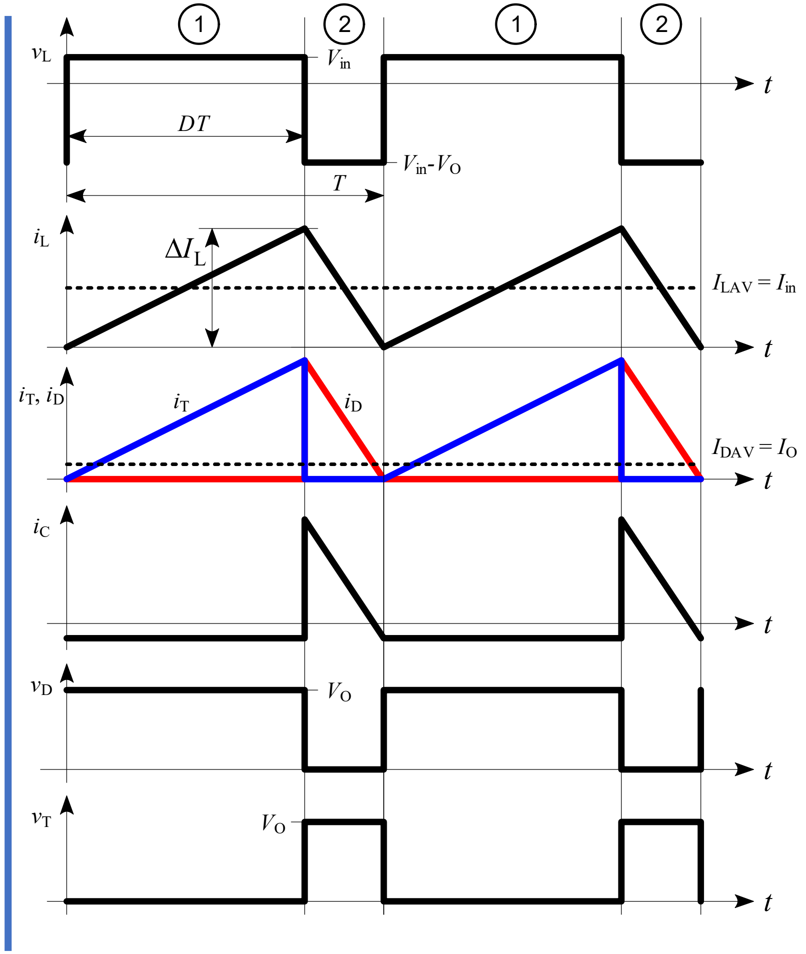
$$I_{Ocrit} \overset{I_{ref}=\frac{V_{in}T}{2L}}{=} I_{ref}D(1-D)$$



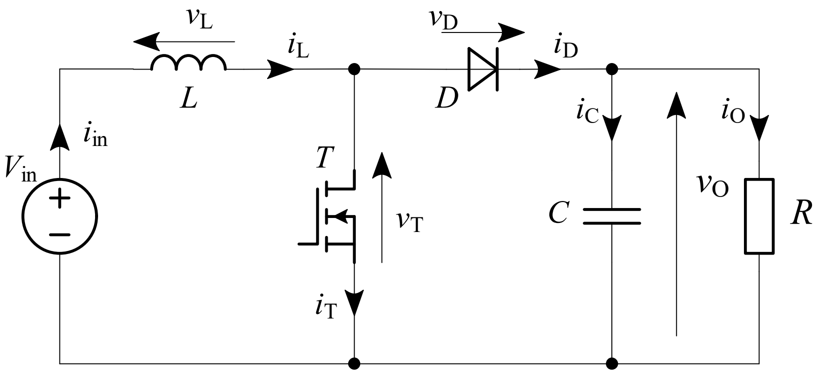
D = 0.5



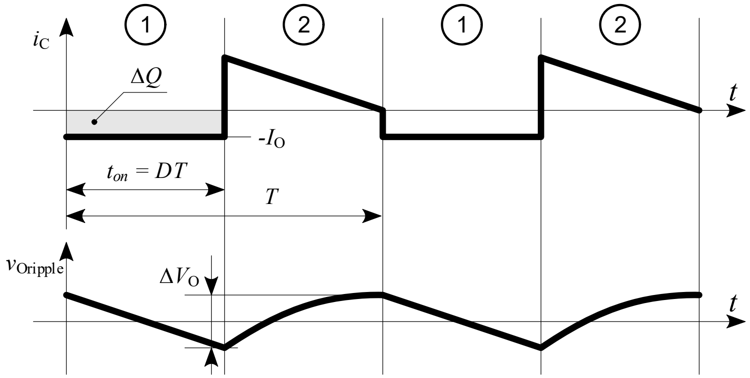
D = 0.75



Capacitor current and voltage



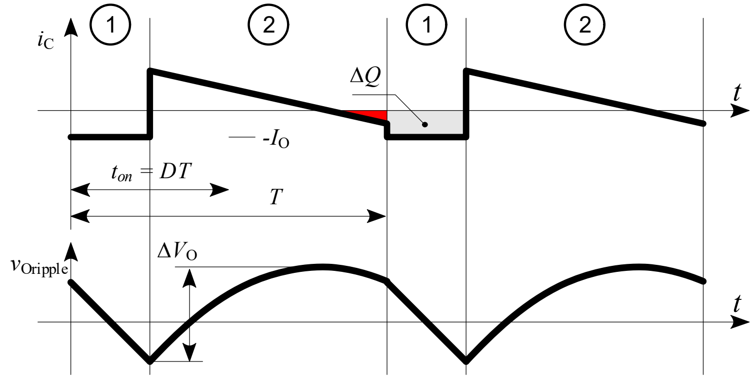
The capacitor current is an ac component of the diode current.



When $I_O D > ½\Delta I_L(1\text{-}D)$ (in CCM) the charge ΔQ during the first equivalent circuit is:

$$\Delta Q = DT \cdot I_O \qquad \Delta V_O = \frac{\Delta Q}{C} = \frac{DTI_O}{C}$$

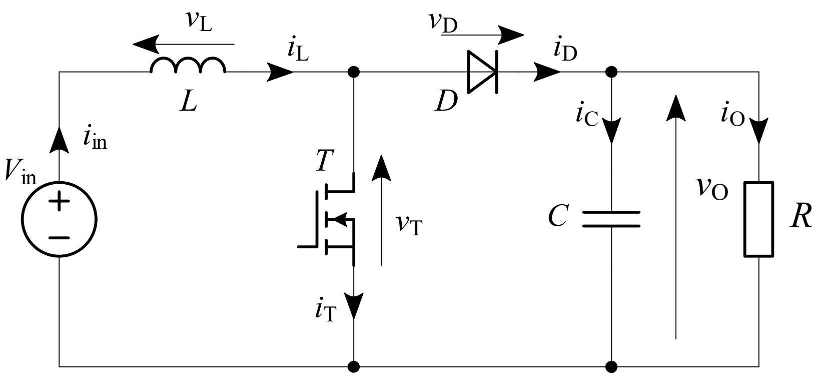
When $I_O D < ½\Delta I_L(1\text{-}D)$ (in CCM) a small portion of the charge also occur at the second equivalent circuit and $\Delta Q > DTI_O$



Due to simplification of the analysis, it is assumed that $\Delta Q = DTI_O$

DCM operation

When the output current $I_{\mathrm{O}} < I_{\mathrm{Ocrit}}$ the converter operates in discontinuous conduction mode (DCM)



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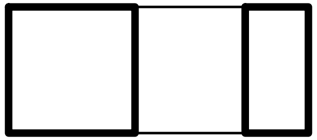
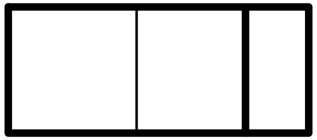


$t \in (0, DT)$

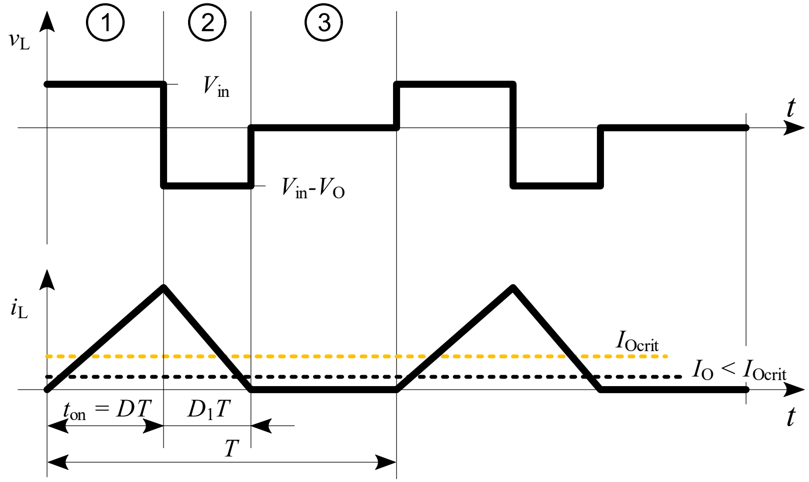
2



$t \in (DT, D_1T)$

3

$t \in (D_1T, T)$



$$V_{\mathrm{LAV}} = V_{\mathrm{in}} D + \left(V_{\mathrm{in}} - V_{\mathrm{O}}\right) D_1 = 0$$

$$D_1 = \frac{V_{\mathrm{in}}}{V_{\mathrm{O}} - V_{\mathrm{in}}} D$$

$$I_{\mathrm{in}} = \frac{1}{2} \Delta I_{\mathrm{L}} \left(D + D_1\right), \text{ where } \Delta I_{\mathrm{L}} = \frac{V_{\mathrm{in}}}{L} DT$$

$$I_{\mathrm{O}} = \frac{V_{\mathrm{in}}}{V_{\mathrm{O}}} I_{\mathrm{in}} = \frac{V_{\mathrm{in}}}{V_{\mathrm{O}}} \frac{1}{2} \frac{V_{\mathrm{in}}}{L} DT\left(D + D_1\right) = \frac{V_{\mathrm{in}}}{2L} \frac{V_{\mathrm{in}}}{V_{\mathrm{O}}} D^2 T \left(1 + \frac{V_{\mathrm{in}}}{V_{\mathrm{O}} - V_{\mathrm{in}}}\right)$$

$$I_{\mathrm{O}} = \frac{V_{\mathrm{in}}}{2L} \frac{V_{\mathrm{in}}}{V_{\mathrm{O}}} T D^2 \left(\frac{V_{\mathrm{O}}}{V_{\mathrm{O}} - V_{\mathrm{in}}}\right) \overset{I_{\mathrm{ref}} = \frac{V_{\mathrm{in}} T}{2L}}{=} I_{\mathrm{ref}} \left(\frac{V_{\mathrm{in}}}{V_{\mathrm{O}} - V_{\mathrm{in}}}\right) D^2$$

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Output characteristics

In DCM
$$I_O = I_{ref}\left(\frac{V_{in}}{V_O - V_{in}}\right)D^2$$

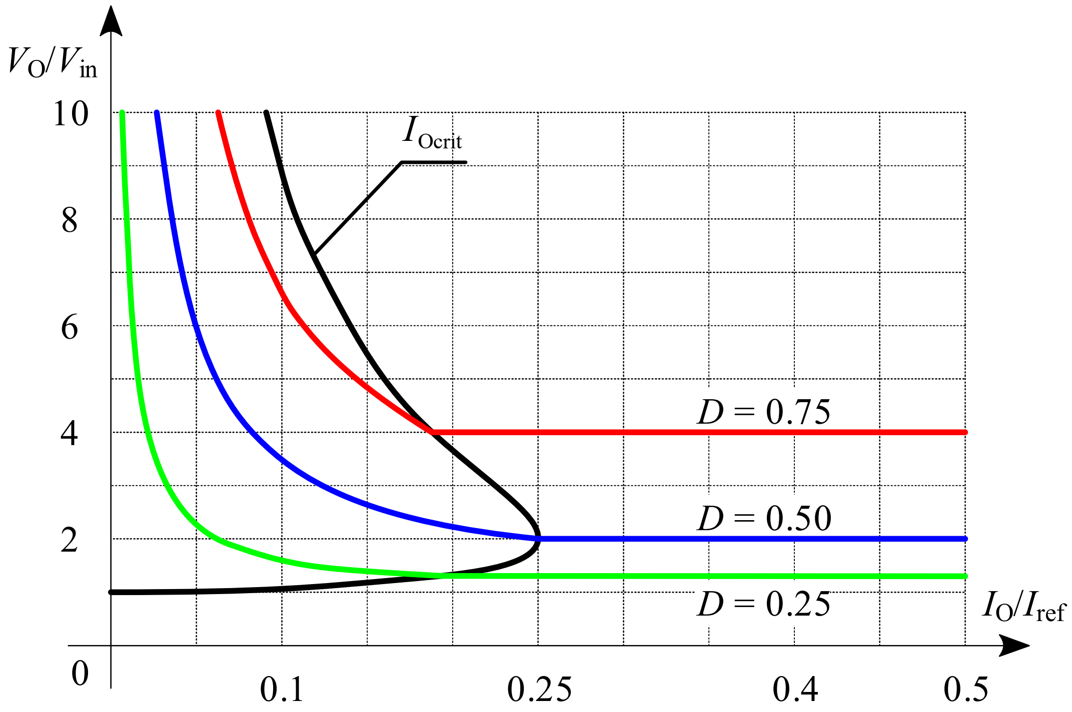
$$\frac{I_O}{I_{ref}}\frac{1}{D^2} = \left(\frac{V_{in}}{V_O - V_{in}}\right) \Rightarrow \frac{V_O}{V_{in}} - 1 = \frac{1}{\frac{I_O}{I_{ref}}\frac{1}{D^2}}$$

$$\frac{V_O}{V_{in}} = \frac{1}{\frac{I_O}{I_{ref}}\frac{1}{D^2}} + 1$$

In CCM
$$\frac{V_O}{V_{in}} = \frac{1}{1-D}$$

At CCM/DCM boundary

$$I_{Ocrit} = I_{ref}D(1-D) \overset{D=\frac{V_O - V_{in}}{V_O}}{=} I_{ref}\frac{V_O - V_{in}}{V_O}\left(1 - \frac{V_O - V_{in}}{V_O}\right) = I_{ref}\frac{\frac{V_O}{V_{in}} - 1}{\left(\frac{V_O}{V_{in}}\right)^2}$$



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Boost with V_O = const

$$I_O = \frac{V_{in}}{2L}\frac{V_{in}}{V_O}TD^2\left(\frac{V_O}{V_O - V_{in}}\right) \overset{I_{Oref}=\frac{V_O T}{2L}}{=} I_{Oref}\frac{V_{in}}{V_O}\left(\frac{V_{in}}{V_O - V_{in}}\right)D^2$$

In DCM

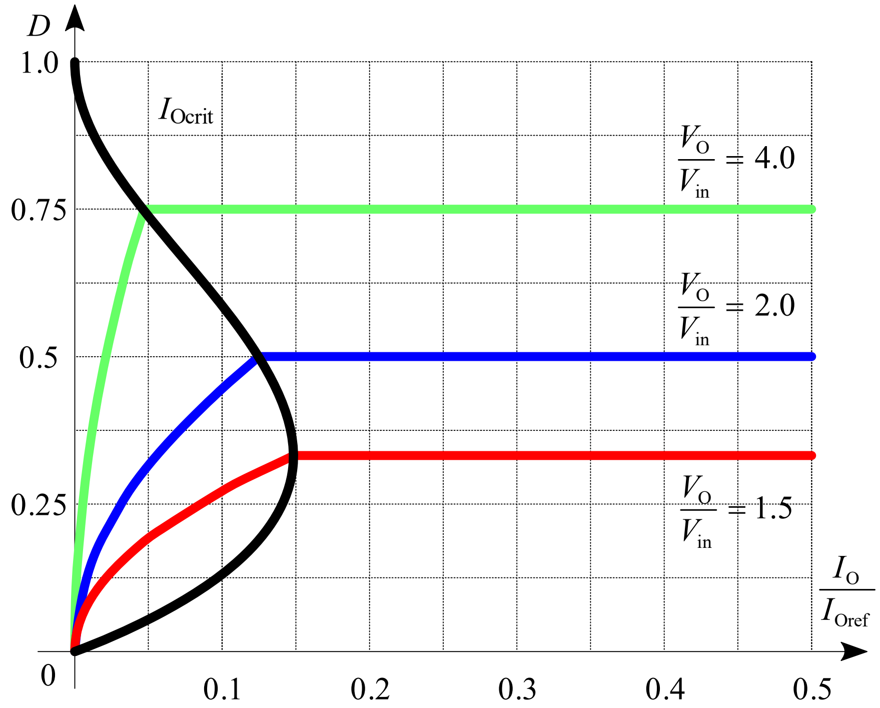
$$D^2 = \frac{\dfrac{I_O}{I_{Oref}}}{\dfrac{V_{in}}{V_O}\left(\dfrac{V_{in}}{V_O - V_{in}}\right)} \Rightarrow D = \sqrt{\frac{\dfrac{I_O}{I_{Oref}}}{\dfrac{V_{in}}{V_O}\left(\dfrac{V_{in}}{V_O - V_{in}}\right)}}$$

In CCM $\quad D = 1 - \dfrac{V_{in}}{V_O}$

At CCM/DCM boundary

$$I_{Ocrit} = \frac{V_{in}T}{2L}\frac{V_O}{V_O}D(1-D)$$

$$I_{Ocrit} \overset{I_{Oref}=\frac{V_O T}{2L}}{=} I_{Oref}\frac{V_{in}}{V_O}D(1-D) = I_{Oref}(1-D)D(1-D) = I_{Oref}D(1-D)^2$$



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Boost converter example #1

Boost converter example as a MPPT converter in solar systems.

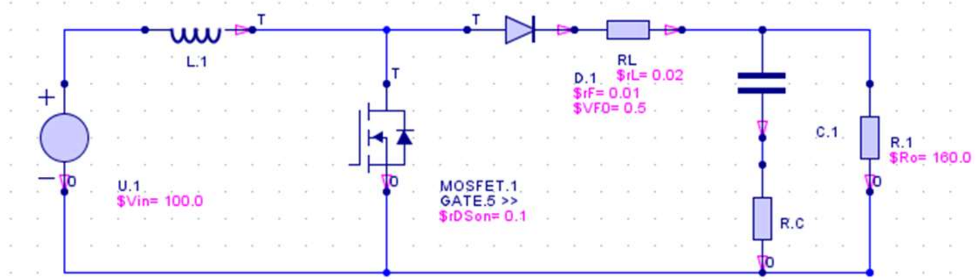
Input voltage:	$V_{\mathrm{in}} = 100$ V,
Output voltage:	$V_{\mathrm{out}} = 400$ V,
Switching frequency:	$f = 100$ kHz, ($T = 10$ μs)
Output capacitance:	$C = 10$ uF,
Output power:	$P_{\mathrm{O}} = 1000$ W,
Inductor current pk-pk	$\Delta I_{\mathrm{L}} = 50\% I_{\mathrm{LAV}}$.

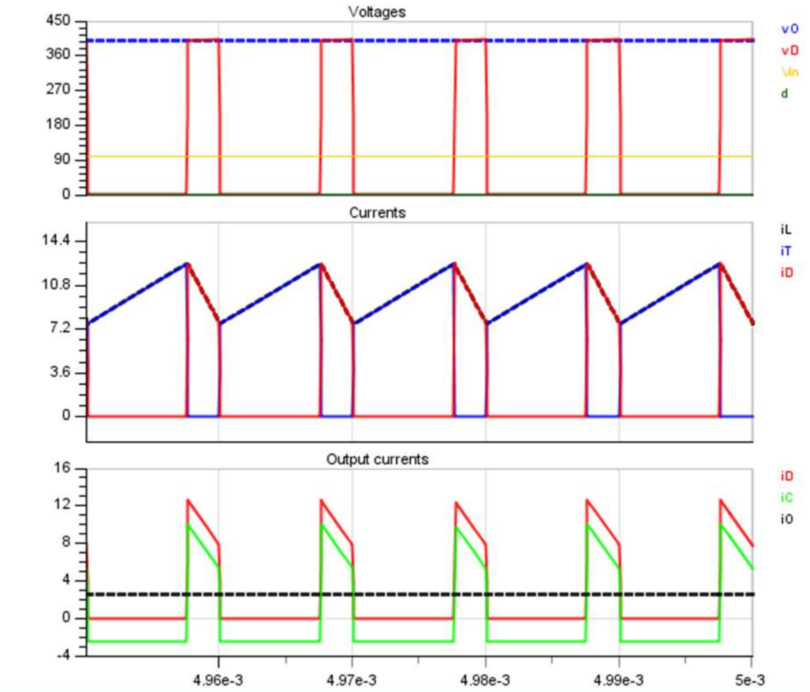
$$I_{\mathrm{O}} = \frac{P_{\mathrm{O}}}{V_{\mathrm{O}}} = \frac{1000\ \mathrm{W}}{400\ \mathrm{V}} = 2.5\ \mathrm{A} \qquad R = \frac{V_{\mathrm{O}}}{I_{\mathrm{O}}} = \frac{400\ \mathrm{V}}{2.5\ \mathrm{A}} = 160\ \Omega$$

$$I_{\mathrm{in}} = \frac{P_{\mathrm{in}}}{V_{\mathrm{in}}} \overset{P_{\mathrm{in}}=P_{\mathrm{O}}}{=} \frac{1000\ \mathrm{W}}{100\ \mathrm{V}} = 10\,\mathrm{A} \qquad D = 1 - \frac{V_{\mathrm{in}}}{V_{\mathrm{O}}} = 1 - \frac{100\,\mathrm{V}}{400\,\mathrm{V}} = 0.75$$

$$\Delta I_{\mathrm{L}} = \frac{V_{\mathrm{in}}}{L} DT \Rightarrow L = \frac{V_{\mathrm{in}}}{\Delta I_{\mathrm{L}}} DT = \frac{100\,\mathrm{V}}{0.5 \cdot 10\,\mathrm{A}} 0.75 \cdot 10\ \mu\mathrm{s} = 150\,\mu\mathrm{H}$$

GeckoCIRCUITS model of the boost converter





Boost converter example #2

Boost converter example in battery level voltage applications

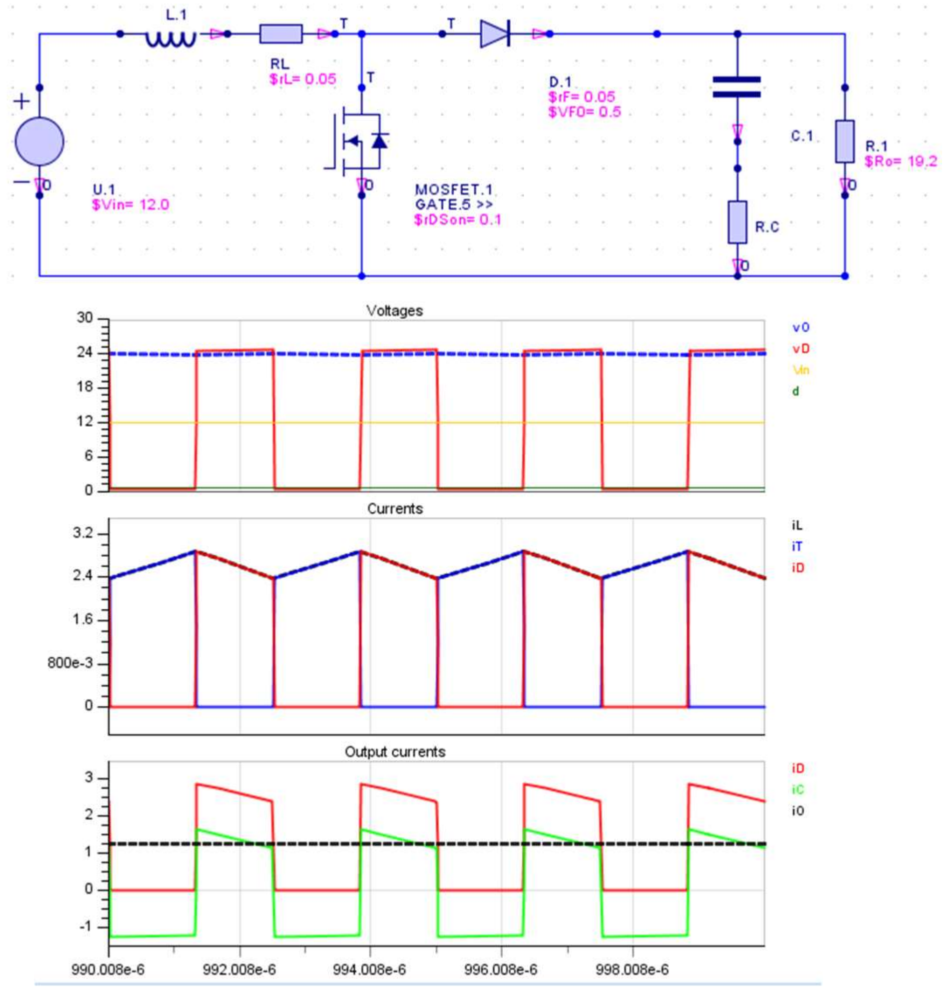
Input voltage:	$V_{\text{in}} = 12\text{ V},$
Output voltage:	$V_{\text{out}} = 24\text{ V},$
Switching frequency:	$f = 400\text{ kHz}, (T = 2.5\ \mu\text{s})$
Output capacitance:	$C = 10\text{ uF},$
Output power:	$P_{\text{O}} = 30\text{ W},$
Inductor current pk-pk	$\Delta I_{\text{L}} = 20\% I_{\text{LAV}}.$

$$I_{\text{O}} = \frac{P_{\text{O}}}{V_{\text{O}}} = \frac{30\text{ W}}{24\text{ V}} = 1.25\text{ A} \qquad R = \frac{V_{\text{O}}}{I_{\text{O}}} = \frac{24\text{ V}}{1.25\text{ A}} = 19.2\ \Omega$$

$$I_{\text{in}} = \frac{P_{\text{in}}}{V_{\text{in}}} \overset{P_{\text{in}}=P_{\text{O}}}{=} \frac{30\text{ W}}{12\text{ V}} = 2.5\text{A} \qquad D = 1 - \frac{V_{\text{in}}}{V_{\text{O}}} = 1 - \frac{12\text{ V}}{24\text{ V}} = 0.5$$

$$\Delta I_{\text{L}} = \frac{V_{\text{in}}}{L} DT \Rightarrow \underline{L = \frac{V_{\text{in}}}{\Delta I_{\text{L}}} DT = \frac{12\text{ V}}{0.2 \cdot 2.5\text{ A}} 0.5 \cdot 2.5\ \mu\text{s} = 30\mu\text{H}}$$

GeckoCIRCUITS model of the boost converter



Output voltage of other DC-DC converters

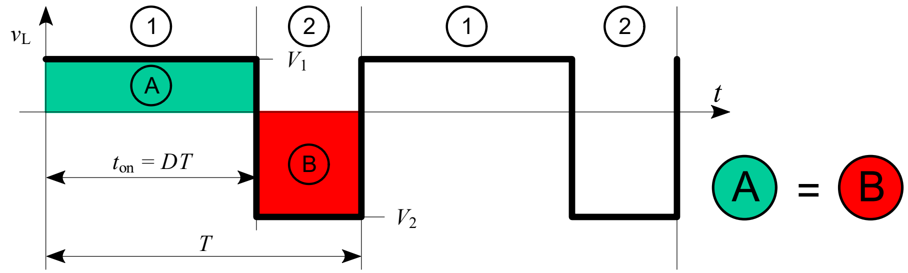
In basic DC-DC converter topologies voltage across the inductor can be equal to V_{in}, $-V_O$, $V_{in}-V_O$.

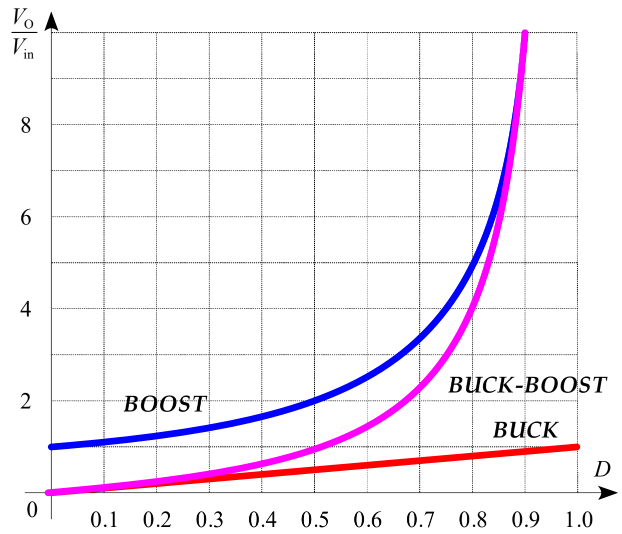
It is always true, that in the first equivalent circuit voltage is positive whereas this voltage is negative in the second equivalent circuit.

The possible combinations:

Topology	V_1	V_2	V_O/V_{in} (CCM)
Step down	$V_{in}-V_O$	$-V_O$	D
Step up	V_{in}	$V_{in}-V_O$	$\frac{1}{1-D}$
Step up-down	V_{in}	$-V_O$	$\frac{D}{1-D}$

Inductor voltage is rectangular





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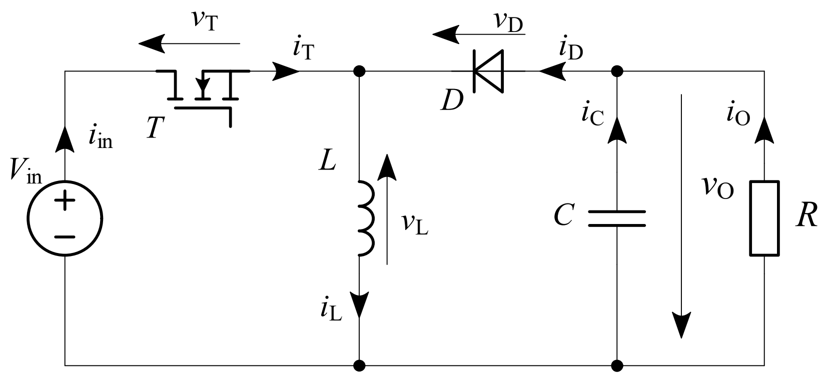
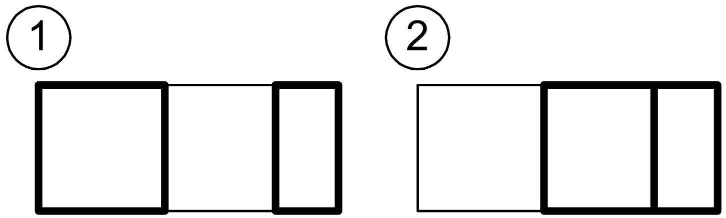
Buck-boost converter

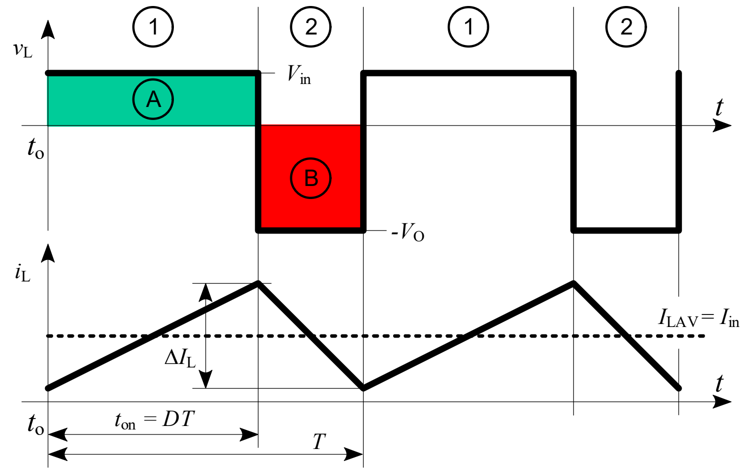
The buck-boost converter (a step up-down converter) has negative output voltage referred to negative terminal of the input source.



Transistor *T* is switched between the on- and off-states. For CCM there are two equivalent circuits



Average value of the inductor voltage in the steady state operation is equal to zero



A = B

$$V_{in}DT + (-V_O)(1-D)T = 0$$

$$V_{in}D - V_O(1-D) = 0$$

$$\boxed{V_O = V_{in}\frac{D}{1-D}}$$

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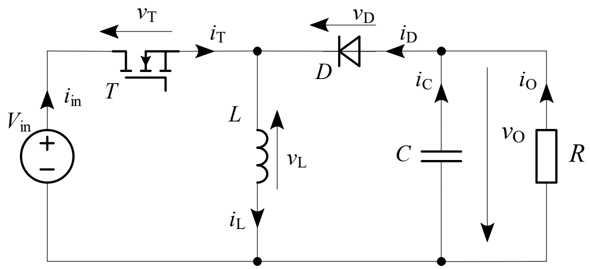
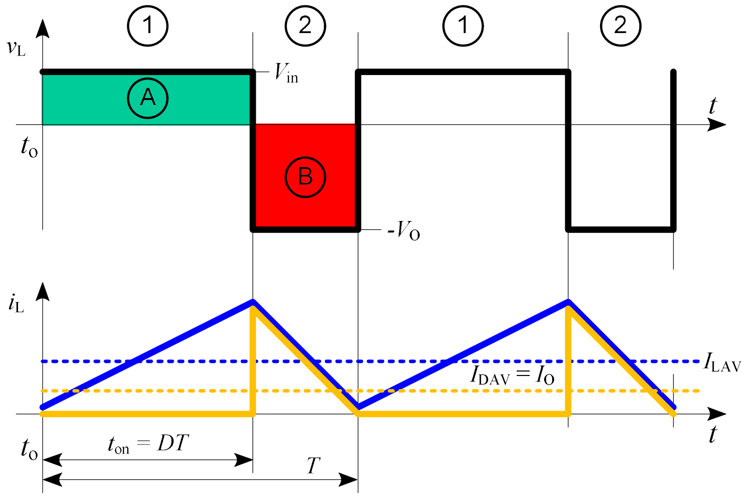
Buck-boost converter

Inductor current average value is not equal to the average value of the output current as in buck converter.





Critical inductor current I_{Lcrit} is at the boundary between CCM & DCM

$$I_{Lcrit} = I_{LAVcrit} = \frac{V_{in}DT}{2L}$$

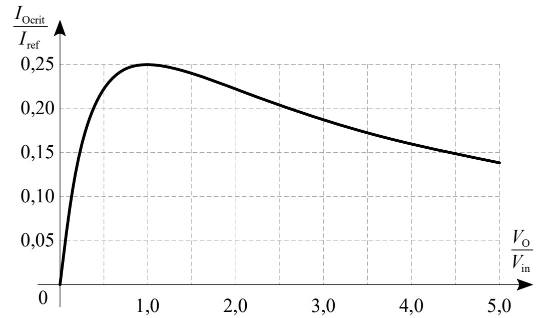
Critical output current I_{Ocrit} is the output current at which the inductor current is equal to I_{Lcrit}

$$I_{Ocrit} = I_{Lcrit}(1-D) = \frac{V_{in}T}{2L}D(1-D)$$

The same as in buck and boost converters

$$\frac{V_O}{V_{in}} = \frac{D}{1-D} = x \Rightarrow D = \frac{x}{x+1}$$

$$I_{Ocrit} = \frac{V_{in}T}{2L}\frac{x}{x+1}\left(1-\frac{x}{x+1}\right) = \frac{V_{in}T}{2L}\frac{x}{(x+1)^2}$$



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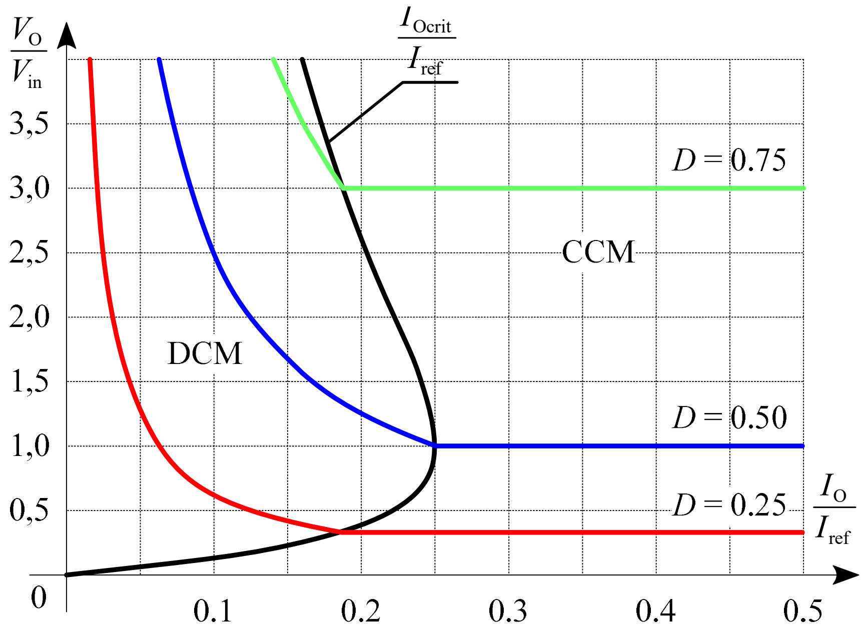
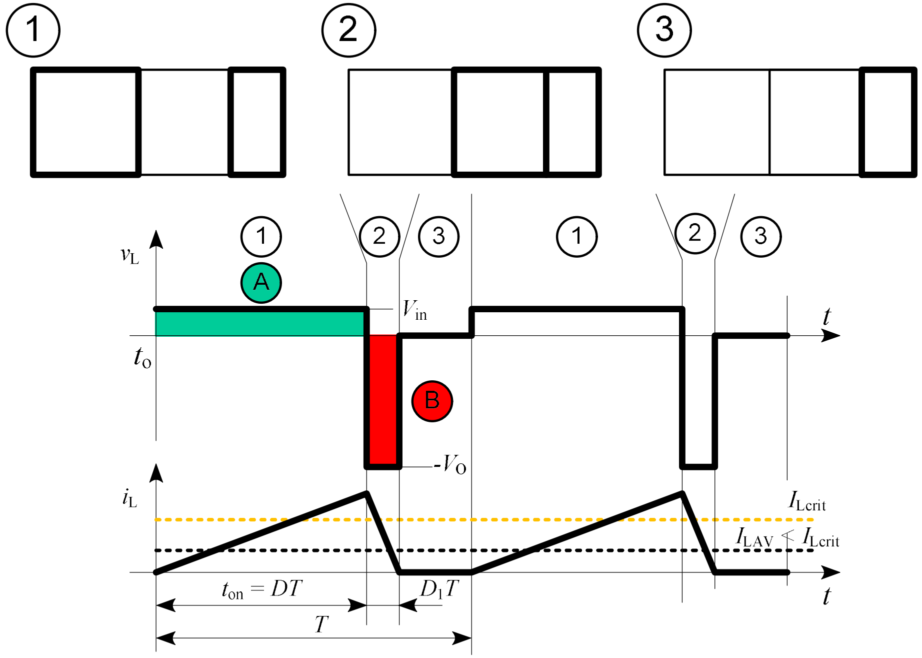
Buck-boost converter

The output characteristic for V_{in} = const



For DCM it is needed to calculate duty cycle D_1



$$V_{LAV} = V_{in}D + (-V_O)D_1 = 0 \Rightarrow D_1 = \frac{V_{in}}{V_O}D \ or \ D_1 = \frac{1}{\frac{V_O}{V_{in}}}D$$

$$I_O = \frac{1}{2}\Delta I_L D_1 = \frac{1}{2}\frac{V_{in}T}{L}DD_1 = \frac{1}{2}\frac{V_{in}T}{L}D^2\frac{V_{in}}{V_O} \Rightarrow \frac{V_O}{V_{in}} = \frac{1}{2}\frac{V_{in}T}{LI_O}D^2 = D^2\frac{I_{ref}}{I_O}$$

Buck-boost converter

The output characteristic for V_O = const

Defining output reference current: $$I_{\mathrm{Oref}} = \frac{V_O T}{2L}$$

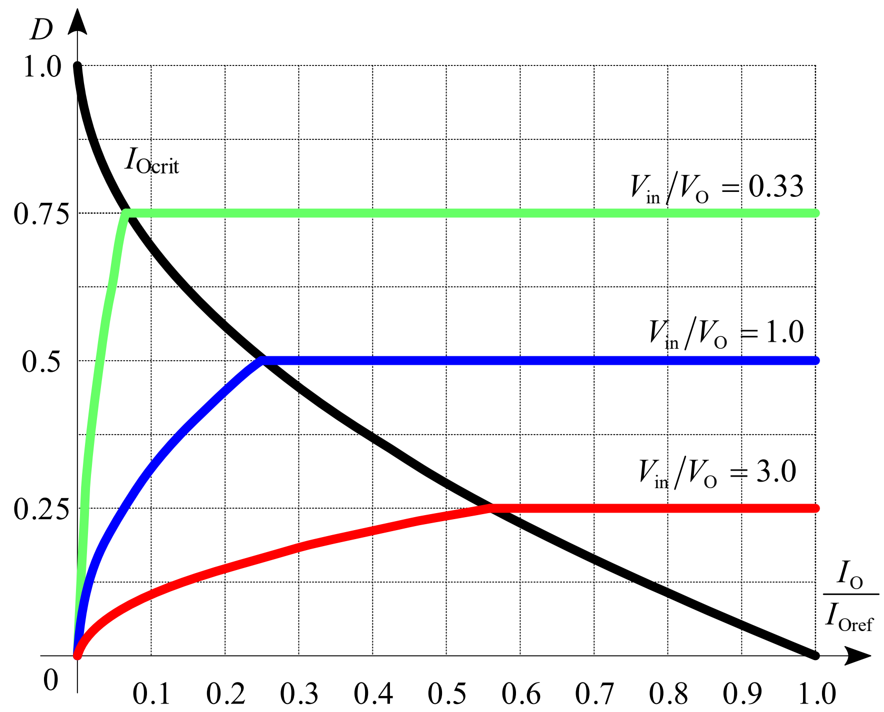
The critical otuput current is:

$$I_{\mathrm{Ocrit}} = \frac{V_{\mathrm{in}} T}{2L} D(1-D) \overset{V_{\mathrm{in}}\frac{D}{1-D}=V_O}{=} \frac{V_O T}{2L}(1-D)^2 = I_{\mathrm{Oref}}(1-D)^2$$

The relation between the output current and duty cycle in DCM is:

$$I_O = \frac{1}{2}\Delta I_L D_1 = \frac{V_{\mathrm{in}} T}{2L} D D_1 \overset{D_1=\frac{1}{\frac{V_O}{V_{\mathrm{in}}}}D}{=} I_{\mathrm{Oref}} D^2 \left(\frac{V_{\mathrm{in}}}{V_O}\right)^2$$

The output characteristic for V_O = const



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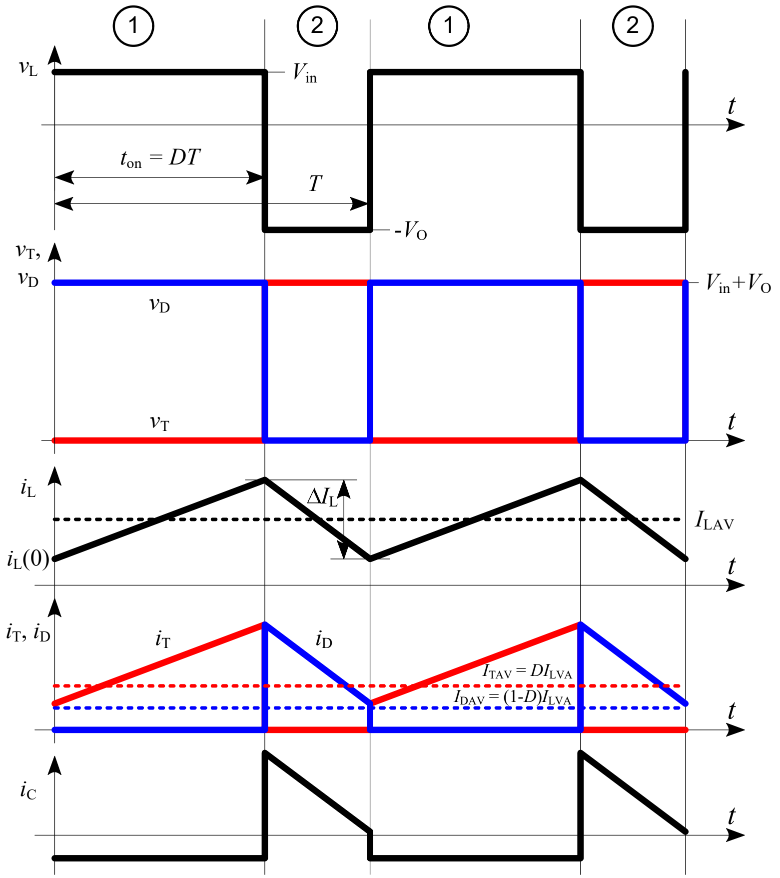
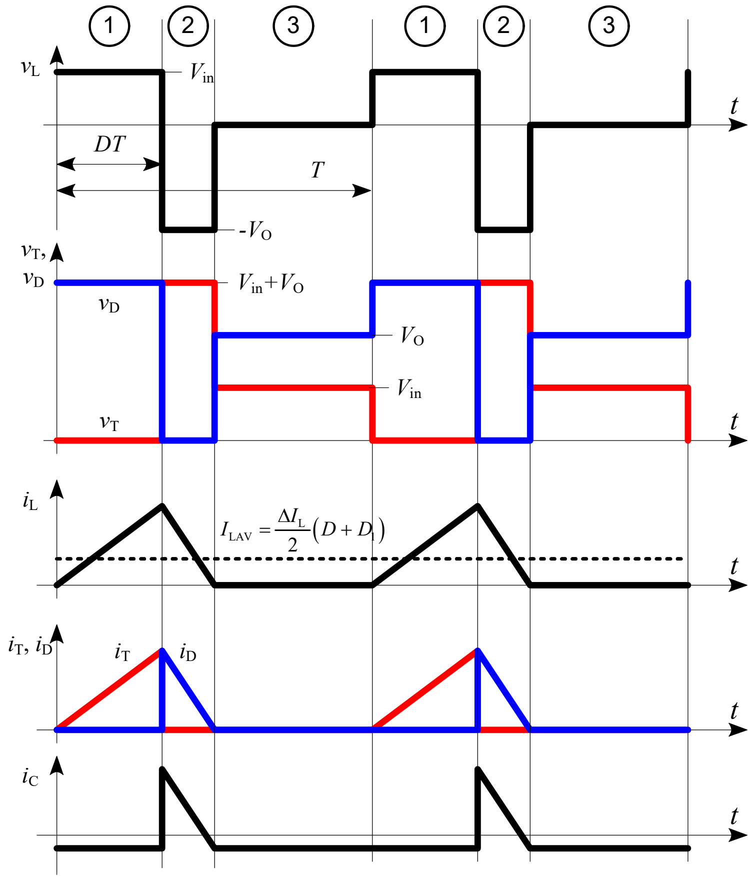
Buck-boost converter

Waveforms in CCM



Waveforms in DCM



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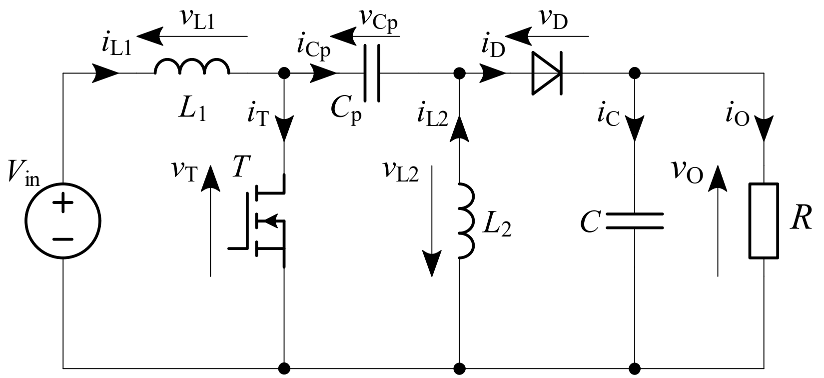
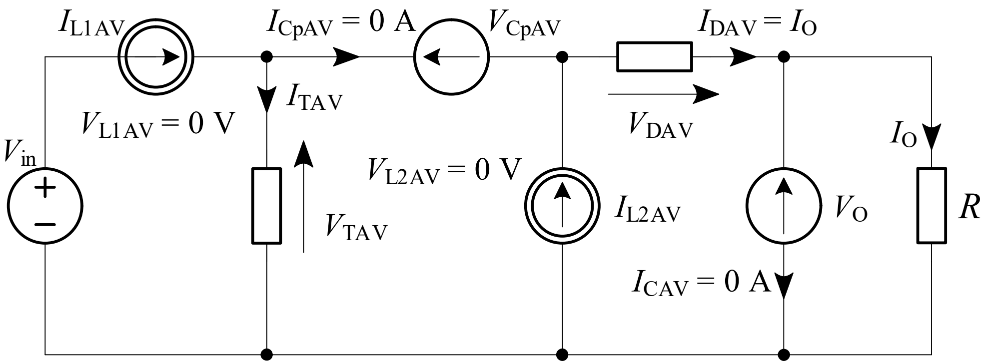
SEPIC converter

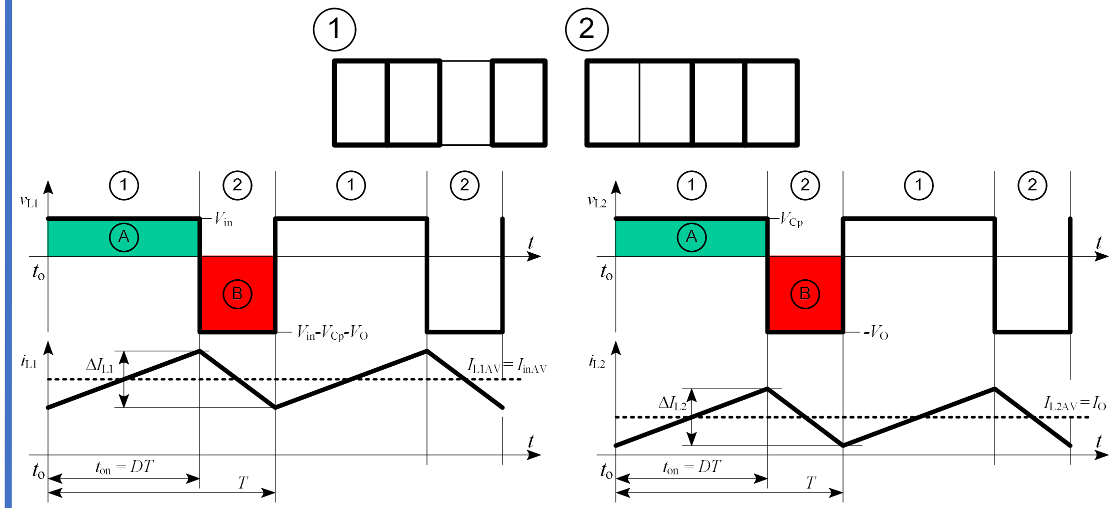
The SEPIC converter topology consists of two inductors



Averaged model of SEPIC converter



Derivation of the output voltage characteristic for CCM



$$V_{L1AV}: V_{in}D + \left(V_{in} - V_{Cp} - V_O\right)(1-D) = 0$$

$$V_{L2AV}: V_{Cp}D - V_O(1-D) = 0$$

$$V_{in}D + \left(V_{in} - V_{Cp}\right)(1-D) - V_{Cp}D = 0$$

$$\underline{V_{in}D} + V_{in} - \underline{V_{in}D} - V_{Cp} + \underline{\underline{V_{Cp}D}} - \underline{\underline{V_{Cp}D}} = 0$$

$$V_{Cp} = V_{in}$$

$$\frac{V_O}{V_{in}} = \frac{D}{1-D}$$

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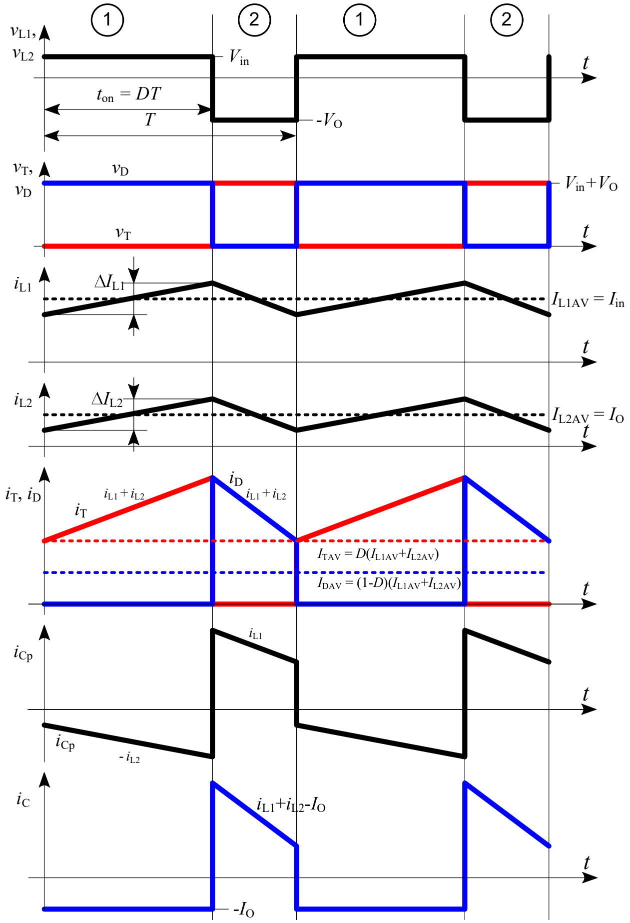
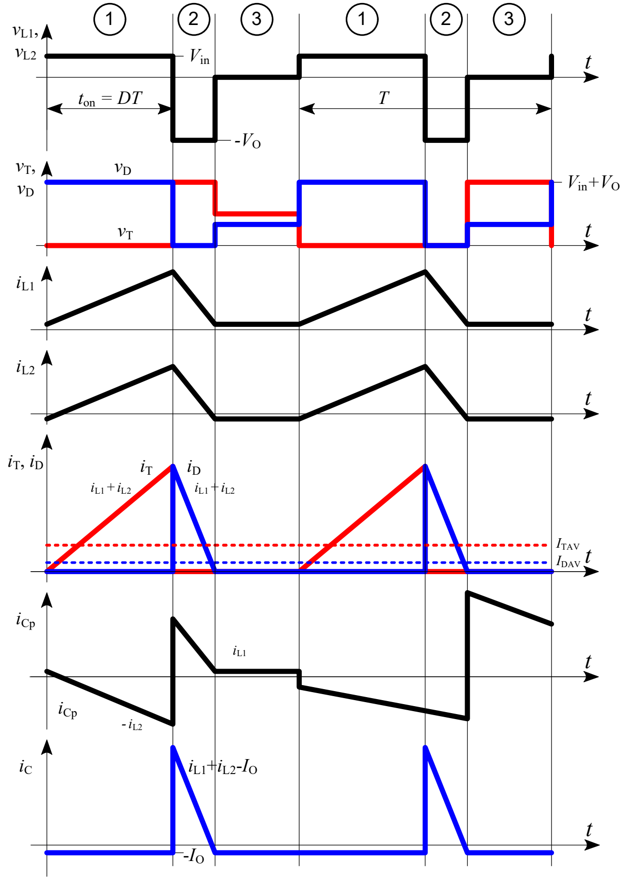
SEPIC converter

Waveforms in CCM

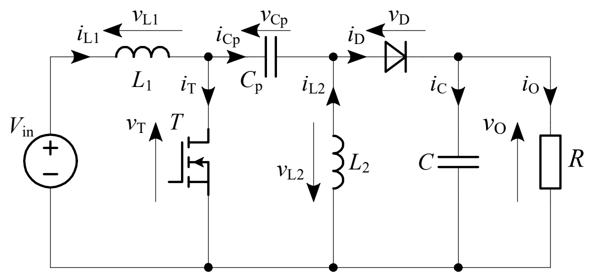


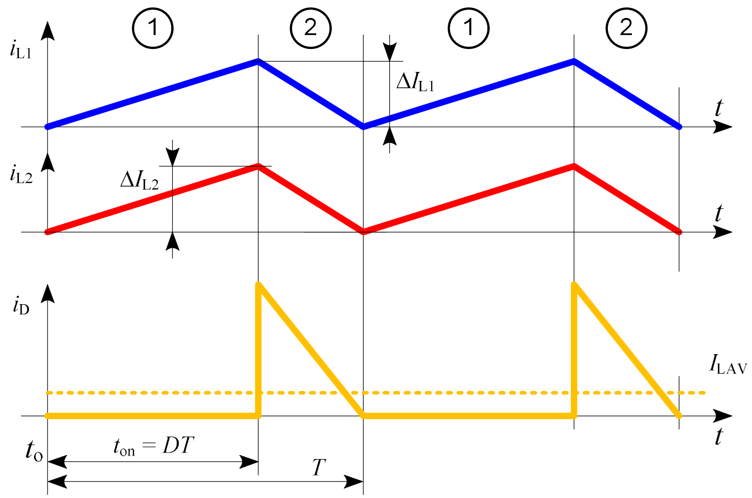
Waveforms in DCM



SEPIC converter

Boundary current occurs when diode current reaches zero value at the end of the switching period





Critical output current I_{Ocrit} is at the boundary between CCM & DCM

$$I_{Ocrit} = \frac{1}{2}(\Delta I_{L1} + \Delta I_{L2})(1-D) = \frac{V_{in}T}{2}\left(\frac{1}{L_1} + \frac{1}{L_2}\right)D(1-D)$$

$$I_{Ocrit} = \frac{V_{in}T}{2L_1}\left(\frac{L_1 + L_2}{L_2}\right)D(1-D) = I_{ref}\left(\frac{L_1 + L_2}{L_2}\right)D(1-D)$$

$$= I_{ref}\left(\frac{L_1 + L_2}{L_2}\right)\frac{\frac{V_O}{V_{in}}}{1 + \frac{V_O}{V_{in}}}\left(\frac{1}{1 + \frac{V_O}{V_{in}}}\right)$$

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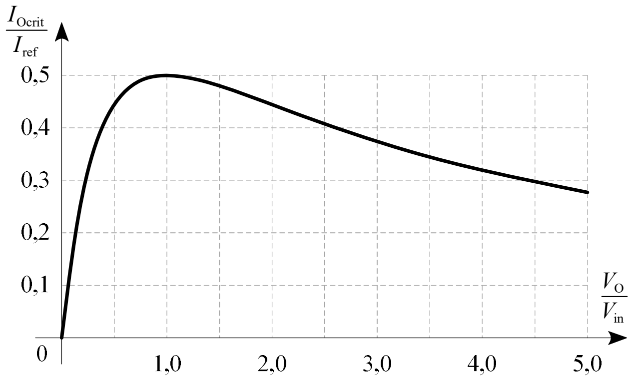
SEPIC converter

Boundary between DCM and CCM in sepic converter can be shown as a characteristic of VO/Vin = f(IO)

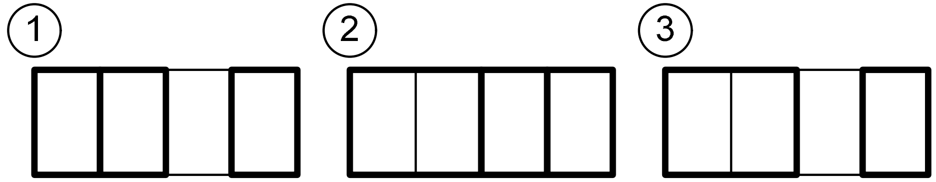
$$I_{\mathrm{Ocrit}} = \frac{V_{\mathrm{in}}T}{2L_1}\left(\frac{L_1+L_2}{L_2}\right)D(1-D)$$

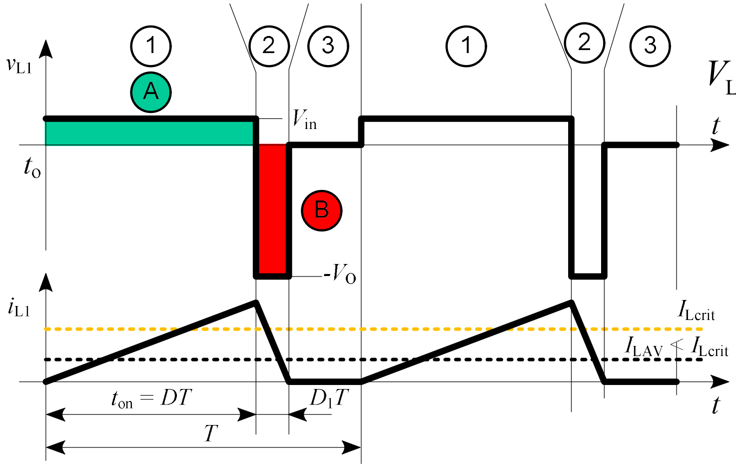
$$\frac{V_{\mathrm{O}}}{V_{\mathrm{in}}} = \frac{D}{1-D} = x \Rightarrow D = \frac{x}{x+1}$$

$$I_{\mathrm{Ocrit}} = \frac{V_{\mathrm{in}}T}{2L_1}\left(\frac{L_1+L_2}{L_2}\right)\frac{x}{x+1}\left(1-\frac{x}{x+1}\right) = \frac{V_{\mathrm{in}}T}{2L_1}\left(\frac{L_1+L_2}{L_2}\right)\frac{x}{(x+1)^2}$$



For DCM it is needed to calculate duty cycle D_1





$$V_{\mathrm{LAV}} = V_{\mathrm{in}}D + (-V_{\mathrm{O}})D_1 = 0$$

$$D_1 = \frac{V_{\mathrm{in}}}{V_{\mathrm{O}}}D$$

or

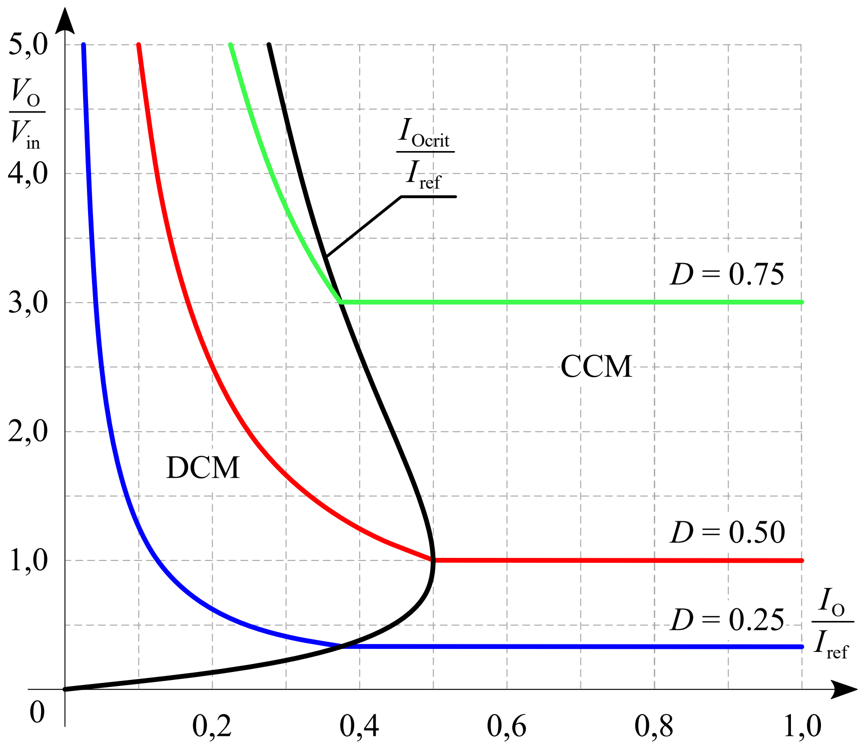
$$D_1 = \frac{1}{\frac{V_{\mathrm{O}}}{V_{\mathrm{in}}}}D$$

$$I_{\mathrm{O}} = \frac{1}{2}(\Delta I_{\mathrm{L1}}+\Delta I_{\mathrm{L2}})D_1 = \frac{1}{2}\frac{V_{\mathrm{in}}T}{L_1}\left(\frac{L_1+L_2}{L_2}\right)DD_1 = \frac{1}{2}\frac{V_{\mathrm{in}}T}{L_1}\left(\frac{L_1+L_2}{L_2}\right)D^2\frac{V_{\mathrm{in}}}{V_{\mathrm{O}}}$$

$$\frac{V_{\mathrm{O}}}{V_{\mathrm{in}}} = \frac{1}{I_{\mathrm{O}}}\frac{1}{2}\frac{V_{\mathrm{in}}T}{L_1}\left(\frac{L_1+L_2}{L_2}\right)D^2 = D^2\frac{I_{\mathrm{ref}}}{I_{\mathrm{O}}}\left(\frac{L_1+L_2}{L_2}\right)$$

SEPIC converter

The output characteristic for V_{in} = const

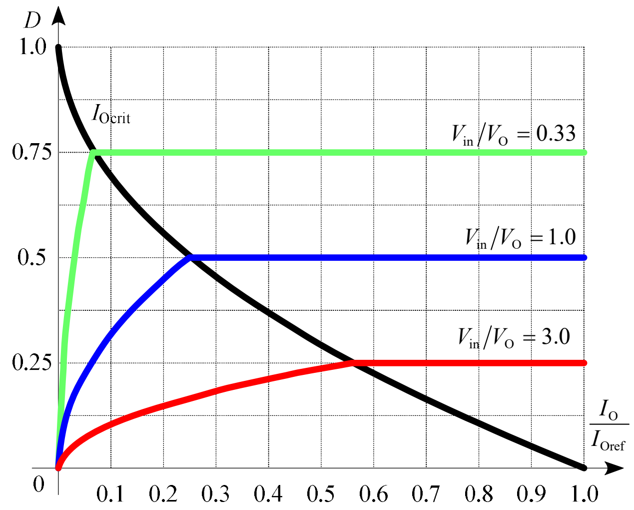


Defining output reference current: $I_{Oref} = \frac{V_O T}{2L_1}$

$$I_{Ocrit} = \frac{V_{in}T}{2L_1}\left(\frac{L_1+L_2}{L_2}\right)D(1-D) \overset{V_{in}\frac{D}{1-D}=V_O}{=} \frac{V_O T}{2L_1}\left(\frac{L_1+L_2}{L_2}\right)(1-D)^2 =$$

$$= I_{Oref}\left(\frac{L_1+L_2}{L_2}\right)(1-D)^2$$

$$I_O = \frac{1}{2}\Delta I_L D_1 = \frac{V_{in}T}{2L} D D_1 \overset{D_1=\frac{1}{V_O/V_{in}}D}{=} I_{Oref} D^2 \left(\frac{V_{in}}{V_O}\right)^2$$



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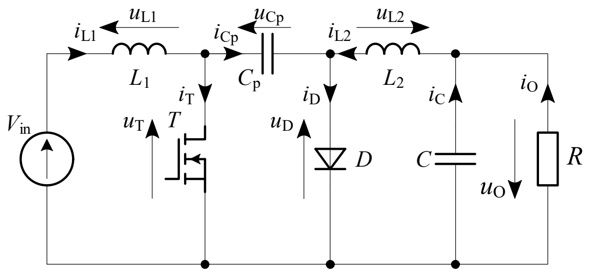
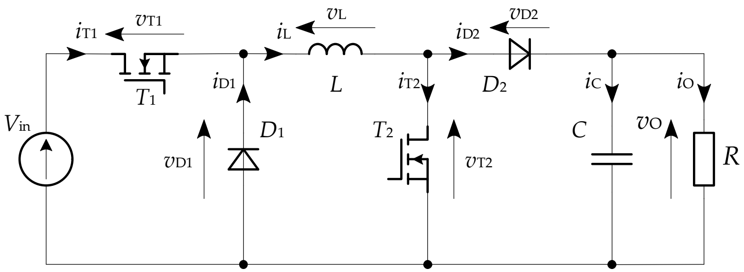
Other Buck-Boost converters

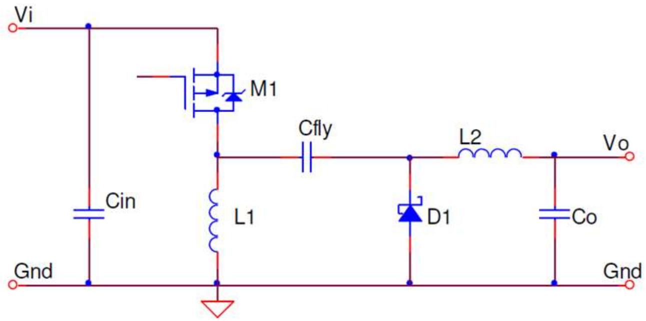
Ćuk converter



Two-transistor buck-boost converter



Zeta converter



Interleaved converter