# Power Electronics
# Lecture 5

## Diode and thyristor rectifiers

01 Half wave diode and thyristor rectifier

02 Full wave rectifiers

03 Six pulse rectifiers


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# Half wave rectifiers

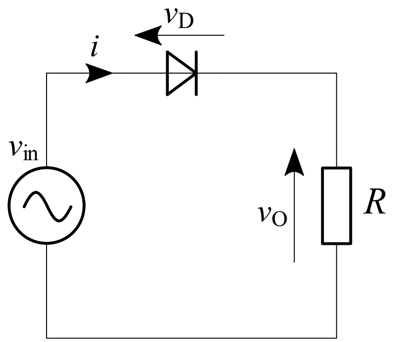
Half-wave rectifiers in practice are used most often in low-power applications.

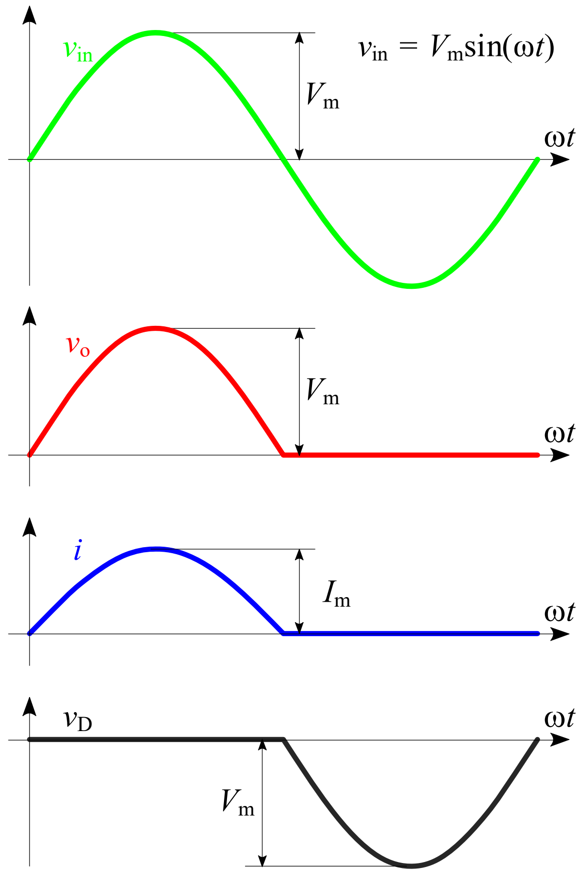
The average current in the supply is not zero, this may cause problems in transformer performance.

Half-wave rectifiers are mainly analyzed because their operation is similar to more complicated circuits (like full-wave multipulse rectifiers).

In the half-wave diode rectifier with resistive load the current $i$ flows when diode is forward biased ($v_{in} > 0$, $v_D = 0$, $v_O = v_{in}$)

$$v_{in} = V_m \sin(\omega t)$$




$$v_{in} = V_m \sin(\omega t)$$

$$v_o = \begin{cases} V_m \sin(\omega t) & \text{for } v_{in} > 0 \\ 0 & \text{for } v_{in} < 0 \end{cases}$$

$$i = \begin{cases} \dfrac{V_m}{R} \sin(\omega t) & \text{for } v_{in} > 0 \\ 0 & \text{for } v_{in} < 0 \end{cases}$$

$$v_D = \begin{cases} 0 & \text{for } v_{in} > 0 \\ V_m \sin(\omega t) & \text{for } v_{in} < 0 \end{cases}$$



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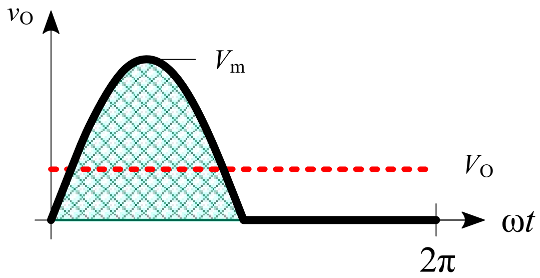
# Half wave rectifiers

The average value of the output voltage $V_O$ is given as




$$V_O = \frac{1}{2\pi}\int_0^{\pi} V_m \sin(\omega t)\, d\omega t = -\frac{V_m}{2\pi}\left[\cos(\omega t)\Big|_0^{\pi}\right] = -\frac{V_m}{2\pi}\left[(-1-1)\right] = \frac{V_m}{\pi}$$

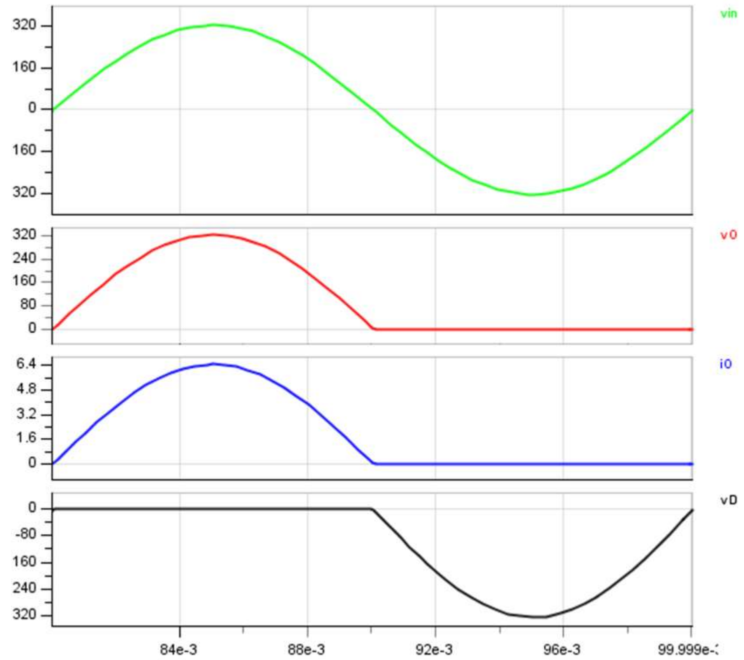
The average value of the current

$$I_{AV} = \frac{V_O}{R} = \frac{V_m}{\pi R}$$

The rms value of the output voltage

$$V_{Orms} = \sqrt{\frac{1}{2\pi}\int_0^{\pi}\left[V_m \sin(\omega t)\right]^2 d\omega t} = \frac{V_m}{2}$$

Example of the diode half-wave diode rectifier with R load ($R = 50\ \Omega$) supplied from sinusoidal voltage source $V_{inrms}$ = 230 V ($V_m = 325$ V).

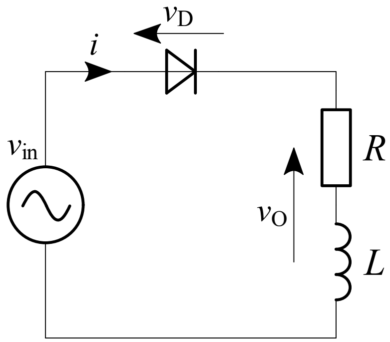



Results

| | avg | rms |
|---|---|---|
| vO | 103.4488 | 162.4969 |
| iO | 2.069 | 3.2499 |
| vD | -103.4488 | 162.4961 |
| vin | -89.0743e-9 | 229.8053 |

# Half wave rectifiers

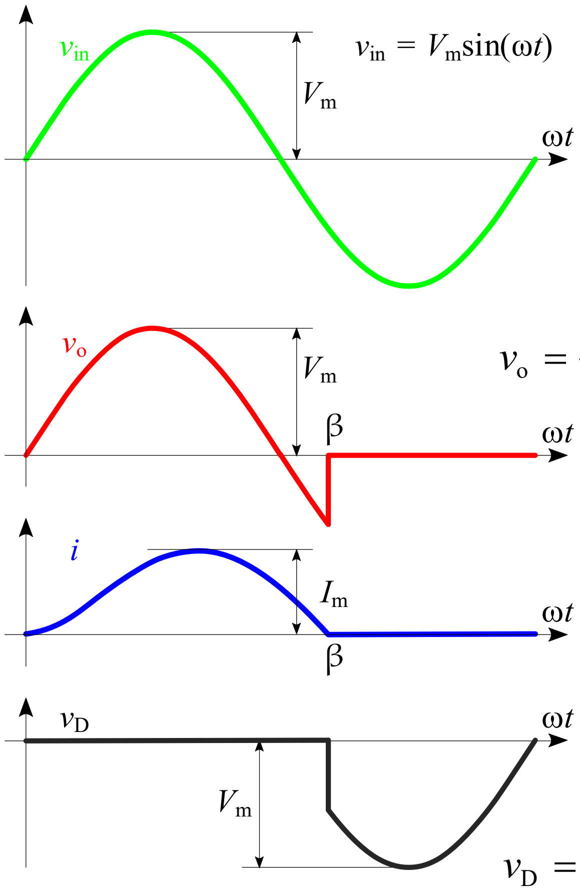
In the half-wave diode rectifier with resistive-inductive load the current $i$ flows for positive input voltage and for a part when the input voltage is negative.




The current $i$ is composed of two componenets, the forced response $i_f$ and natural response $i_n$.

$$i = i_f + i_n = \underbrace{\frac{V_m}{Z}\sin(\omega t - \varphi)}_{i_f} + \underbrace{\frac{V_m}{Z}\sin(\varphi) e^{-\frac{t}{\tau}}}_{i_n}$$

$$where: Z = \sqrt{R^2 + (\omega L)^2};\ \varphi = \arctan\left(\frac{\omega L}{R}\right);\ \tau = \frac{L}{R}$$




$$v_o = \begin{cases} V_m \sin(\omega t) & \text{for } \omega t \in (0, \beta) \\ 0 & \text{for } \omega t \in (\beta, 2\pi) \end{cases}$$

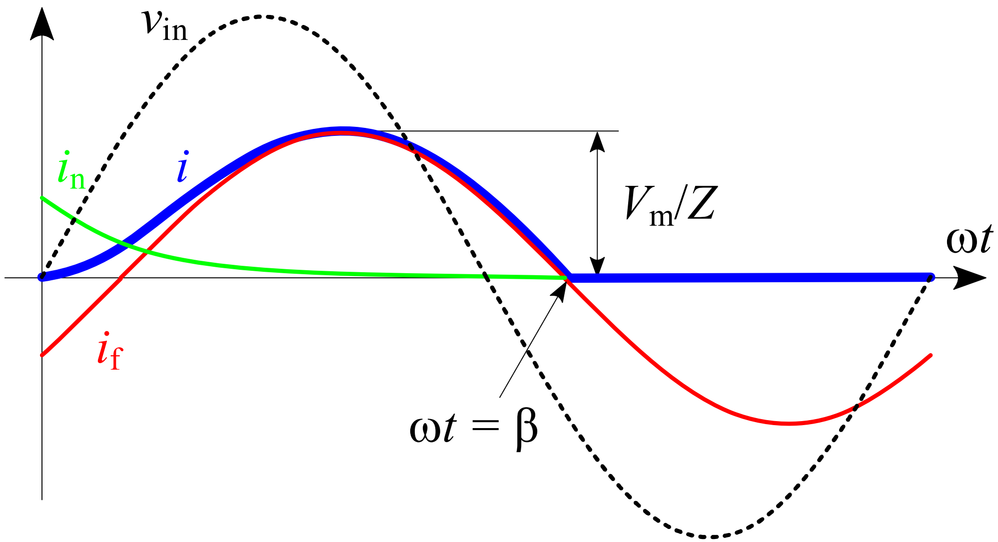
$$i = \begin{cases} i_f + i_n & \text{for } \omega t \in (0, \beta) \\ 0 & \text{for } \omega t \in (\beta, 2\pi) \end{cases}$$

$$v_D = \begin{cases} 0 & \text{for } \omega t \in (0, \beta) \\ V_m \sin(\omega t) & \text{for } \omega t \in (\beta, 2\pi) \end{cases}$$

# Half wave rectifiers

The current $i$ is composed of two componenets, the forced response $i_f$ and natural response $i_n$.

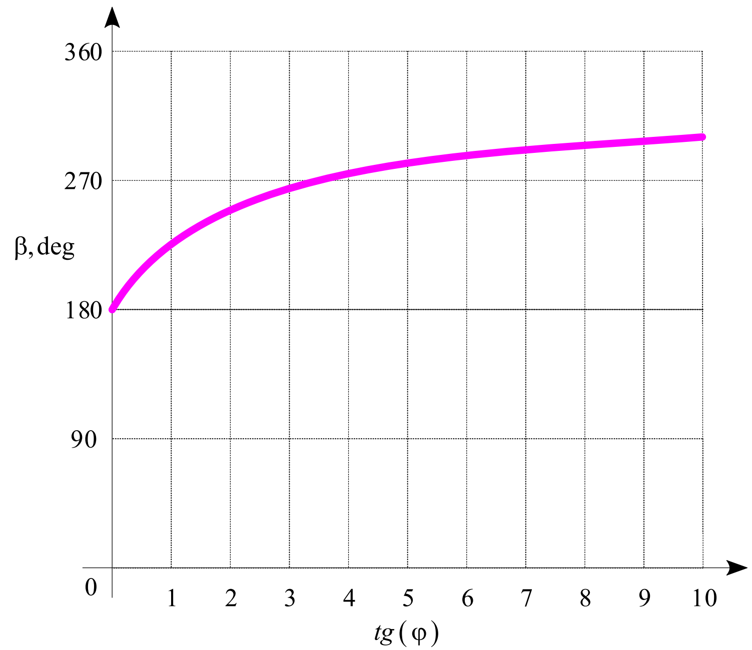
$$i = i_f + i_n = \underbrace{\frac{V_m}{Z}\sin(\omega t - \varphi)}_{i_f} + \underbrace{\frac{V_m}{Z}\sin(\varphi) e^{-\frac{t}{\tau}}}_{i_n}$$




The turn off angle $\beta$ is required for calculating the average value of the output voltage and current. This angle can be numerically computed from the equation

$$i = 0 = \frac{V_m}{Z}\sin(\beta - \varphi) + \frac{V_m}{Z}\sin(\varphi) e^{-\frac{\beta}{\omega\tau}}$$

$$A\left[\sin(\beta - \varphi) + \sin(\varphi) e^{-\frac{\beta}{\omega\tau}}\right] = 0$$





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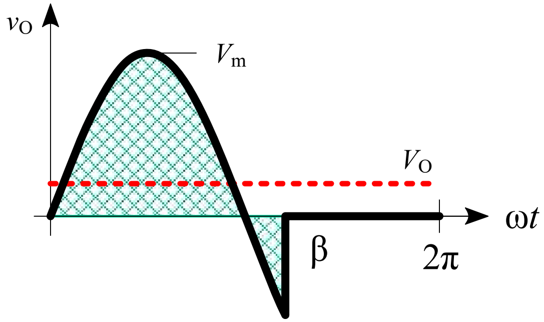
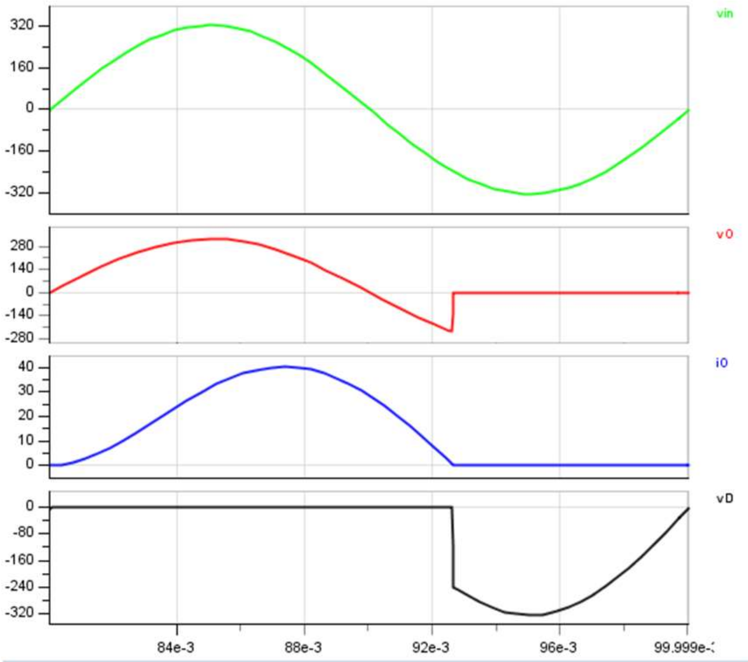
# Half wave rectifiers

The average value of the output voltage $V_O$ for RL load is given as.




$$V_O = \frac{1}{2\pi}\int_0^{\beta} V_m \sin(\omega t)\, d\omega t = -\frac{V_m}{2\pi}\left[\cos(\omega t)\Big|_0^{\beta}\right] = \frac{V_m}{2\pi}\left[1-\cos(\beta)\right]$$

Example of the diode half-wave diode rectifier with RL load ($R = 6\ \Omega$, $L = 20\ \mathrm{mH}$, $\mathrm{tg}(\varphi) \approx 1$) supplied from sinusoidal voltage source $V_{inrms}$ = 230 V ($V_m = 325\ \mathrm{V}$).



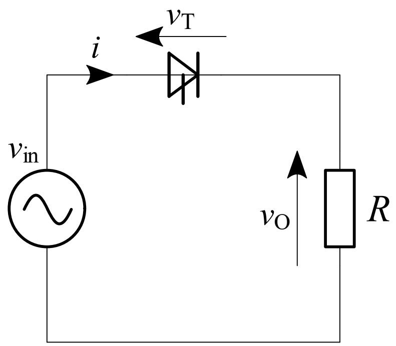

Results

| | avg | rms |
|---|---|---|
| vO | 86.8288 | 170.7296 |
| iO | 14.4715 | 21.1915 |
| vD | -86.8288 | 153.824 |
| vin | -89.0743e-9 | 229.8053 |

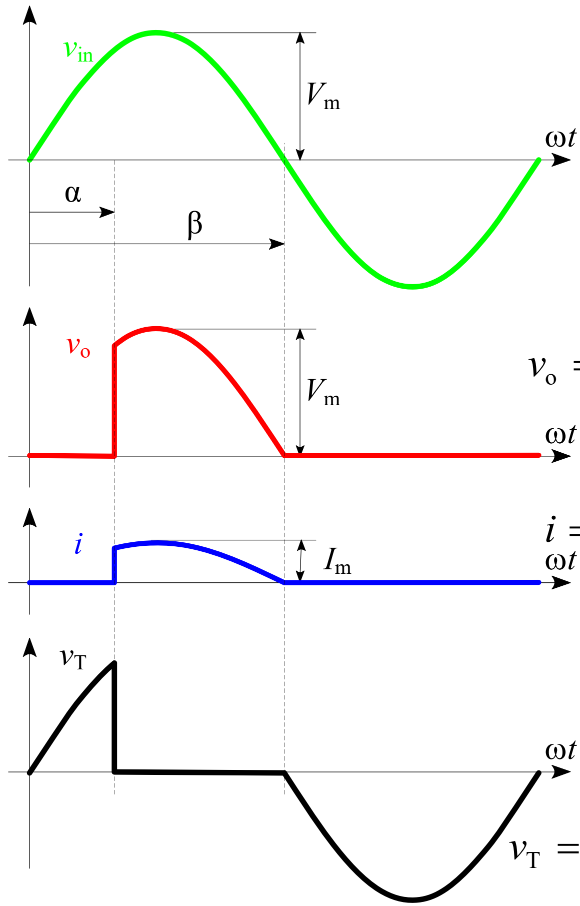
# Controlled half wave rectifiers

In the half-wave thyristor rectifier, the current i flows when thyristor is forward biased ($v_{in} > 0$) and to SCR gate current signal is applied.

The gate signal is applied synchronously with the fundamental period of $v_{in}$ at the delay angle $\alpha$.




In the half-wave thyristor rectifier with resitive load the turn-off angle β = $\pi$.




$$v_o = \begin{cases} V_m \sin(\omega t) & \text{for } \omega t \in (\alpha, \beta) \\ 0 & \text{for } \omega t \notin (\alpha, \beta) \end{cases}$$

$$i = \begin{cases} \frac{V_m}{R} \sin(\omega t) & \text{for } \omega t \in (\alpha, \beta) \\ 0 & \text{for } \omega t \notin (\alpha, \beta) \end{cases}$$

$$v_T = \begin{cases} 0 & \text{for } \omega t \in (\alpha, \beta) \\ V_m \sin(\omega t) & \text{for } \omega t \notin (\alpha, \beta) \end{cases}$$



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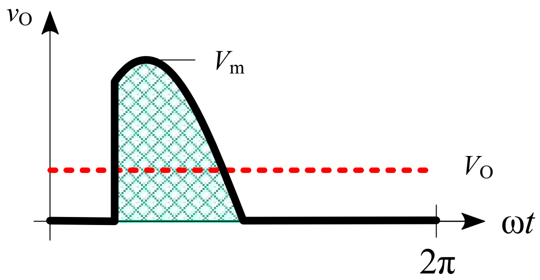
# Controlled half wave rectifiers

The average value of the output voltage $V_O$ of the half-wave thyristor rectifier with resistive load is given as

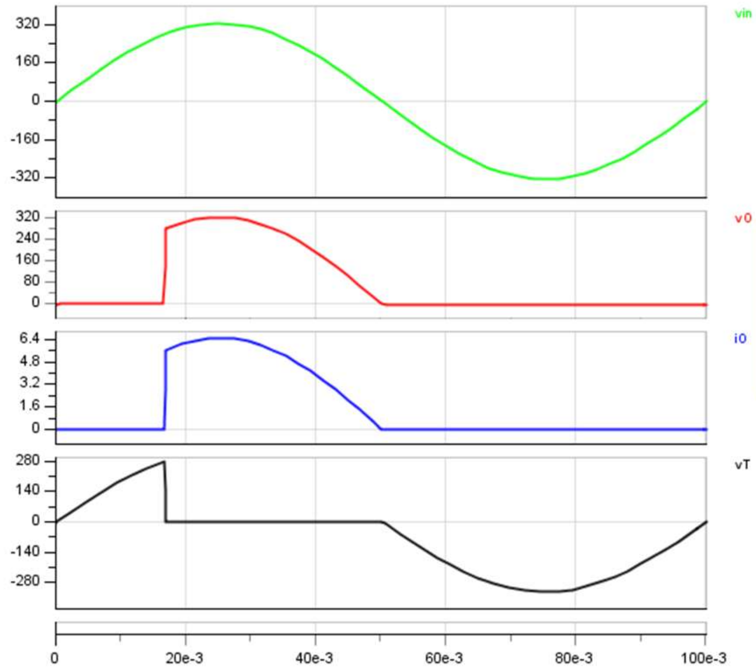



$$V_O = \frac{1}{2\pi}\int_{\alpha}^{\pi} V_m \sin(\omega t)\, d\omega t = -\frac{V_m}{2\pi}\left[\cos(\omega t)\Big|_{\alpha}^{\pi}\right] = \frac{V_m}{2\pi}\left[\left(1+\cos(\alpha)\right)\right]$$

The average value of the current

$$I_{AV} = \frac{V_O}{R} = \frac{V_m}{2\pi R}\left[\left(1+\cos(\alpha)\right)\right]$$

Example of the thyristor half-wave diode rectifier with R load ($R = 50\ \Omega$) supplied from sinusoidal voltage source $V_{inrms}$ = 230 V ($V_m = 325$ V) with a delay angle $\alpha = 60^o$.




Results

| | avg | rms |
|---|---|---|
| vO | 77.5694 | 145.719 |
| iO | 1.5514 | 2.9144 |
| vT | -77.5694 | 177.6729 |
| vG | 50.0005e-3 | 223.6079e-3 |
| vin | -89.0743e-9 | 229.8053 |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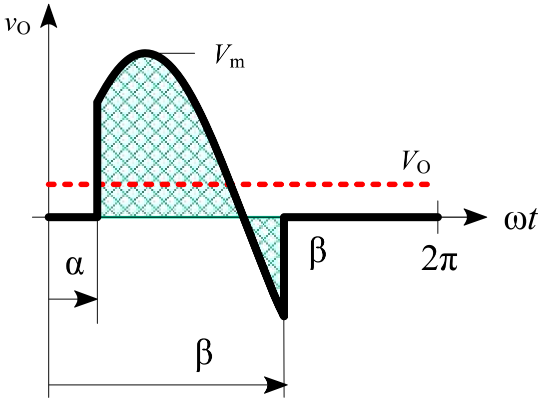
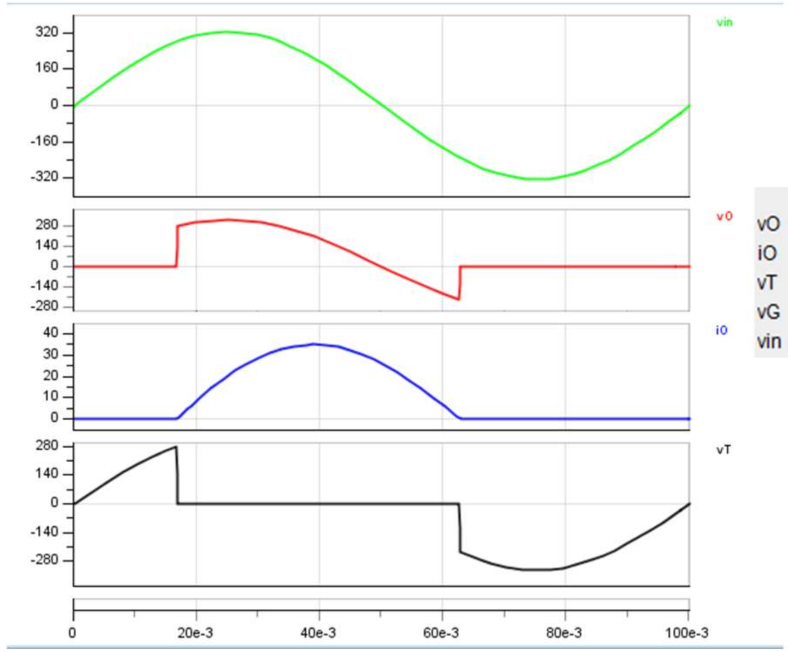
# Controlled half wave rectifiers

The average value of the output voltage $V_O$ of the half-wave thyristor rectifier with resistive-inductive load is given as




$$V_O = \frac{1}{2\pi}\int_{\alpha}^{\beta} V_m \sin(\omega t)\, d\omega t = -\frac{V_m}{2\pi}\left[\cos(\omega t)\Big|_{\alpha}^{\beta}\right] = \frac{V_m}{2\pi}\left[\left(\cos(\alpha) - \cos(\beta)\right)\right]$$

Example of the diode half-wave thyristor rectifier with RL load ($R = 6\ \Omega$, $L = 20\ \text{mH}$, $\text{tg}(\varphi) \approx 1$) supplied from sinusoidal voltage source $V_{\text{inrms}}$ = 230 V ($V_m = 325$ V) with a delay angle $\alpha = 60^{o}$.




Results

| | avg | rms |
|---|---|---|
| vO | 62.0667 | 153.8446 |
| iO | 10.3445 | 16.891 |
| vT | -62.0667 | 170.6104 |
| vG | 50.0005e-3 | 223.6079e-3 |
| vin | -89.0743e-9 | 229.8053 |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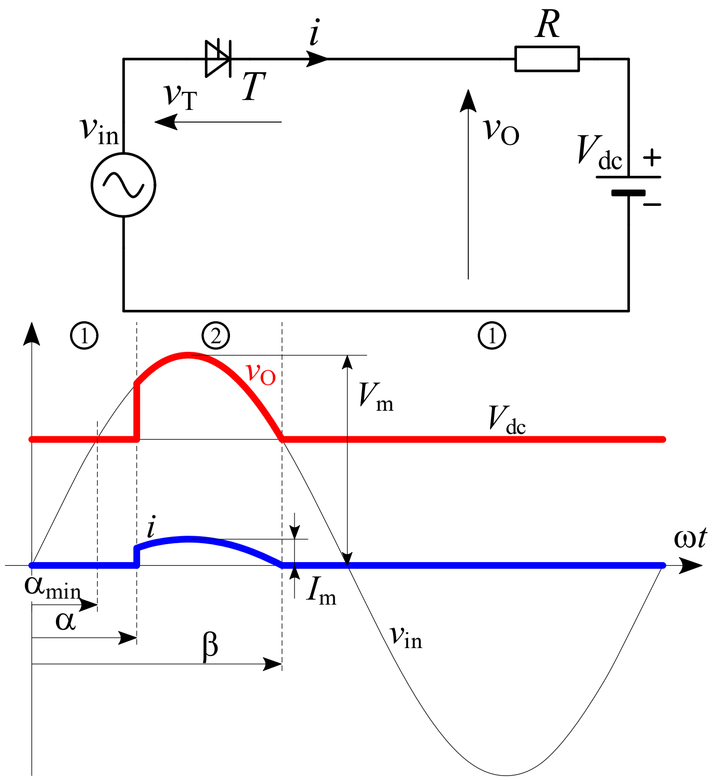
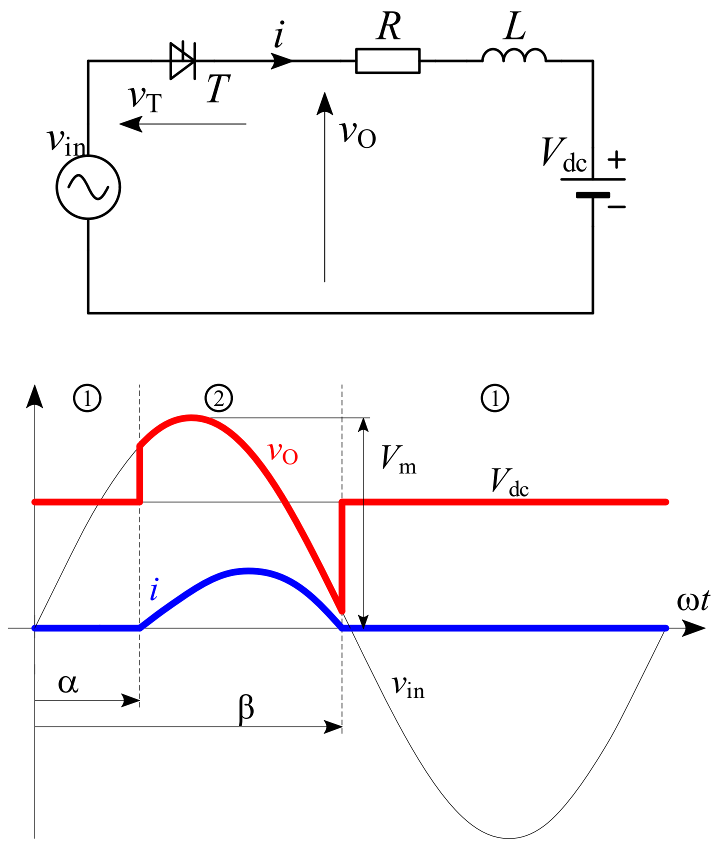
# Half-wave rectifier

Single-phase half-wave thyristor rectifier with $RV_{dc}$ load




$$V_O = \frac{V_m}{2\pi}(\cos\alpha - \cos\beta) + V_{dc}\left(1 - \frac{\beta - \alpha}{2\pi}\right)$$

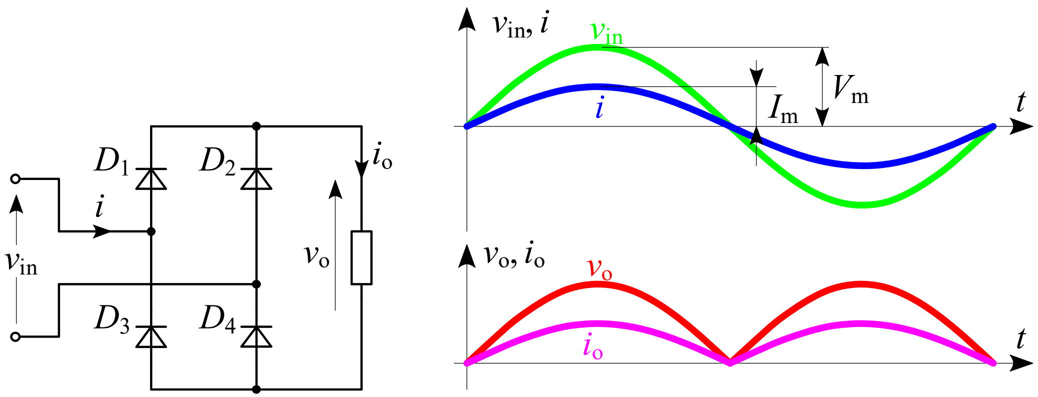
Single-phase half-wave thyristor rectifier with $RLV_{dc}$ load




$$V_O = \frac{V_m}{2\pi}(\cos\alpha - \cos\beta) + V_{dc}\left(1 - \frac{\beta - \alpha}{2\pi}\right)$$

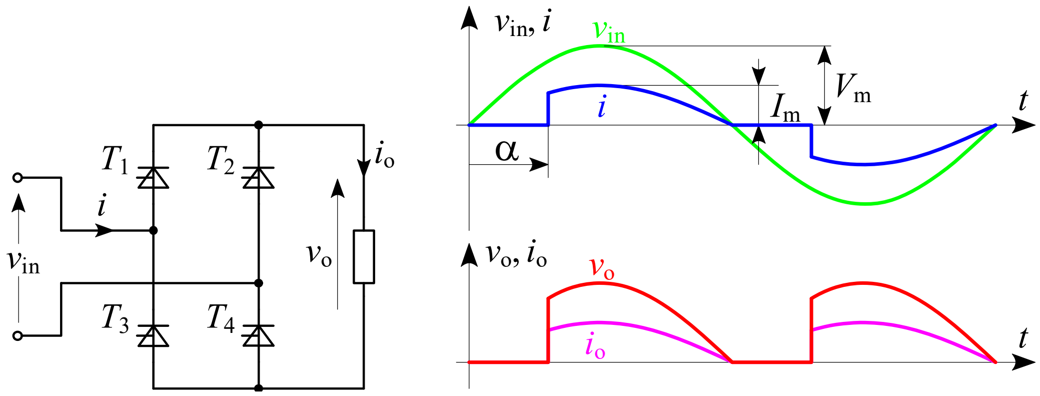
# Full-wave rectifier

Single-phase full-wave diode rectifier with R load




$$V_O = \frac{1}{\pi}\int_0^{\pi} V_m \sin(\omega t)\, d\omega t = \frac{2V_m}{\pi}$$

Single-phase full-wave thyristor rectifier with R load




$$V_O = \frac{1}{\pi}\int_{\alpha}^{\pi} V_m \sin(\omega t)\, d\omega t = \frac{V_m}{\pi}\left[\left(1+\cos(\alpha)\right)\right]$$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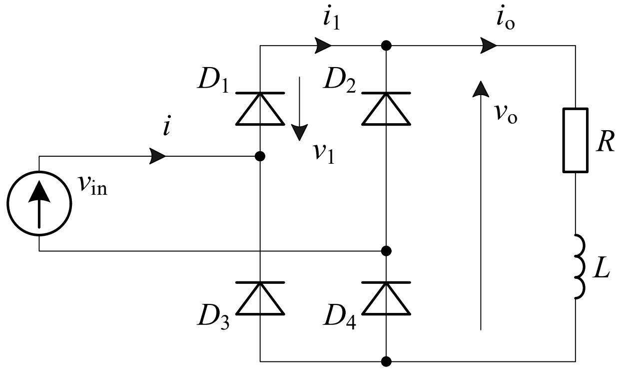
# Full-wave rectifier RL

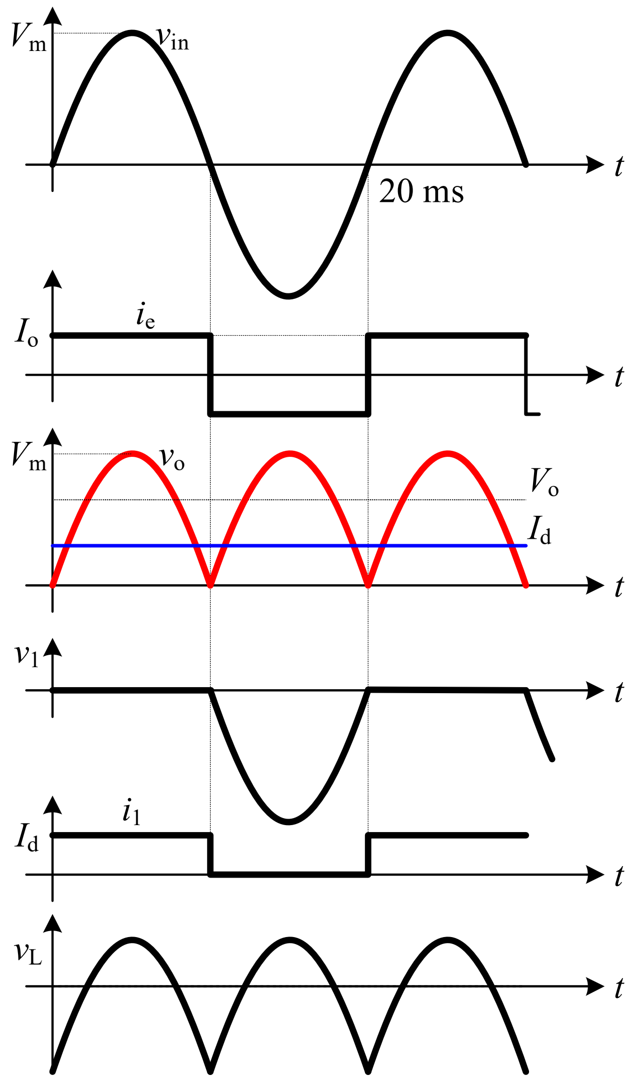
Single-phase full-wave diode rectifier with RL load




Load current could be constant with limited ripple

The full-wave bridge rectifier can operate in CCM with RL load

RL load is limited to DC drive systems, welding machines, electrolysis reactors





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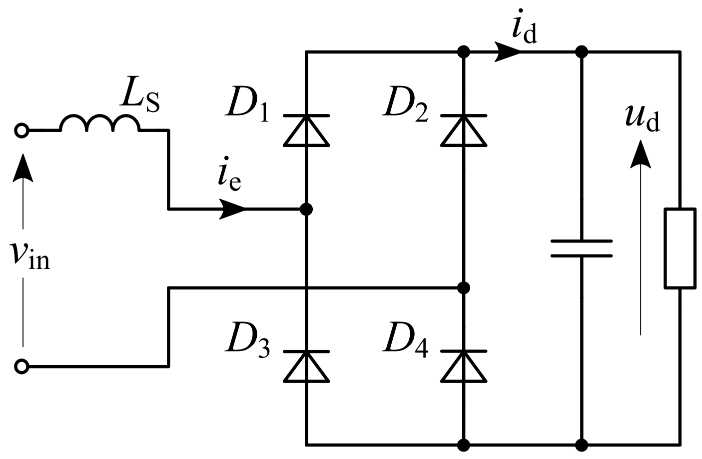
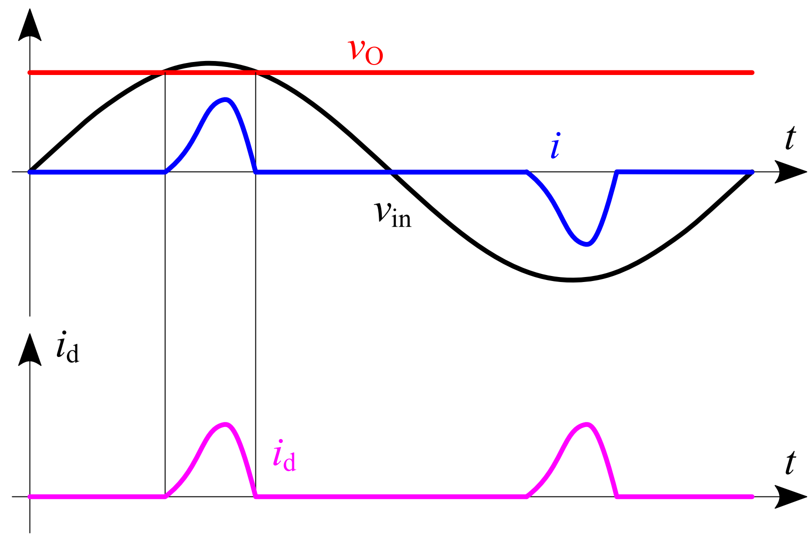
# Full-wave rectifier RC

Single-phase full-wave diode rectifier with RC load




Load current is not continuous thus converter always operates in DCM.

Conduction angles depend on RLC parameters and phase currents can be more distorted than currents of rectifiers with RL loads.

RC load is commonly used in domestic low power appliances and medium and high-power adjustable drive systems.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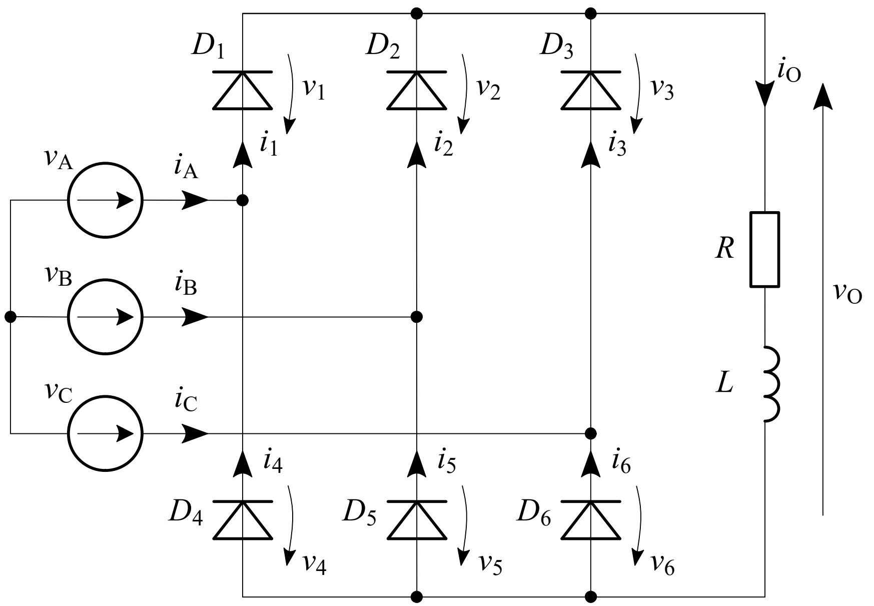
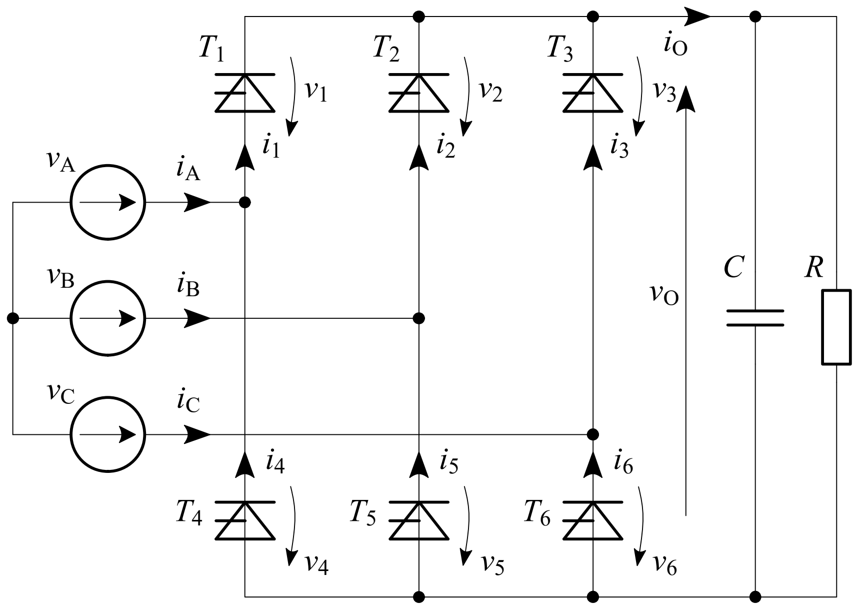
# Six-pulse rectifiers

Three-phase full-wave rectifiers are also known as six-pulse rectifiers. There are similar options of rectifiers as for half-wave rectifiers or single-phase rectifiers.

Six-pulse diode rectifier with RL load




Six-pulse thyristor rectifier with RC load



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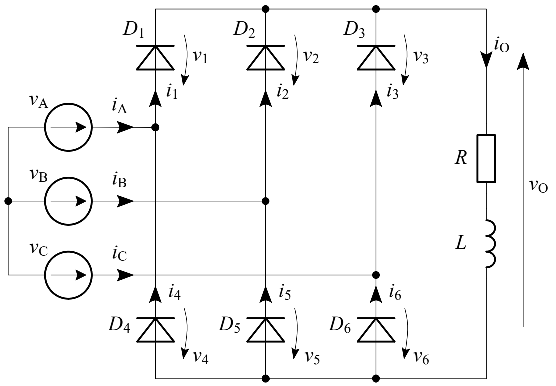
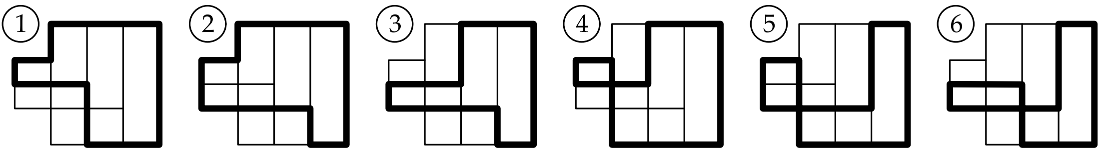
# Six-pulse diode rectifier – output voltage

The operation of six-pulse diode rectifier with continuous output current $i_O$.

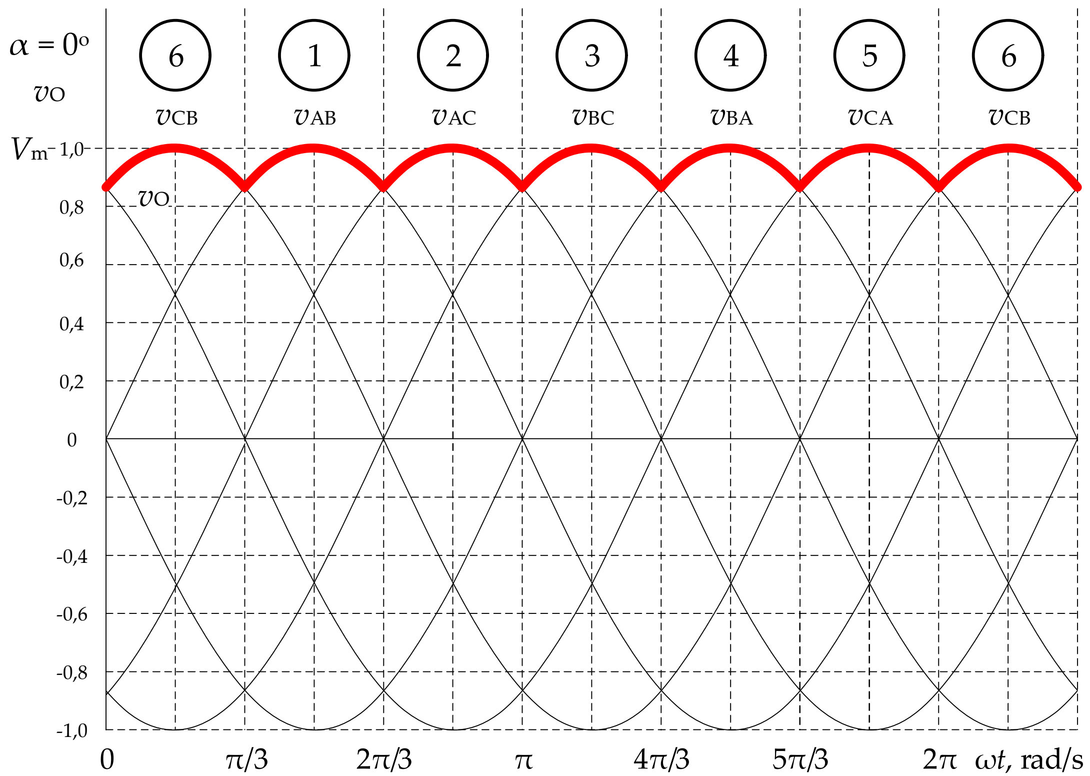



There are 6 equivalent circuits




At the same time only two diodes conducts current. One diode is from upper group ($D_1$, $D_2$, $D_3$) and the second is from lower group ($D_4$, $D_5$, $D_6$)

Output voltage

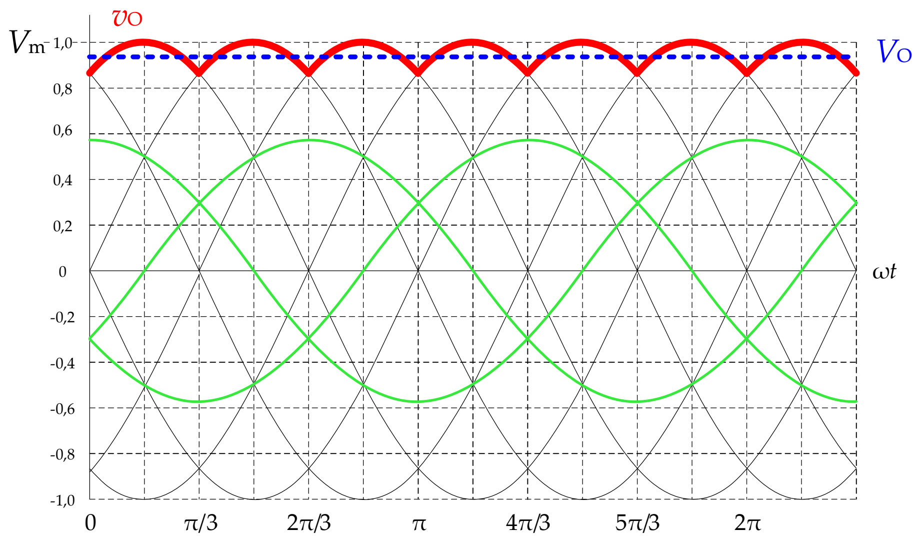


# Six-pulse diode rectifier

The average value of the output voltage of the six-pulse rectifier is calculated as

$$V_O = \frac{1}{2\pi}\cdot 6\cdot\int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} V_m \sin(\omega t)\,d\omega t = \frac{6V_m}{2\pi}\left(\cos\left(\frac{\pi}{3}\right)-\cos\left(\frac{2\pi}{3}\right)\right)=$$

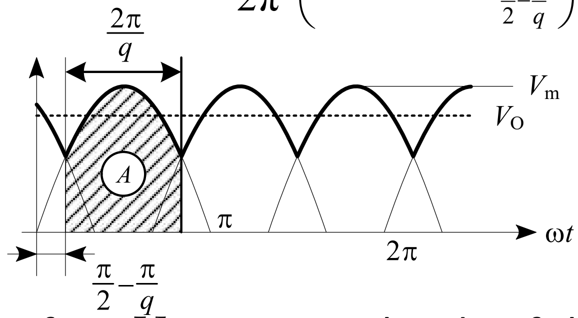
$$=\frac{6V_m}{2\pi}\left(\frac{1}{2}+\frac{1}{2}\right)=\frac{3V_m}{\pi}$$




In general $q$-pulse rectifiers the average value of the output voltage

$$V_O = \frac{q}{2\pi}\int_{\frac{\pi}{2}-\frac{\pi}{q}}^{\frac{\pi}{2}+\frac{\pi}{q}} V_m \sin(\omega t)\,d(\omega t) = \frac{qV_m}{2\pi}\left(-\cos(\omega t)\Big|_{\frac{\pi}{2}-\frac{\pi}{q}}^{\frac{\pi}{2}+\frac{\pi}{q}}\right)=$$

$$=\frac{qV_m}{\pi}\sin\left(\frac{\pi}{q}\right)$$




In the half-wave rectifiers $V_m$ is an amplitude of the phase voltage, but in full-wave rectifiers $V_m$ is an amplitude of the line-to-line voltage.

For 6-pulse rectifier supplied from three-phase grid with line-to-line voltage $V_{ll,rms} = 400\ \mathrm{V}$ the output voltage is:

$$V_O = \frac{6V_m}{\pi}\sin\left(\frac{\pi}{6}\right) \overset{V_m=\sqrt{2}\cdot 400V}{=} \frac{3\sqrt{2}\cdot 400V}{\pi} = 540.2\ \mathrm{V}$$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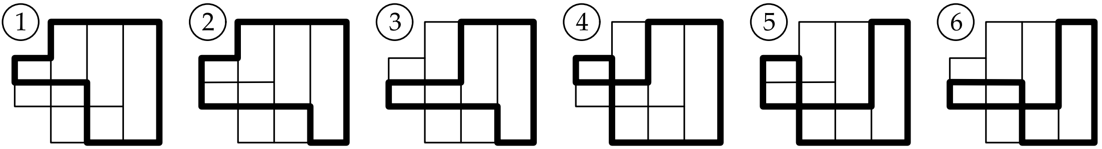
# Six-pulse diode rectifier – diode voltage

The diode voltages in different equivalent circuits are composed of different line-to-line voltages

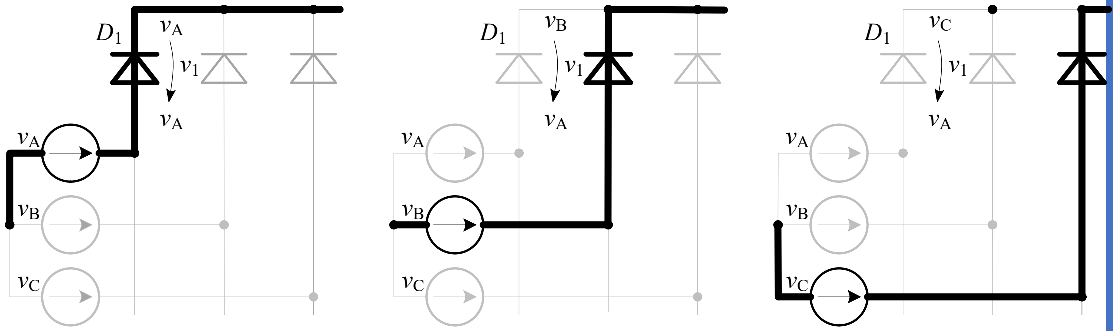



The diode $D_1$ voltage $v_1$ is a difference of the anode and the cathode voltages.

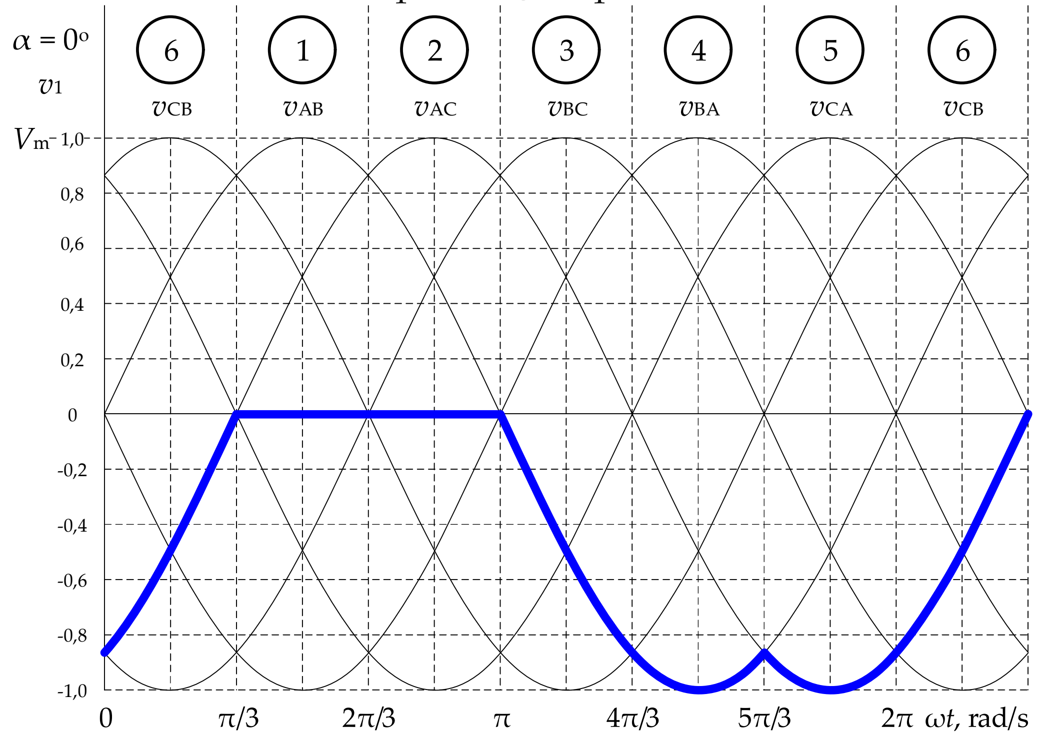
1 2
$$v_1 = v_A - v_A = 0$$

3 4
$$v_1 = v_A - v_B = v_{AB}$$

5 6
$$v_1 = v_A - v_C = v_{AC}$$



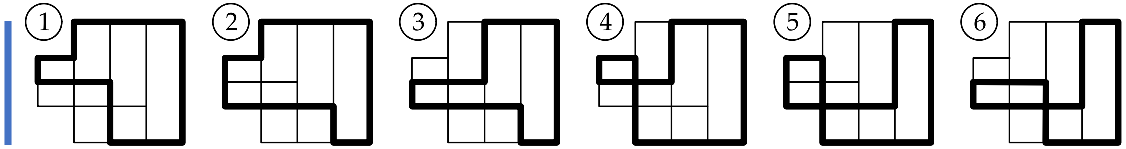

Diode $D_1$ voltage $v_1$ in CCM




Voltages across other diodes are similar to $v_1$ but phase-shifted by multiple of 60$^o$

# Six-pulse diode rectifier – current waveforms

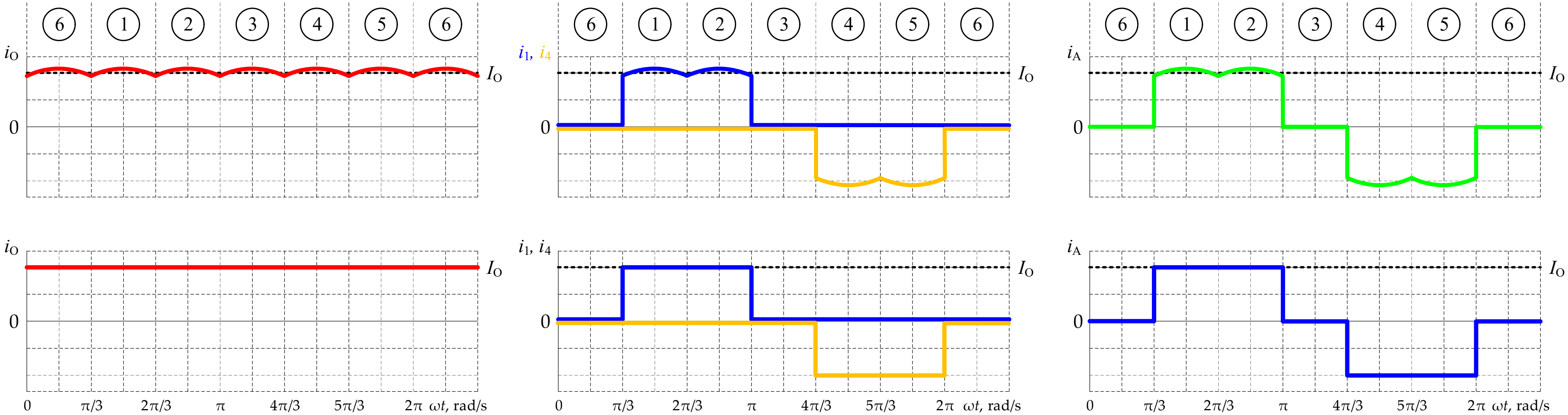
Output current in the six-pulse diode rectifier depends on the load type and its parameters.




load type | output current $i_O$ | diode $D_1$ and $D_4$ currents | phase A current $i_A$

*R*

*RL*




The rectifier currents are calculated from:

$$I_O = \frac{V_O}{R} \qquad I_{1AV} = \frac{1}{3} I_O \qquad I_{AAV} = 0$$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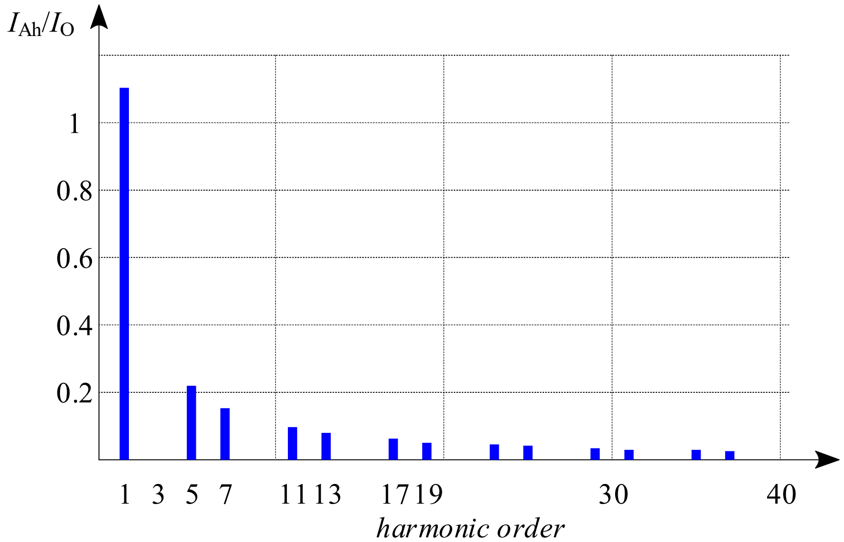
# Six-pulse diode rectifier – phase current harmonic spectrum

Phase current $i_{\mathrm{A}}$ consists of harmonic components $I_{\mathrm{A}h}$

$$I_{\mathrm{A}h} = \begin{cases} \frac{4}{\pi}\frac{I_{\mathrm{O}}}{h}\cos\left(h\frac{\pi}{6}\right) & \text{for odd harmonics} \\ 0 & \text{for even harmonics} \end{cases}$$




Only odd harmonics which are not multiples of three exist in the phase current ($h$ = k×6±1)

The amplitude of the phase current fundamental component $I_{\mathrm{A1}}$ is transferring active and reactive power.

$$I_{\mathrm{A1}} = \frac{4}{\pi} I_{\mathrm{O}} \frac{\sqrt{3}}{2} = \frac{2\sqrt{3}}{\pi} I_{\mathrm{O}}$$

The output power $P_{\mathrm{O}}$ is

$$P_{\mathrm{O}} = I_{\mathrm{O}} V_{\mathrm{O}} = \frac{3}{\pi} I_{\mathrm{O}} V_{\mathrm{m}} = \frac{3\sqrt{2}}{\pi} I_{\mathrm{O}} V_{\mathrm{ll}}$$

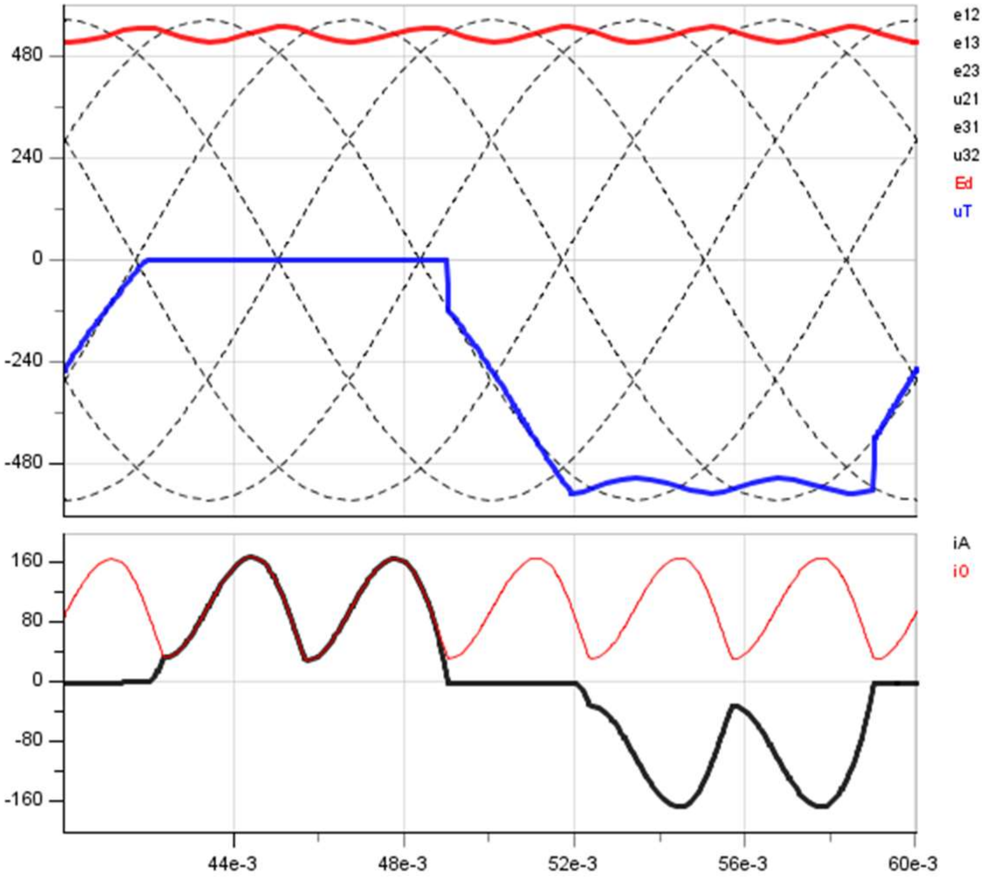
The input power $P_{\mathrm{in}}$ of the three-phase six-pulse rectifier

$$P_{\mathrm{in}} = \sqrt{3}\frac{I_{\mathrm{A1}}}{\sqrt{2}} V_{\mathrm{ll}} = \sqrt{3}\frac{2\sqrt{3}}{\sqrt{2}\pi} I_{\mathrm{O}} V_{\mathrm{ll}} = \frac{3\sqrt{2}}{\pi} I_{\mathrm{O}} V_{\mathrm{ll}}$$

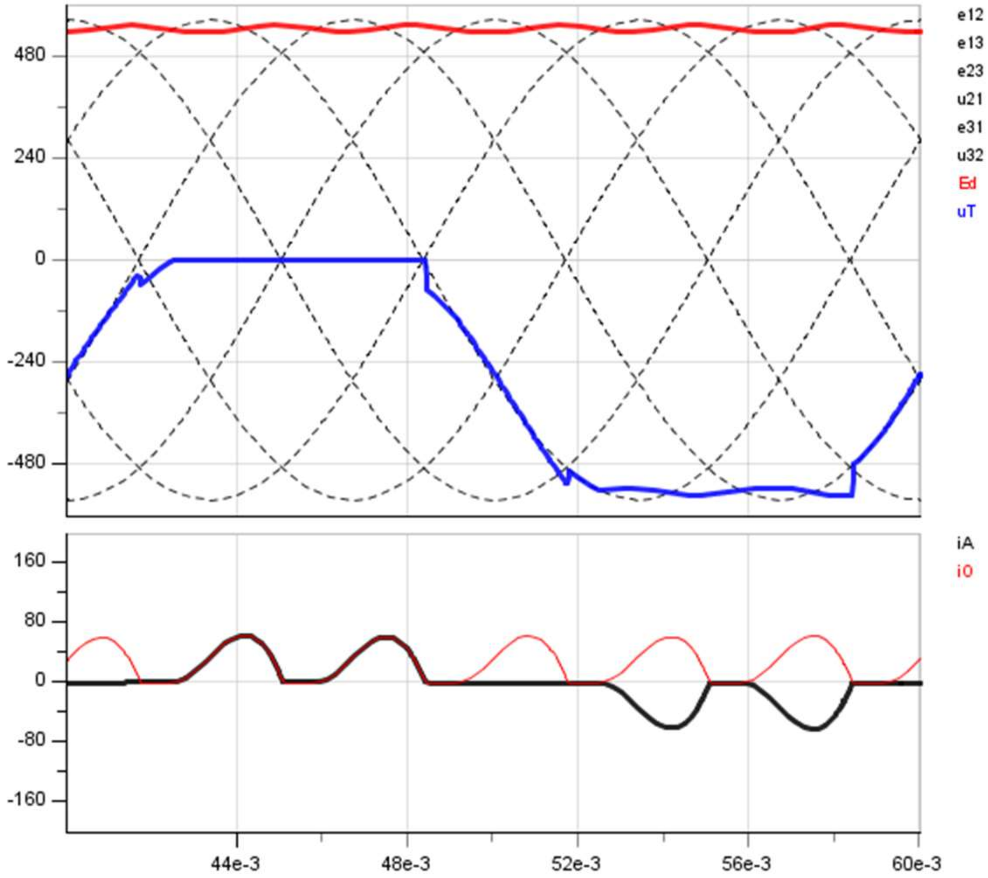
The input power $P_{\mathrm{in}}$ is equal to the output power because it is assumed that rectifier is lossless.

# Six-pulse diode rectifier – RC load

The six-pulse rectifier with RC load operating in CCM

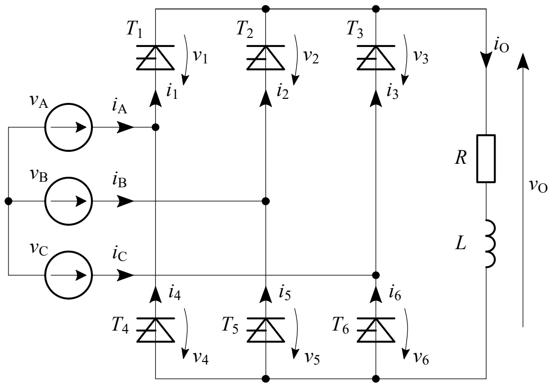



The six-pulse rectifier with RC load operating in DCM



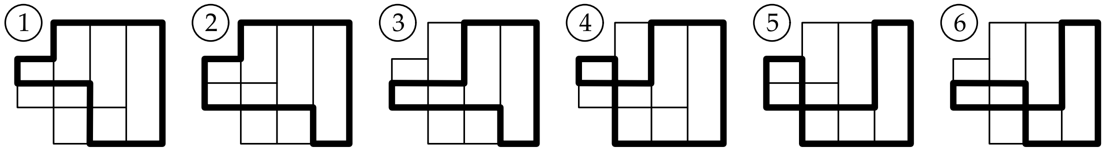
# Six-pulse thyristor rectifier – output voltage

The operation of six-pulse thyristor rectifier with continuous output current $i_O$.

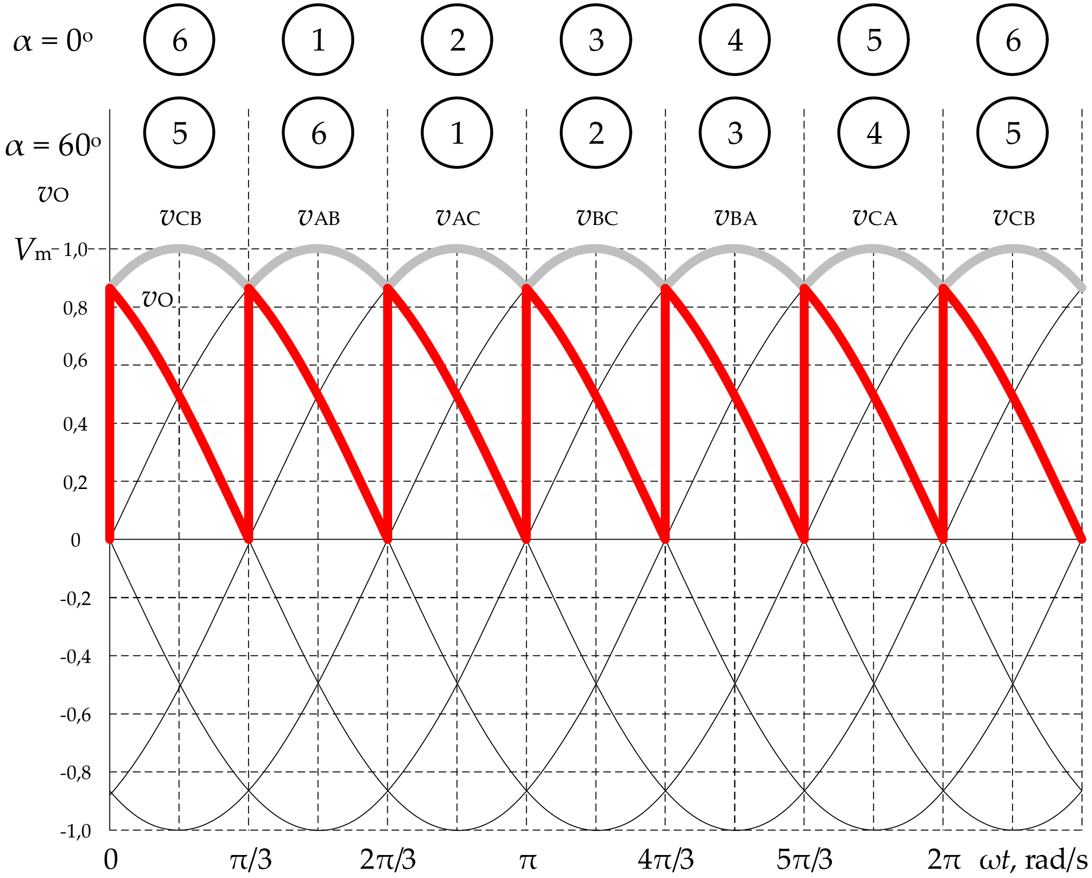



Thyristors are turned on with a delay angle α.

All 6 equivalent circuits are the same as in the six-pulse diode rectifier




Output voltage





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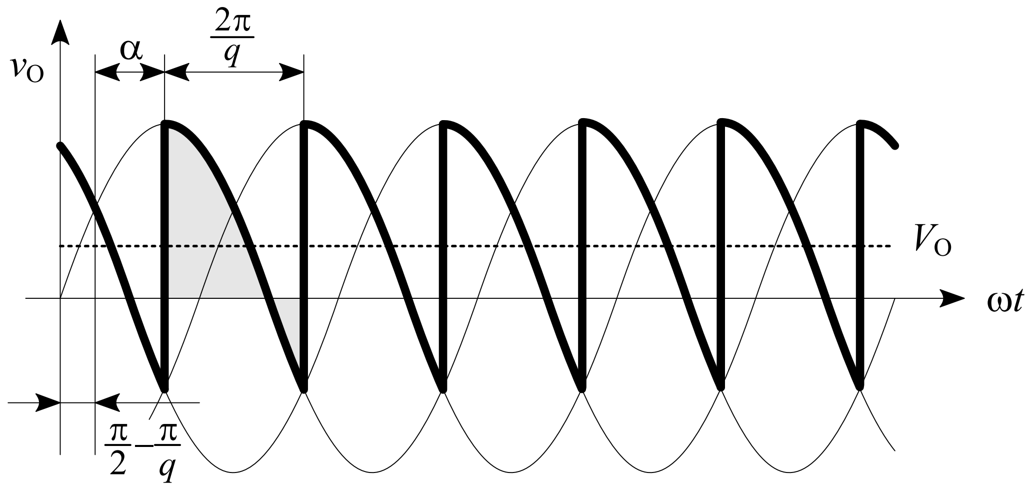
# Six-pulse thyristor rectifier – output voltage

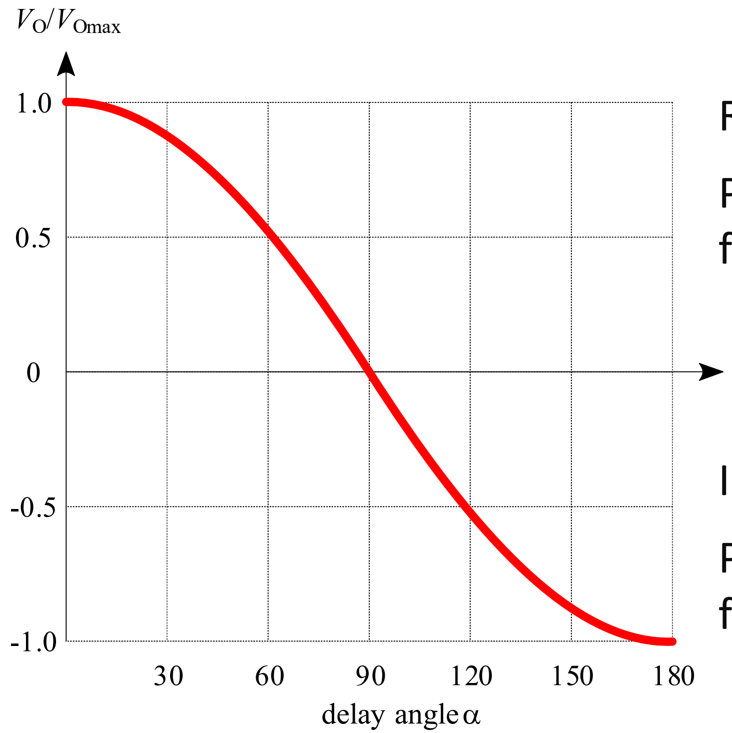
The average value of the output voltage $V_{\mathrm{O}}$ depends on the delay angle $\alpha$.




$$V_{\mathrm{O}} = \frac{q}{2\pi} \int_{\frac{\pi}{2}-\frac{\pi}{q}+\alpha}^{\frac{\pi}{2}+\frac{\pi}{q}+\alpha} V_{\mathrm{m}} \sin(\omega t)\,\mathrm{d}(\omega t) = \frac{qV_{\mathrm{m}}}{2\pi}\left( -\cos(\omega t)\Big|_{\frac{\pi}{2}-\frac{\pi}{q}+\alpha}^{\frac{\pi}{2}+\frac{\pi}{q}+\alpha} \right) =$$

$$= \frac{qV_{\mathrm{m}}}{\pi} \sin\left(\frac{\pi}{q}\right) \cos(\alpha) = V_{\mathrm{Omax}} \cos(\alpha)$$

The output current flows only in one direction but the average value of the output voltage $V_{\mathrm{O}}$ can be with both polarities.




Rectifier mode

Power transfer from AC to DC

Inverter mode

Power transfer from DC to AC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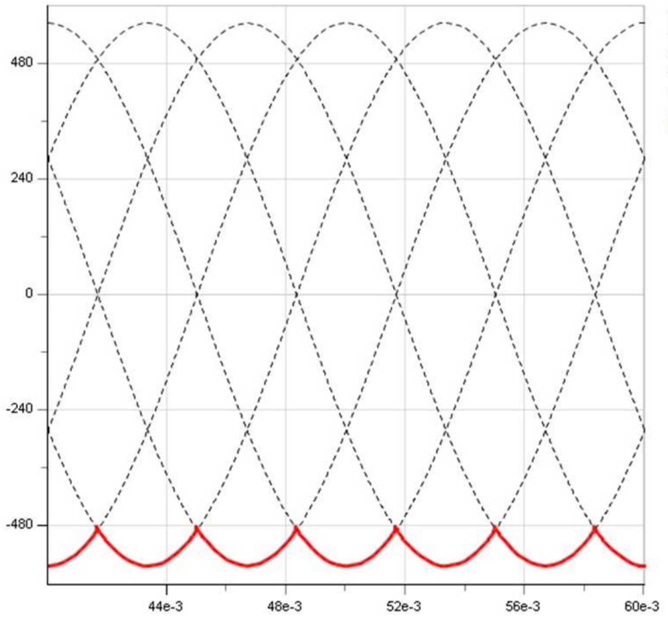
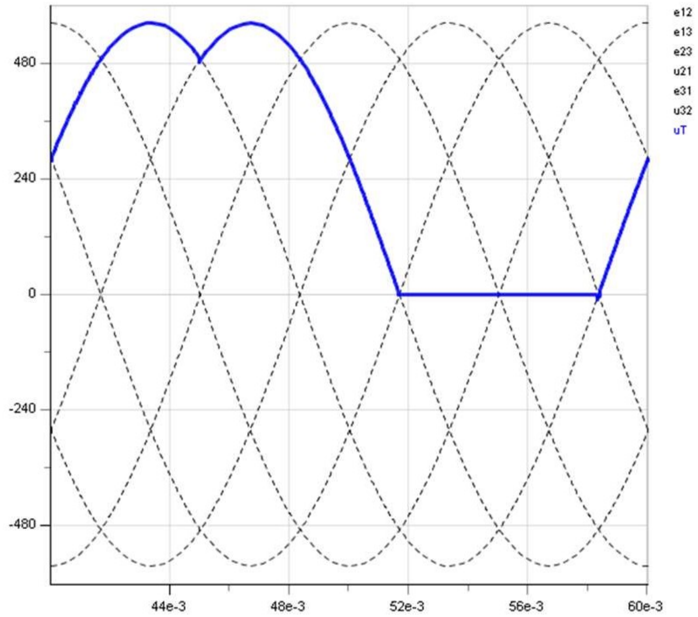
# Six-pulse thyristor rectifier – constant output current

The delay angle $\alpha = 180^\circ$

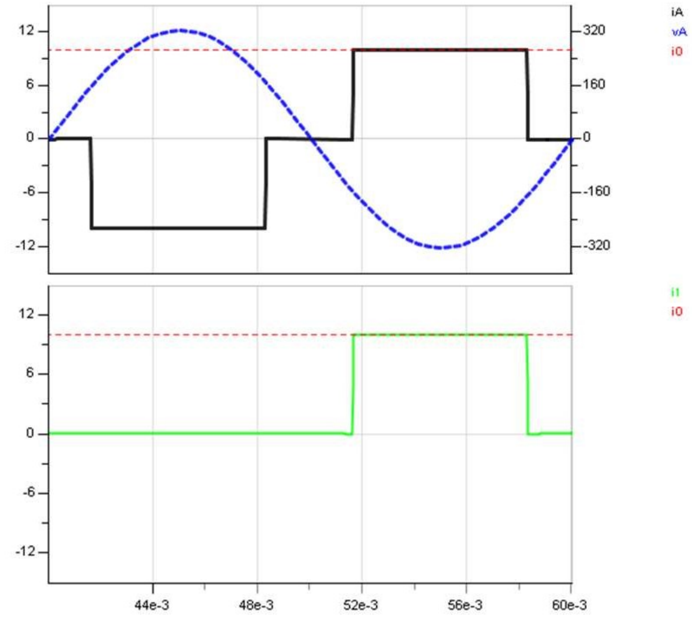
The output voltage




The thyristor $T_1$ voltage




The phase and thyristor currents



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# Six-pulse thyristor rectifier – the operation with high $\alpha$

The load which acts as a constant output current source can occur only in specific applications:

- Electrical drive systems with dc machines
- **H**igh **V**oltage **D**irect **C**urrent Transmission Systems HVDC.

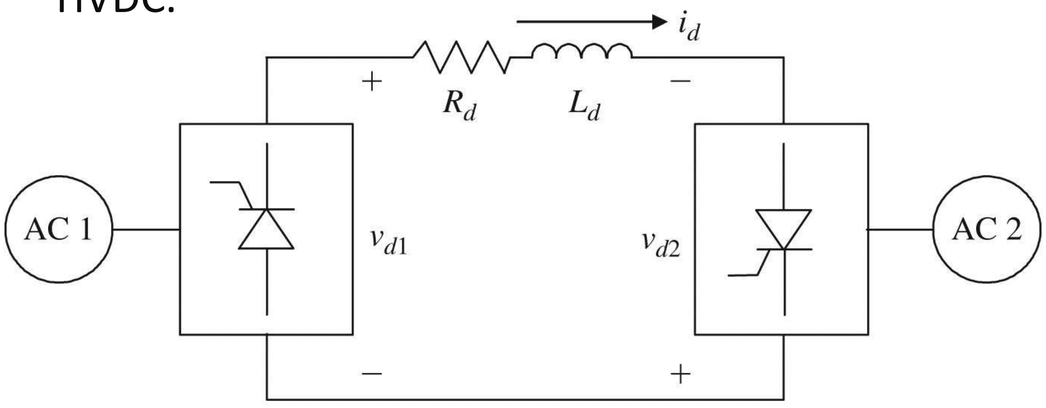
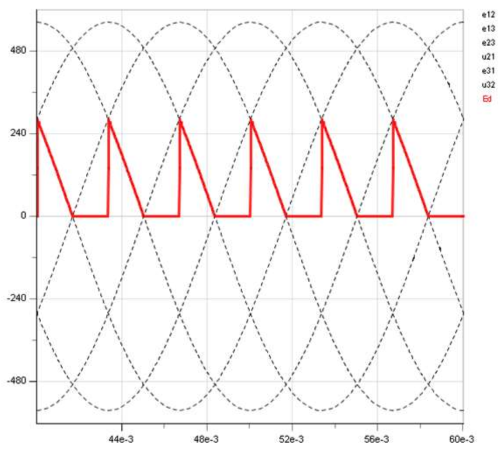



Figure 13.14

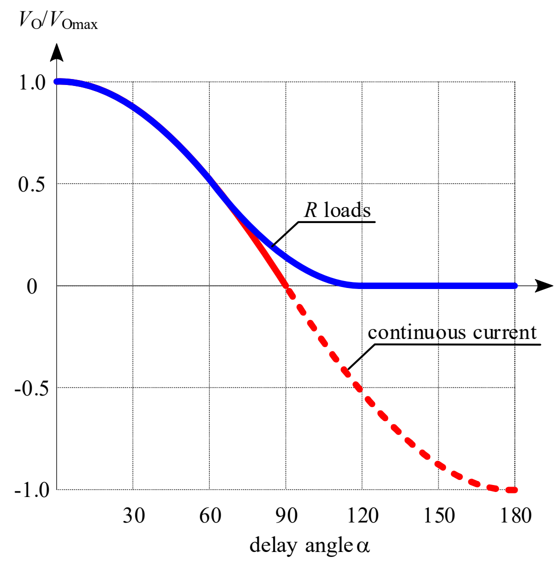

The power flow depends on voltage polarisation only because the current is unidirectional.

The output voltage characteristic $V_O = f(\alpha) = V_{Omax}\cos\alpha$ is only for CCM operation. For converters with R or RL loads DCM when $\alpha > 60^o$

The output voltage for R load with $\alpha = 90^o$




The output voltage characteristic





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# Six-pulse thyristor rectifier – commutation

Why are thyristors or diodes switched off when they were conducting?

A: Other diodes from the same group form a fast-passing short-circuit lasting until the current in new diode is equal $i_O$ (the current in previously conducting diode reaches zero).

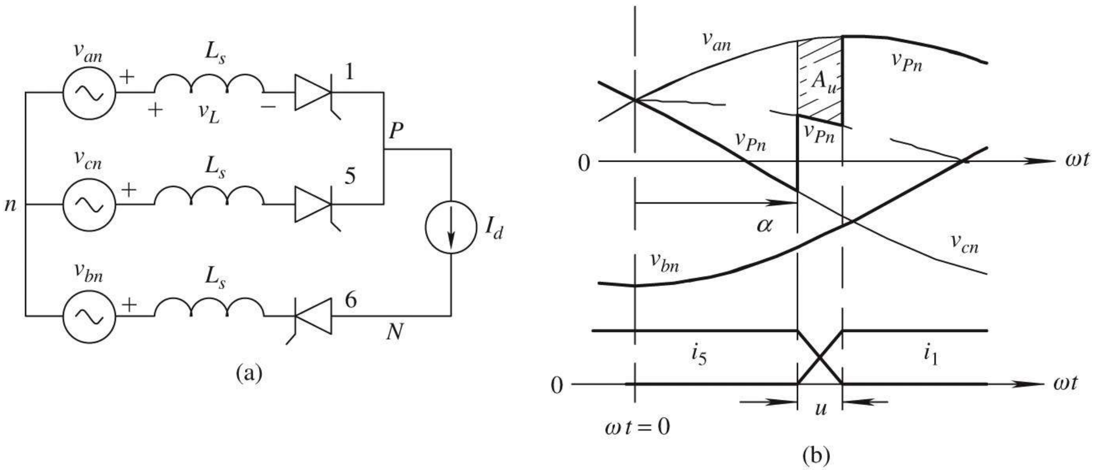
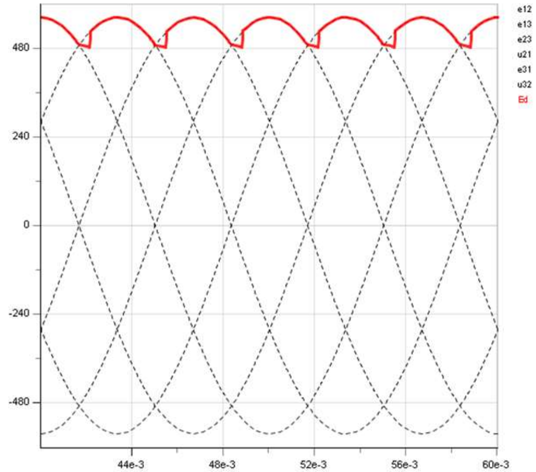


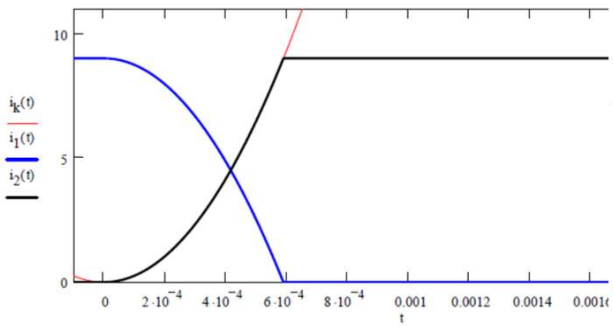

Figure 13.12

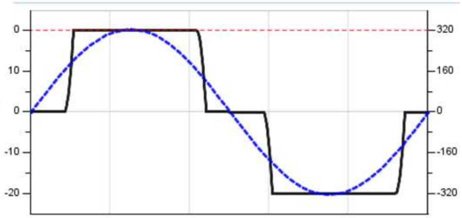


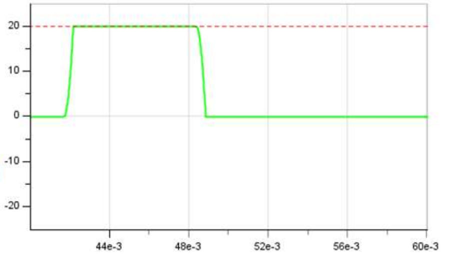


$$V_O = \frac{3}{\pi} V_m - \Delta V_{com}$$

$$\Delta V_{com} = \frac{3}{\pi} I_d L_S$$









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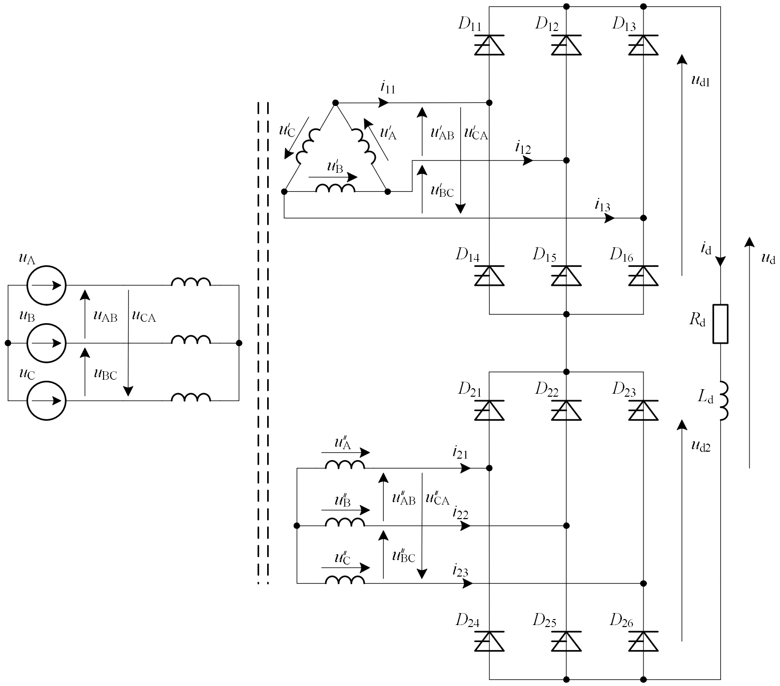
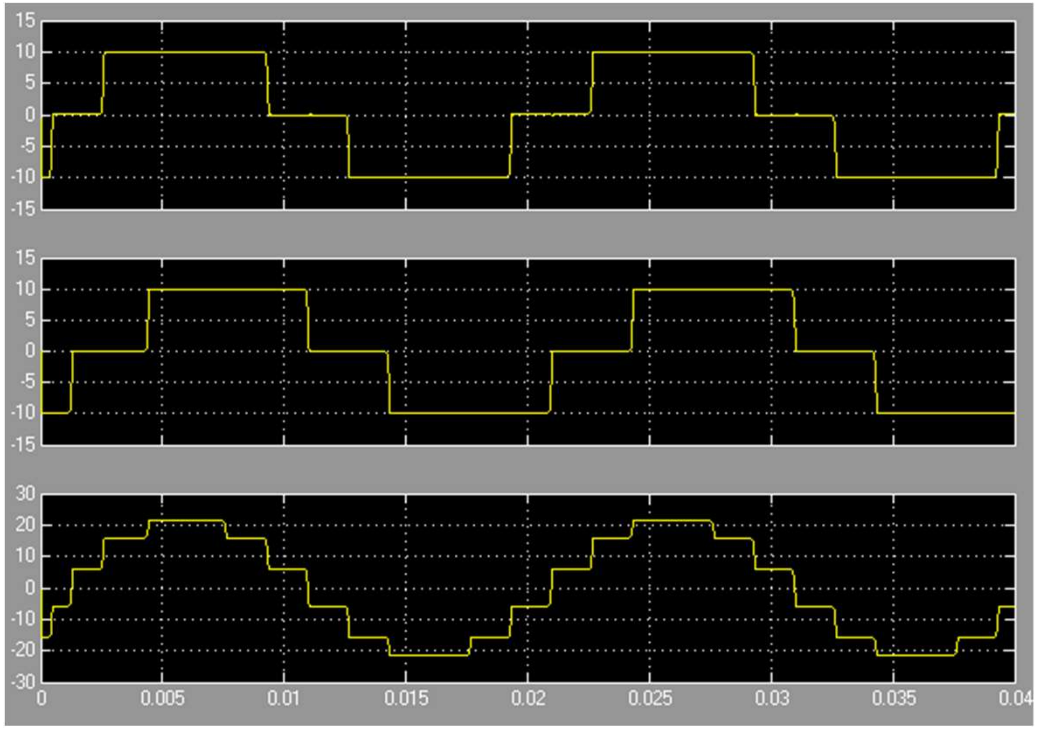
# Multipulse rectifiers

Twelve pulse rectifier is constructed from two six-pulse rectifiers which outputs are connected in series, but supply voltages are separated by using transformers producing phase shifted voltage by 30$^o$





In the phase current of the 12-pulse rectifier the following harmonics exist $h$ = 11, 13, 23, 25…
($h$ = k×12±1)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