YMX0233 Mathematical Analysis II spring semester of 2023/24 academic year

3.0 credits 2 1-0-1 E A
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Brief description of the course: Functions of several variables. Limit and continuity of functions of two variables. Partial derivatives on functions of several variables. Directional derivative. Gradient of scalar field. Divergence and curl of vector field. Local extrema of functions of two variables. Evaluation of double and triple integrals. Line integral of scalar field and line integral of vector field. Number series, convergence and divergence of a series. Power series.

Learning outcomes. Student having passed the course:

- defines the function of several variables;
- draws the graph of function of two variables;
- evaluates the limits of functions of two variables;
- is able to find partial derivatives of functions of several variables;
- evaluates directional derivatives;
- knows the concept of gradient, divergence and curl;
- defines and knows the properties of double and triple integral and evaluates double and triple integrals;
- evaluates line integrals of scalar field and vector field;
- knows what is a series and applies convergence tests to determine whether a series is convergent or divergent.
- knows the power series, Taylor series and Maclaurin series.

Topics of the course.

Following items are the subsections in theoretical material but also the questions in examination.

- 1. Functions of several variables. Graph of function of two variables.
- 2. Partial increments and total increment of function several variables.
- 3. Limit of function of two variables.
- 4. Continuity of function of two variables.
- 5. Partial derivatives of functions of several variables.
- 6. Total increment and total differential.
- 7. Partial derivatives of functions given implicitly.
- 8. Partial derivatives of higher order.
- 9. Directional derivative.
- 10. Gradient.
- 11. Divergence and curl.
- 12. Local extrema of function of two variables.
- 13. Definition and properties of double integral.
- 14. Iterated integrals. Evaluation of double integral.
- 15. Evaluation of volumes and areas by double integrals.
- 16. Definition and properties of triple integral.
- 17. Evaluation of triple integral.
- 18. Line integral with respect to arc length.
- 19. Evaluation of line integral with respect to arc length.
- 20. Line integral with respect to coordinates.
- 21. Evaluation of line integral with respect to coordinates.
- 22. Green's formula.

- 23. Path independence of line integral.
- 24. Number series, partial sums and convergence. Necessary condition for convergence.
- 25. Comparison test of positive number series.
- 26. D'Alembert's and Cauchy tests.
- 27. Integral test.
- 28. Alternating series. Leibnitz's test.
- 29. Series with whatever signs. Absolute and conditional convergence.
- 30. Power series. Abel's theorem.
- 31. Taylor's series. Maclaurin series.

Literature

Trench. W.F. Introduction to real analysis, Prentice Hall, 2003. The lectures by topics are in the Moodle.

Spring semester lasts 16 weeks. Every odd week there take place lectures 2 academic hours and every even week exercises 2 academic hours. In lectures there will be given theoretical material with examples an in exercises the theory will be applied to solve problems. The semester ends with an examination in writing during the examination session.

Standard exercises to prepare for the exercises tests are in Moodle under the title "Individual exercises".

Grading.

To get the credits one has to pass the examination.

Prerequisites for the examination: the student has to pass two exercises tests. First test will take place in week 10 and will be assessed in 100-points system. To pass the test the student has to get at least 51 points. Second exercises test will take place during consultation hours in week 16 or during examination session. Maximal amount of points for test no 2 is 60. To pass second test the student must get at least 31 points.

In the examination there will be written only the theory. An examination ticket contains 4 questions about various items of theory. Every theoretical question gives 20 points. Maximal amount of points for the exam will be 80.

Total amount of points for semester is 240.

The student will be graded according to the following scale.

 $217 \dots \, 240$ points "5" excellent

193 ... 216 points "4" very good

169 ... 192 points "3" good 145 ... 168 points "2" satisfactory

121 ... 144 points "1" poor

... 120 points "0" failed