FLUID MECHANICS 1
LECTURE 2

Tamas Pardy
TalTech Lab-on-a-Chip

13.01.2022

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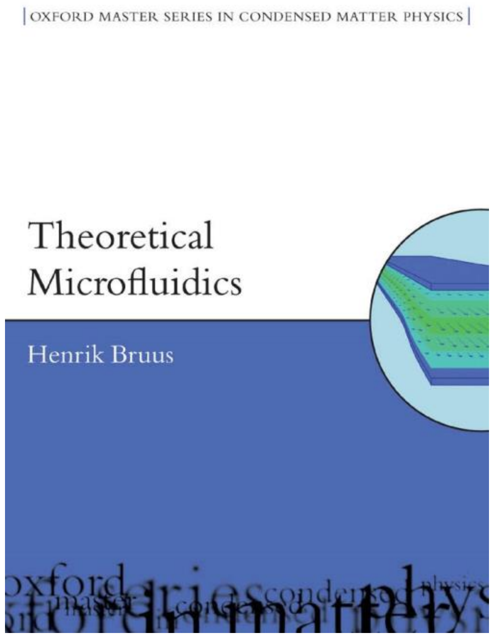
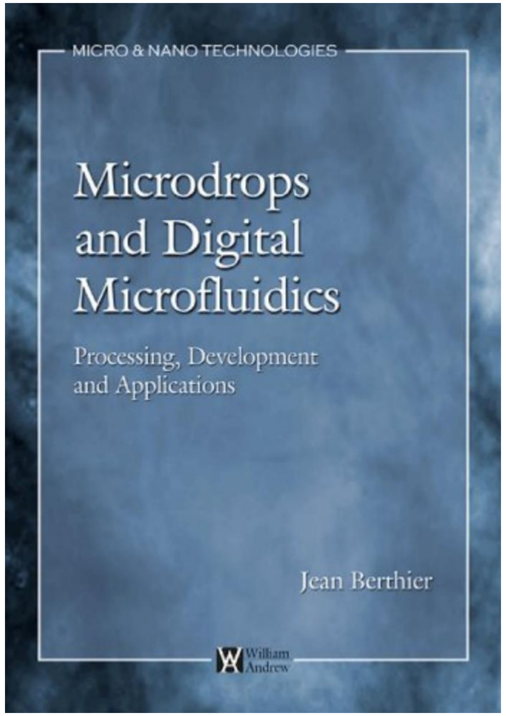
ADDITIONAL MATERIALS

- Theoretical Microfluidics
- **Henrik Bruus**
- ISBN: 9780199235087

- Microdrops and Digital Microfluidics
- **J. Berthier (editor)**
- ISBN: 978-0-8155-1544-9

- **Additional materials:**
  - Nguyen, Nam-Trung, Steven T. Wereley, and Seyed Ali Mousavi Shaegh. Fundamentals and applications of microfluidics. Artech house, 2019.
  - https://www.youtube.com/watch?v=clVwKynHpB0
  - IEE1720 lecture 1 by Natalja Sleptsuk
  - Tamas Pardy, “Development of a silicon-based digital microfluidics system for sample preparation tasks,” Master Thesis, Faculty of IT, Peter Pazmany Catholic University, Budapest, 2013. (special thanks to the HAS and PPCU FIT)
  - Utada, A. S., et al. "Dripping, jetting, drops, and wetting: The magic of microfluidics." *Mrs Bulletin* 32.9 (2007): 702-708.





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OVERVIEW

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SCALING LAWS

- Context
- Scaling of forces
- Effect of forces on microscale flows

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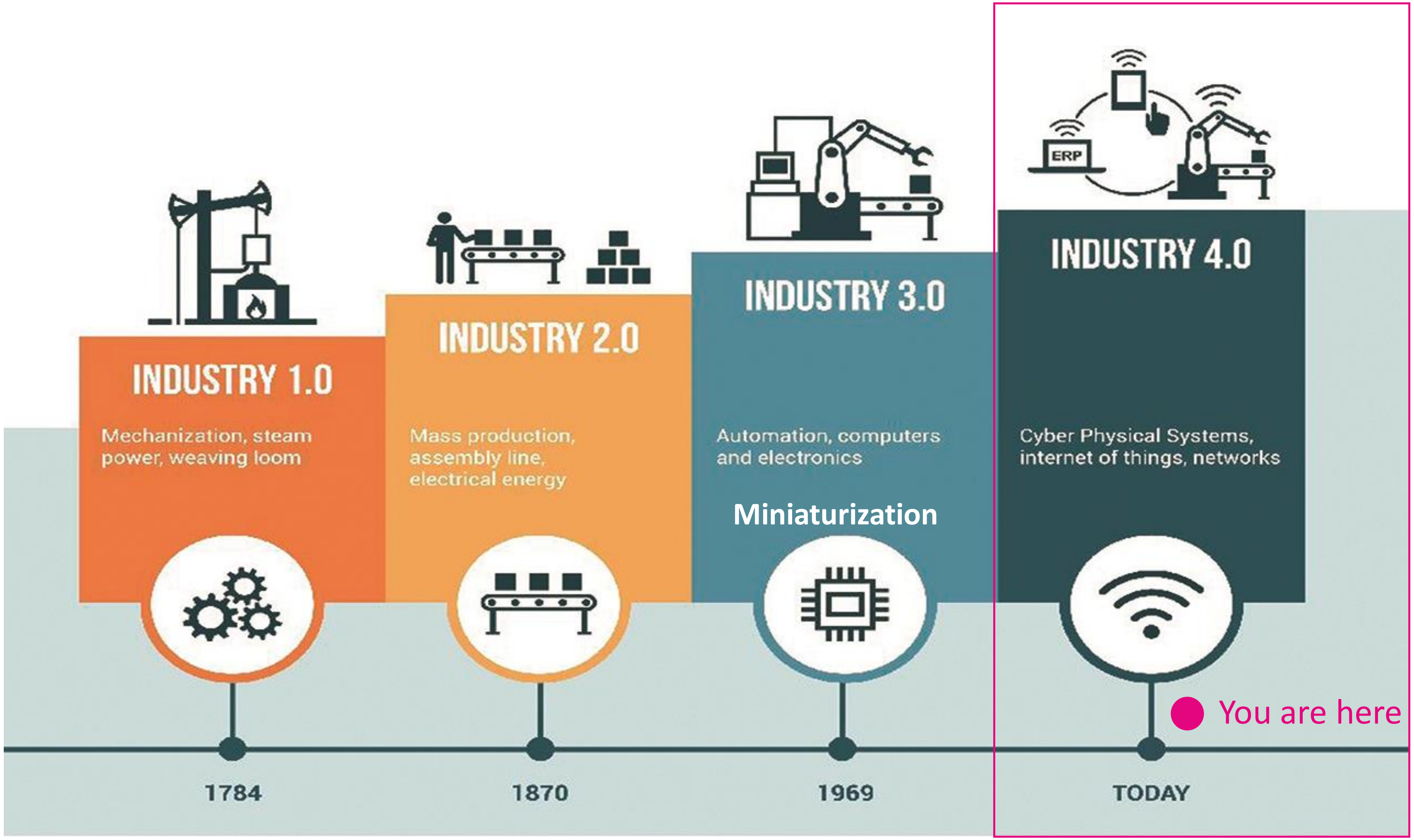
TO PUT THINGS INTO PERSPECTIVE



Industry 5.0

- Human-centric
- Sustainable
- Resilient

Near future

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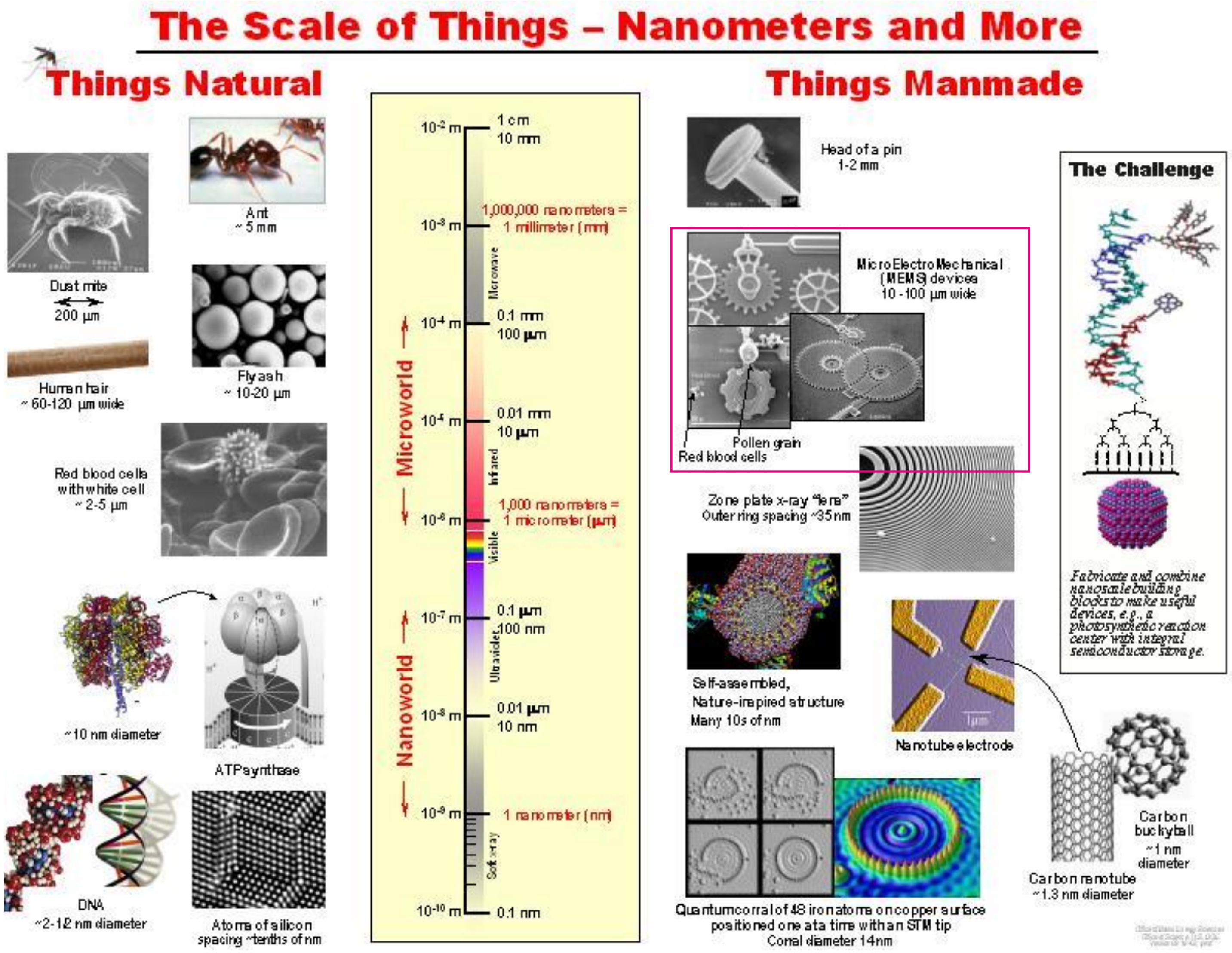
https://www.cadm.com/the-fourth-industrial-revolution/

MINIATURIZATION AND SCALING



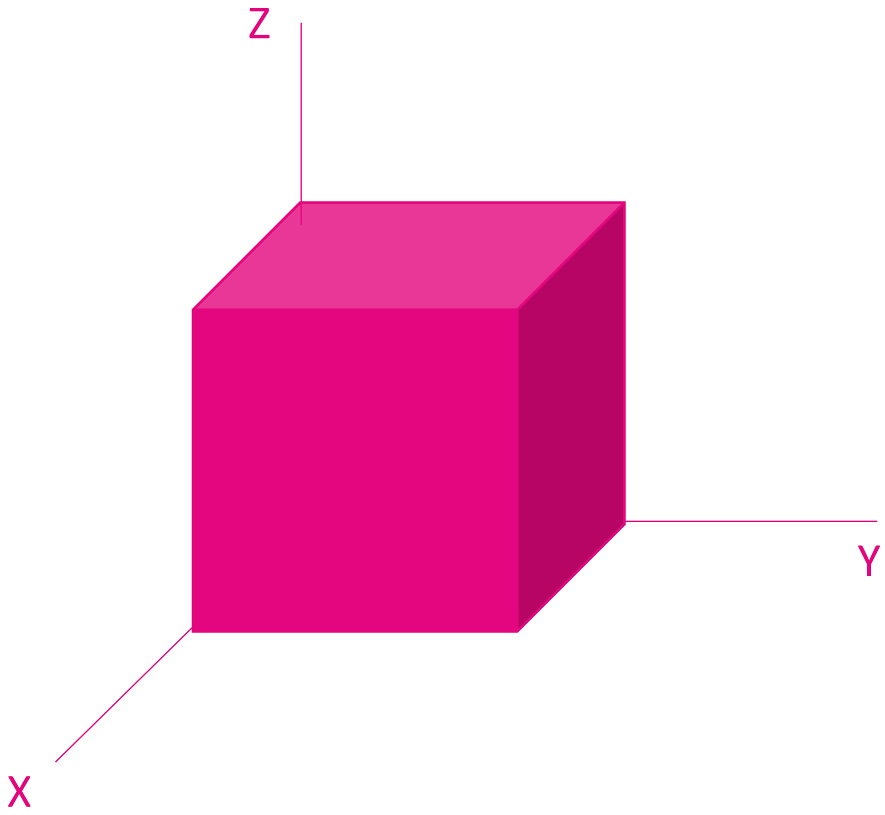
During the 3rd industrial revolution…

Miniaturization
↓
Microelectronics
↓
MEMS
↓
BioMEMS
↓
Lab-on-a-Chip

SCALING LAWS: LENGTH DIMENSION

- Length dimension: L [m]
- $Volume = L_x L_y L_z$

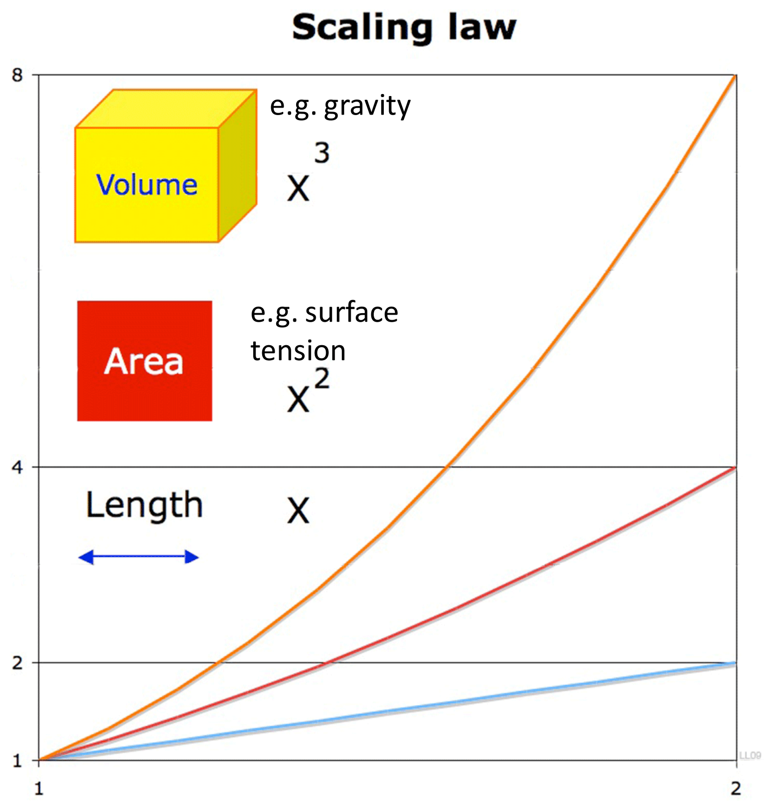
- Effects studied
  - Mechanical
  - Electromagnetic
  - Thermal



SCALING LAWS

- Scaling-down can cause unforeseen effects…
  laws are the same, but they "are enforced with different strength"
  ☺
- *Note: fluid = liquid / gas*

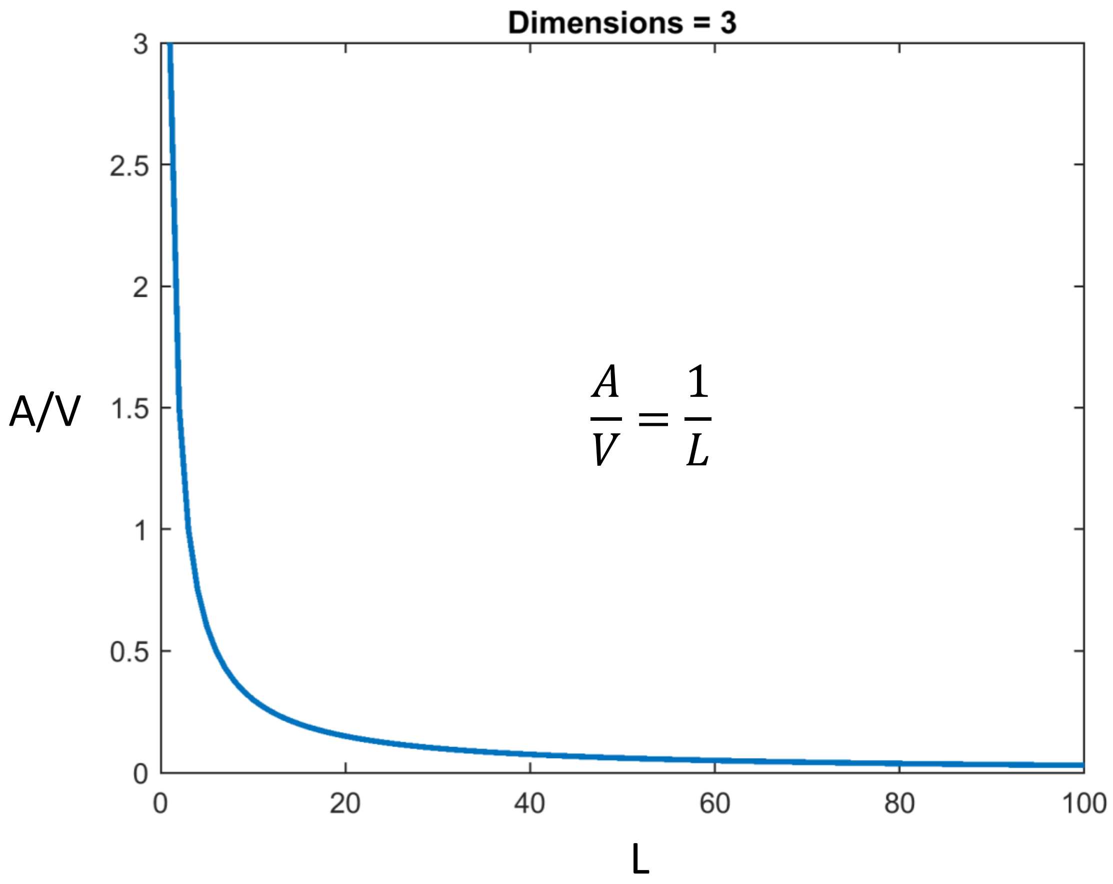
- Microscale fluidics:
  - **Gravity is negligible** compared to surface tension, friction (fluidic resistance)
  - **Co-flowing fluids don't mix** apart from diffusion through interfaces



SURFACE AREA TO VOLUME RATIO (SAV)

- $Volume: V = L_x \cdot L_y \cdot L_z = L^3$
- $Surface: A = L^2;$

- Microscale: high SAV
  - (+) At microscale, surface tension dominates, inertial and body forces less prominent ➔ droplet microfluidics
  - (-) surface adsorption increases ➔ clogging



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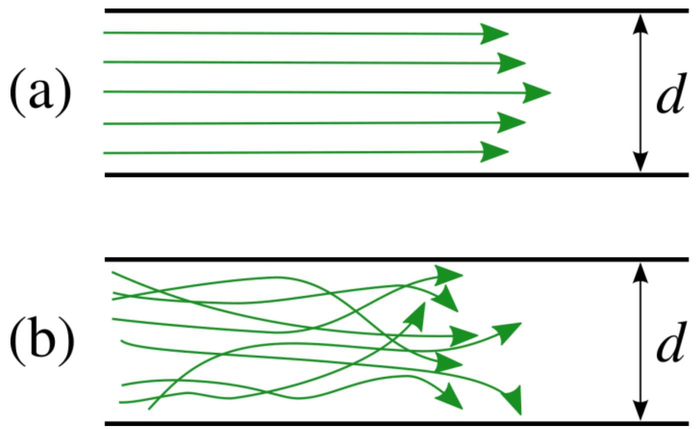
https://en.wikipedia.org/wiki/Surface-area-to-volume_ratio#/media/File:SAV_n3.png

Understanding Laminar and Turbulent Flow - YouTube

SCALING FLOWS

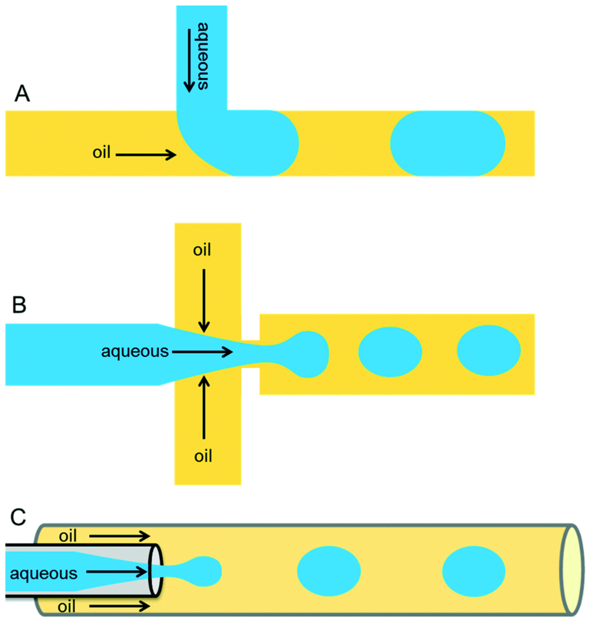
- Macroscale: Laminar, turbulent
- **Microscale: Laminar**

- **Time-dependence**
  (based on whether/how flow depth changes in time): steady/unsteady, whether flow depth changes over time
- **Space-dependence**
  based on whether/how flow depth changes in space): uniform, varied, continuous, spatially varied (discontinuous)



Laminar

Turbulent



Discrete
Discountinuous
Multi-phase

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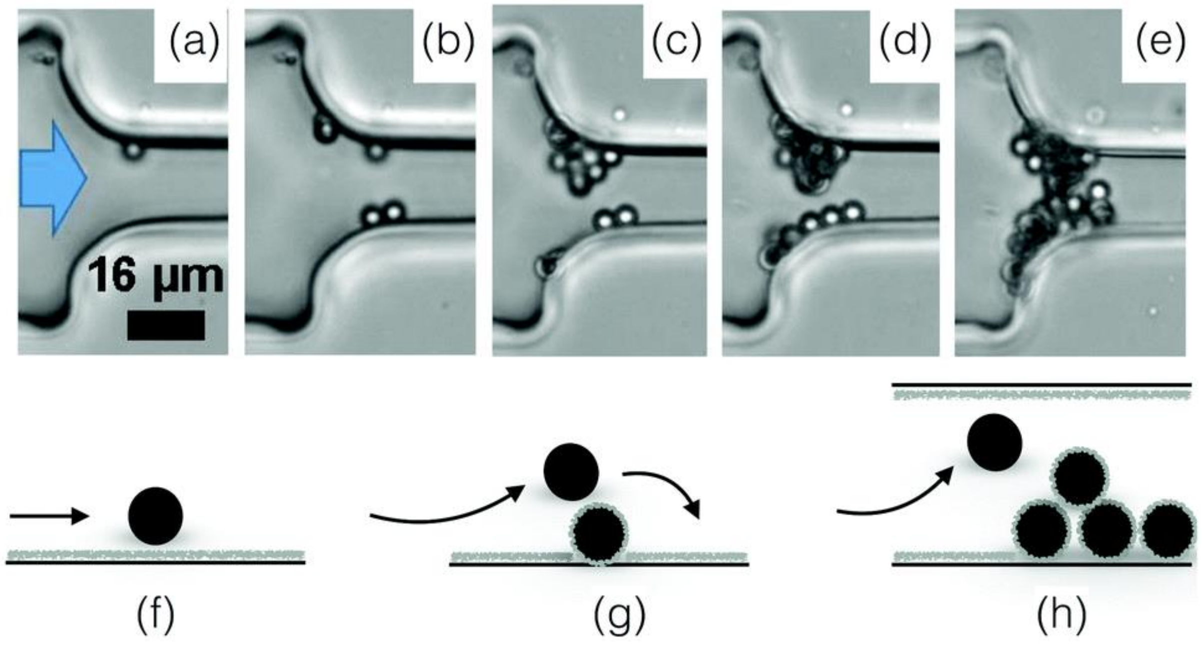
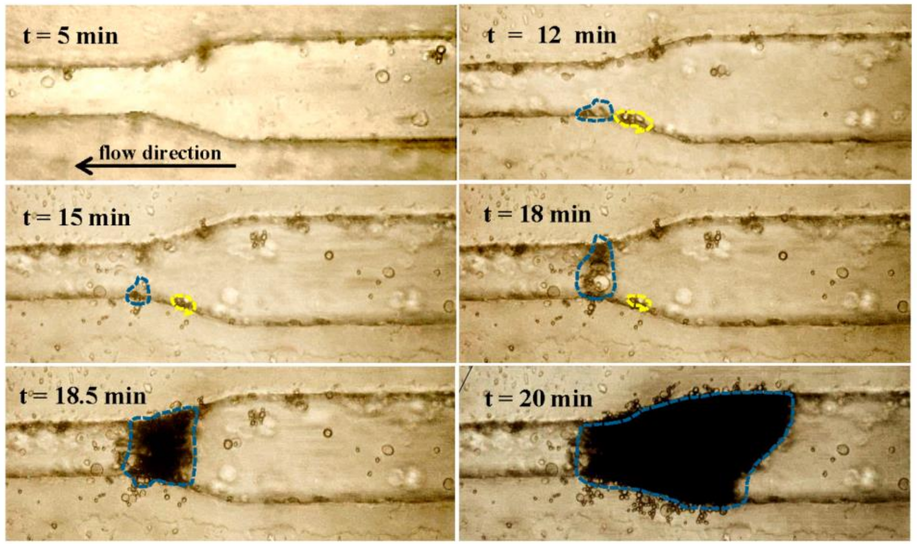
SCALING OF LAWS

- Speed: $v = \frac{s}{t}$
  - Scaling v: $v \sim L$
- Acceleration: $a = \frac{s}{t^2}$
  - *Scaling a:* $a \sim L^{-1}$
- Force: $F = m \cdot a$
  - *Scaling F:* $F \sim L^2$

- ***10x reduction of L → 100x reduction of F***
  (thus body forces are largely irrelevant in microfluidics)



https://pubs.rsc.org/image/article/2017/SM/c6sm01879c/c6sm01879c-f6_hi-res.gif



https://www.mdpi.com/1424-8220/17/1/106/htm

Recall that L: characteristic dimension (e.g. length)

SCALING OF LAWS

Property	Scaling factor
Velocity	$\sim L$
Time	1
Inertia	$\sim L^3$
Pressure	$\sim L$
Gravity	**$\sim L^3$**
Hydrostatic resistance	**$\sim L^{-4}$**
Capillary rise, diffusion constant	**$\sim L^{-1}$**

- Inertial forces and gravity **$\sim L^3$**
  - If inertial force scales relative to characteristic length L by a factor of L^3 then if length decreases 10 times, force decreases 1000 times
  - Similarly, the effect of gravity decreases 1000 times ($1/1000^{th}$ if length is $1/10^{th}$)
  - Thus, the effects of body forces and gravity are minimal on the microscale
- Viscoelastic forces **$\sim L^{-4}$**
  - However, the effect of viscoelastic forces is significantly amplified (for a length of $1/10^{th}$, 10000 times)

SCALING LAWS

- Context
- Scaling of forces
- Effect of forces on microscale flows

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BASIC FLUID MECHANICS

- Continuum assumption
- Newtonian and non-newtonian fluids
- Flow types
- Diffusion
- Surface tension
- Contact angle

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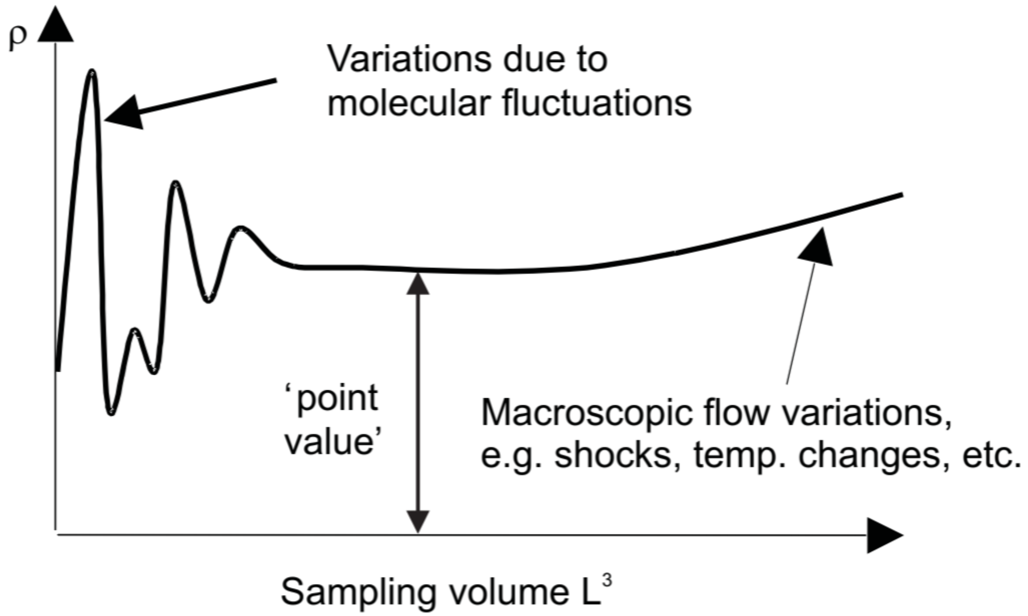
FLUID MECHANICS

Continuum assumption:
properties are defined everywhere in space and vary continuously over the flow between two points

- Dominant properties of fluid flow:
  - **Kinematic**
    (angular velocity, acceleration etc.)
  - **Transport**
    (viscosity, thermal conductivity, diffusivity)
  - **Thermodynamic**
    (pressure, temperature, density)
  - **Miscellaneous**
    (surface tension, etc.)



Continuum assumption is valid on macroscale and mostly valid on microscale (NB! effect of intermolecular forces becomes dominant as we downscale)

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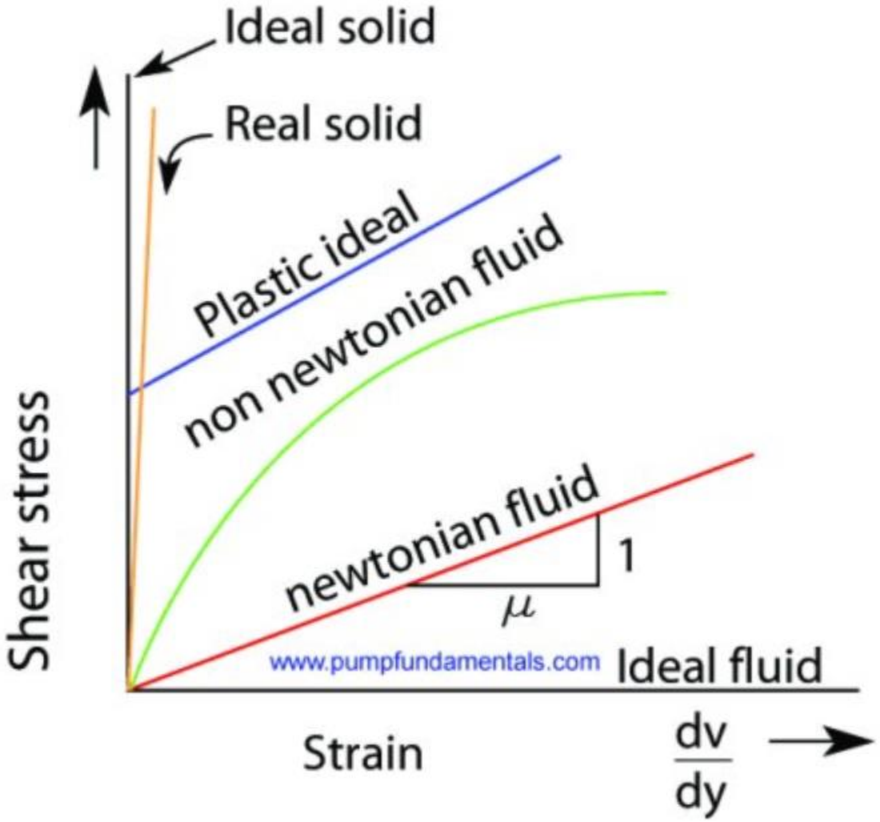
Nguyen, Nam-Trung, Steven T. Wereley, and Seyed Ali Mousavi Shaegh. Fundamentals and applications of microfluidics. Artech house, 2019.

BASIC PROPERTIES OF FLUIDS

- Newtonian fluid: constant viscosity
  - Viscosity remains constant, irrespective of shear stress applied at constant temperature (viscosity-shear stress relation linear)

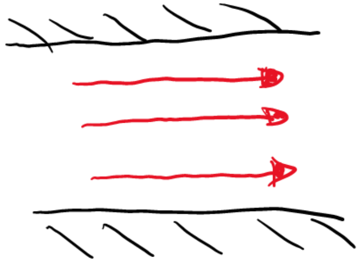
- Non-Newtonian: non-constant viscosity
  - Time-independent viscosity (e.g. ketchup, grease)
  - Time dependent as function of deformation and time history (e.g. yoghurt)
  - Viscoelastic: able to recover part of deformation energy (e.g. lubricants)



https://www.pumpfundamentals.com/pump_glossary.htm

Stress: force over area
Strain: change in length / original length

FLOW TYPES

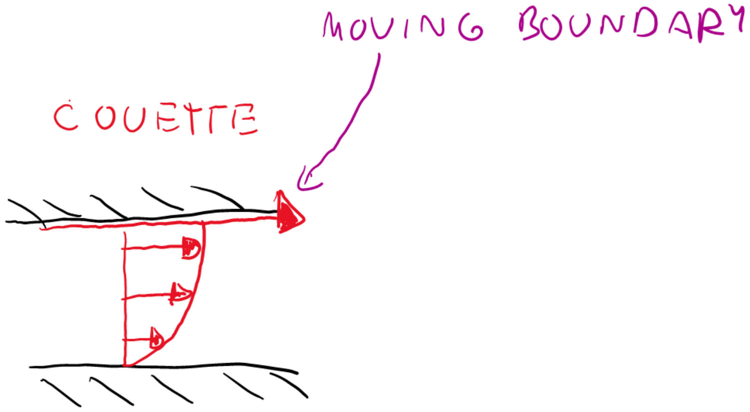


UNIFORM FLOW

~ ELECTROOSMOTIC

[Uniform Flow | Fluid Mechanics - YouTube]

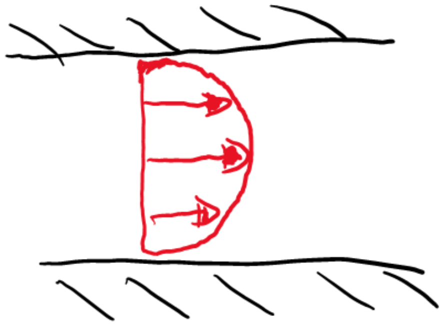
COUETTE



SHEAR FLOW

RELATIVE MOTION BETWEEN BOUNDARIES

POISEUILLE

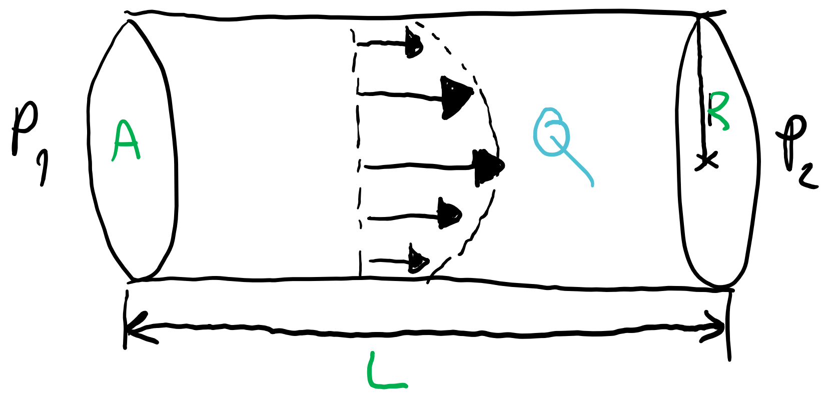


PARABOLIC FLOW

PRESSURE-DRIVEN FLOW

[Viscosity and Poiseuille flow | Fluids | Physics | Khan Academy - YouTube]

HAGEN-POISEUILLE EQUATION



- Parabolic flow profile:
  - Flow is pressure-driven
  - No-slip boundary condition applies (boundary is solid, flow velocity zero at boundary)

$$\Delta p = \frac{8\pi\mu L Q}{A^2} = \frac{8\mu L Q}{\pi R^4}$$

μ: dynamic viscosity $[Pa \cdot s]$

p: pressure $[Pa]$

A: pipe cross section $[m^2]$

R: pipe radius $[m]$

Q: flow rate $\left[\frac{m^3}{s}\right]$

- Pressure-driven flow types:
  - Negative pressure (e.g. vacuum at outlet)
  - Positive pressure (e.g. pump on inlet)

$\boldsymbol{\mu}: dynamic\ viscosity [Pa \cdot s]$

$T: temperature\ [K]$

$D: diffusion\ coefficient\ \left[\frac{m^2}{s}\right]$

DIFFUSION

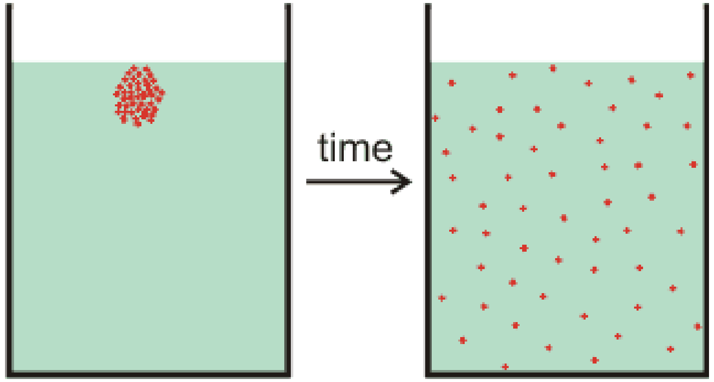
- Estimation of diffusion length:

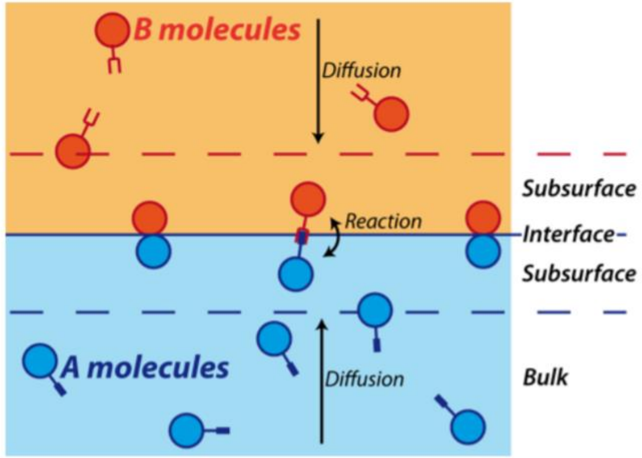
$$L_D = \sqrt{4Dt} = 2\sqrt{Dt}$$

- Estimation of diffusion coefficient

$$\frac{D_{T_1}}{D_{T_2}} = \frac{T_1}{T_2}\frac{\mu_{T_2}}{\mu_{T_1}}$$

- What's the point of all this? In microfluidics, mixing mostly happens by diffusion, through the fluid interfaces





https://arxiv.org/pdf/1708.02804.pdf

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https://www.comsol.com/multiphysics/diffusion-coefficient

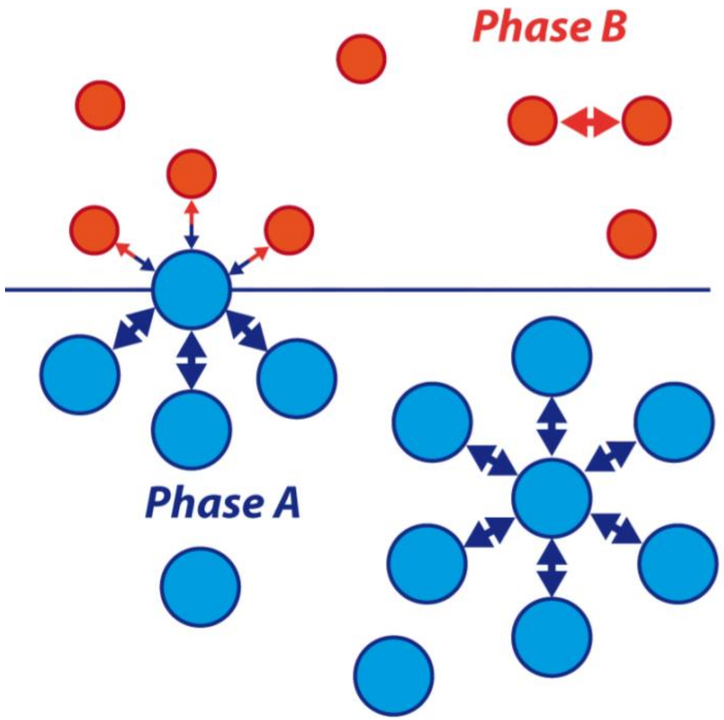
INTERFACES AND SURFACE TENSION

- **Van der Waals attraction**
  - ***In organic liquids, hydrogen bonds in polar liquids hold molecules close to each other***
  - ***However, at an interface, half of the interactions are with molecules from the other phase (gas/liquid)***
  - ***On a macroscopic level, this results in a clear interface (there's "tension between the surfaces of the two phases")***

- Surface tension:

$$\gamma \approx \frac{U}{2\delta^2} \left[\frac{J}{m^2}\right]$$

Where U: total cohesive energy per molecule [J], δ characteristic molecular dimension (δ^2 area of same [m^2])



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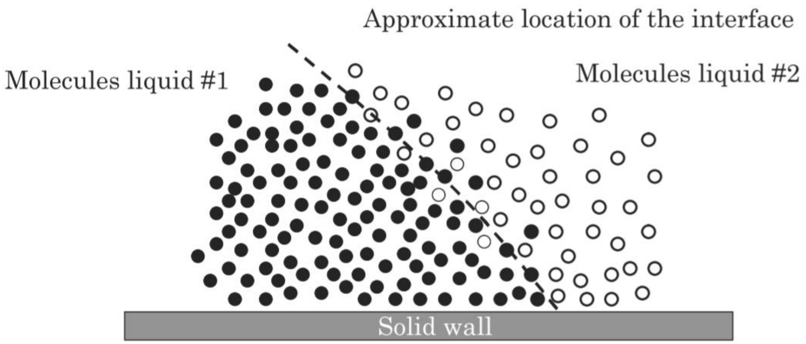

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Berthier, J. (2012). *Micro-drops and digital microfluidics*. William Andrew.

INTERFACES AND SURFACE TENSION

- Van der Waals attraction
  - In organic liquids, hydrogen bonds in polar liquids hold molecules close to each other
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- **Surface tension:**

$$\gamma \approx \frac{U}{2\delta^2} \left[\frac{J}{m^2}\right]$$

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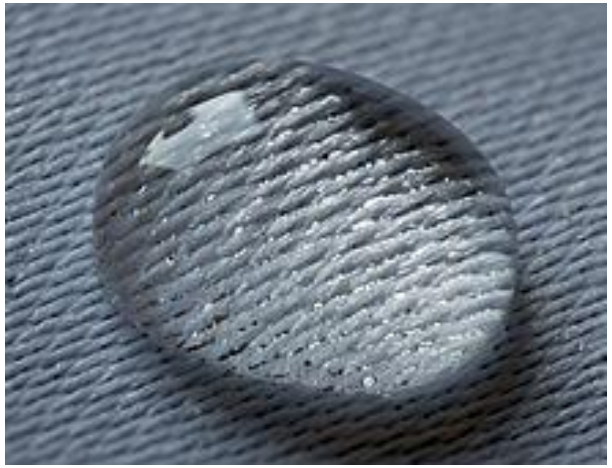


Berthier, J. (2012). *Micro-drops and digital microfluidics*. William Andrew.

INTERFACES AND SURFACE TENSION

- **Surface tension:**

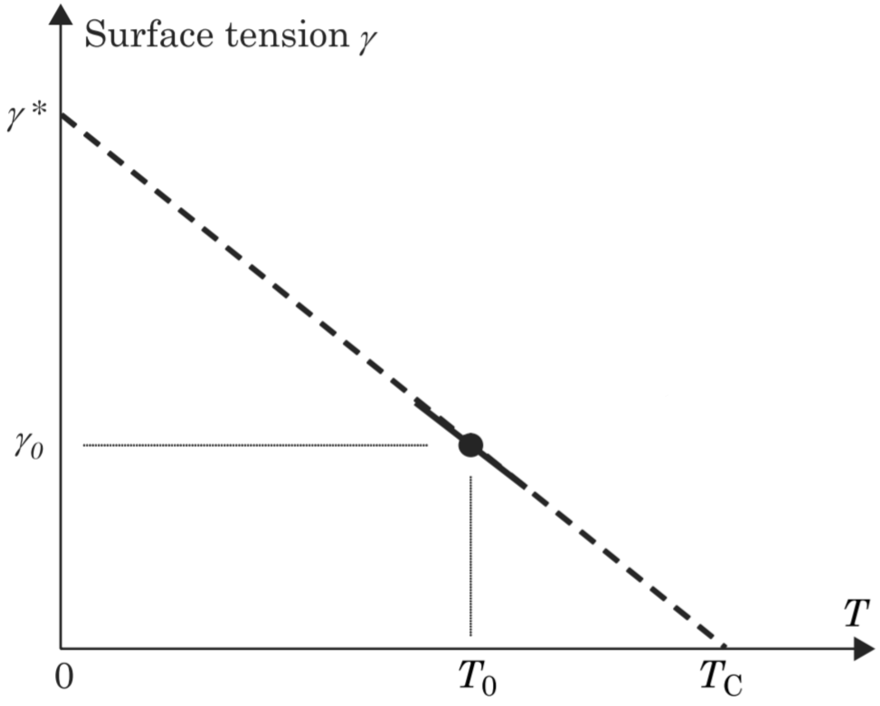
$$\gamma \approx \frac{U}{2\delta^2} = \frac{\Delta E_f}{\Delta A} \left[\frac{J}{m^2}\right]$$

Where U: total cohesive energy per molecule [J], δ characteristic molecular dimension (δ^2 area of same [m^2])
Aka. the E_f surface energy [J] required to maintain surface area A [m^2].

- **Temperature dependence**

$$\gamma_0 = \gamma^* \frac{T_C - T_0}{T_C}$$

  - Where: γ_0: reference surface tension (in air at 20 °C)



Liquid	γ_0
Water	72.8
Mercury	425.4
Ethanol	22.1
Glycerol	64.0
PDMS	19.0

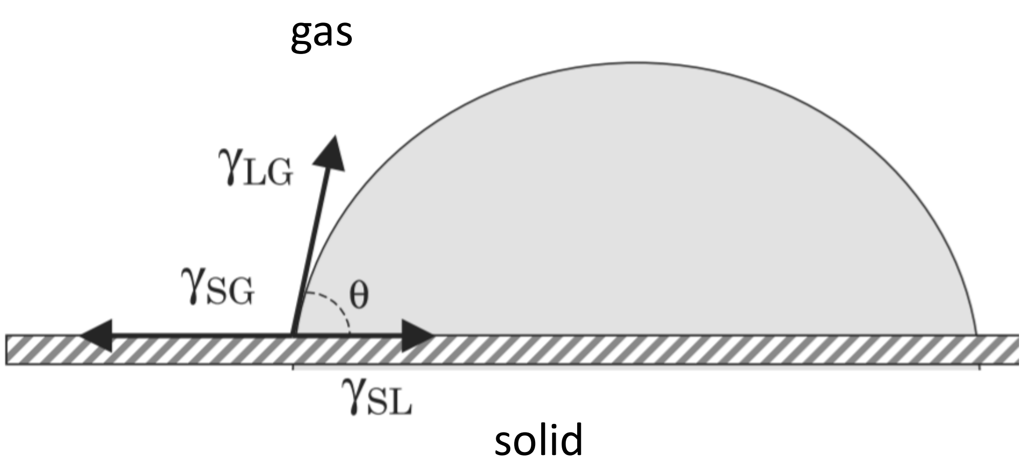
Berthier, J. (2012). *Micro-drops and digital microfluidics*. William Andrew.

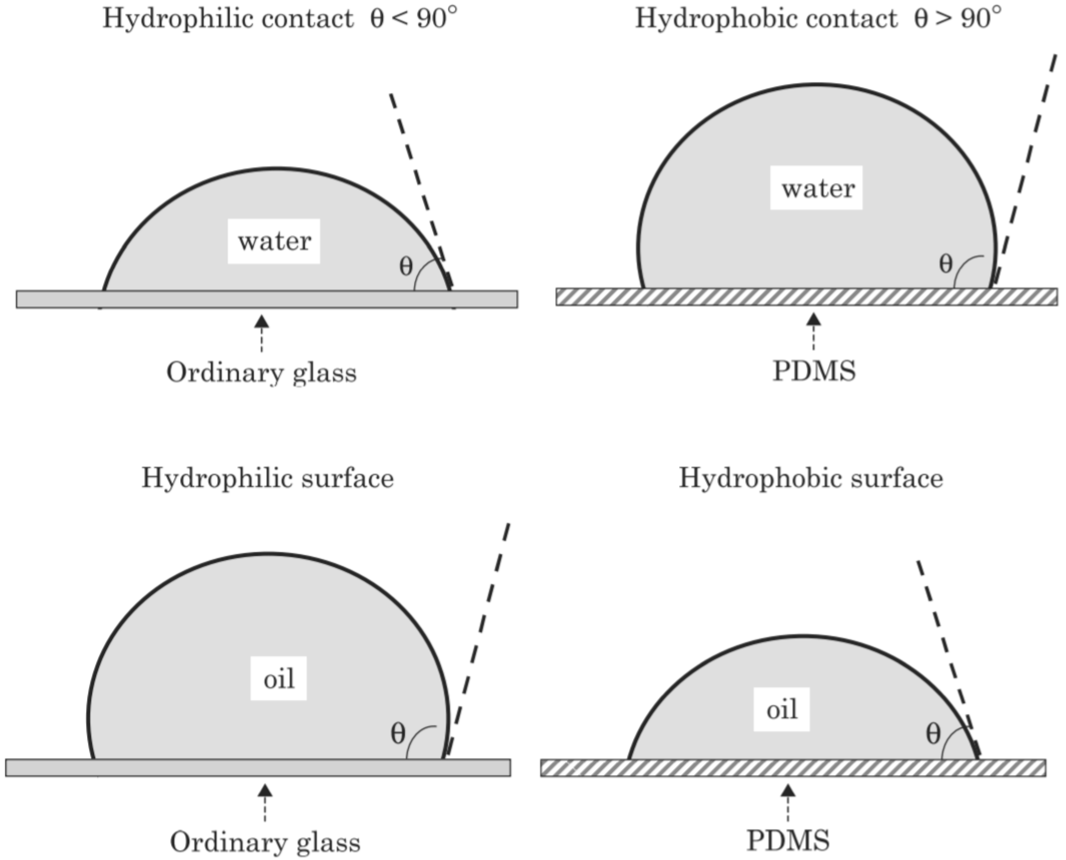
CONTACT ANGLE AND WETTING

$$\gamma_{LG} \cdot cos\theta = \gamma_{SG} - \gamma_{SL}$$

$$\theta = \arccos \frac{\gamma_{SG} - \gamma_{SL}}{\gamma_{LG}}$$

- **Triple contact line**
  - Between solid, gas and liquid, surface tensions at equilibrium are related as on the right
  - The contact angle that the liquid interface closes with the solid surface, is denoted θ





CAPILLARY AND HYDROSTATIC PRESSURE

- Capillary pressure in a tube:

$$\Delta p = \frac{2\gamma}{R}$$

- Capillary rise in a tube:

$$h = \frac{2\gamma cos\theta}{\rho g a}$$

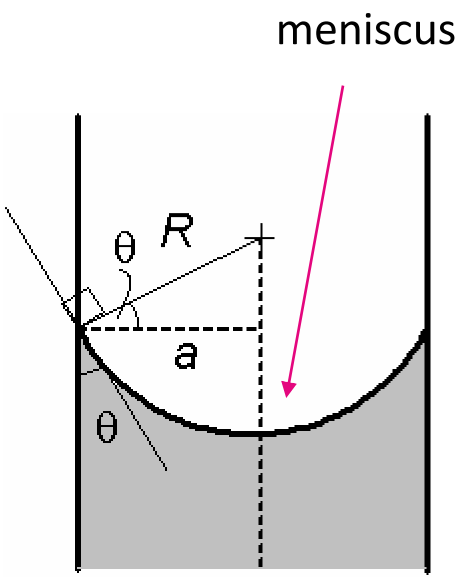
- Hydrostatic pressure

$$p = \rho g h$$

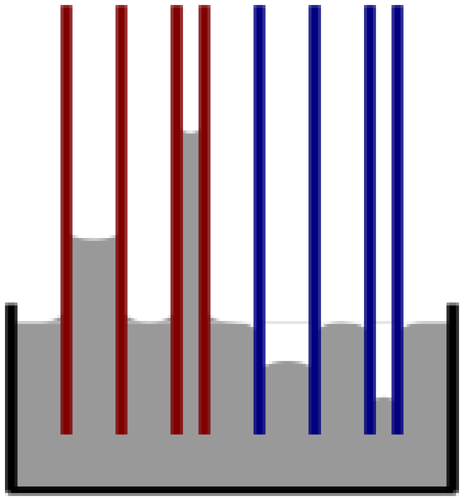
h: height/depth of fluid column [m]

p: pressure $[Pa]$

ρ: density $\left[\frac{kg}{m^3}\right]$



https://upload.wikimedia.org/wikipedia/commons/7/75/Spherical_meniscus.PNG



https://upload.wikimedia.org/wikipedia/commons/thumb/8/85/CapillaryAction.svg/270px-CapillaryAction.svg.png

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BASIC FLUID MECHANICS

- Continuum assumption
- Newtonian and non-newtonian fluids
- Flow types
- Diffusion
- Surface tension
- Contact angle

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INTRODUCTION TO DROPLETS

- Young’s law
- Laplace law
- Capillary pressure
- Surfactants

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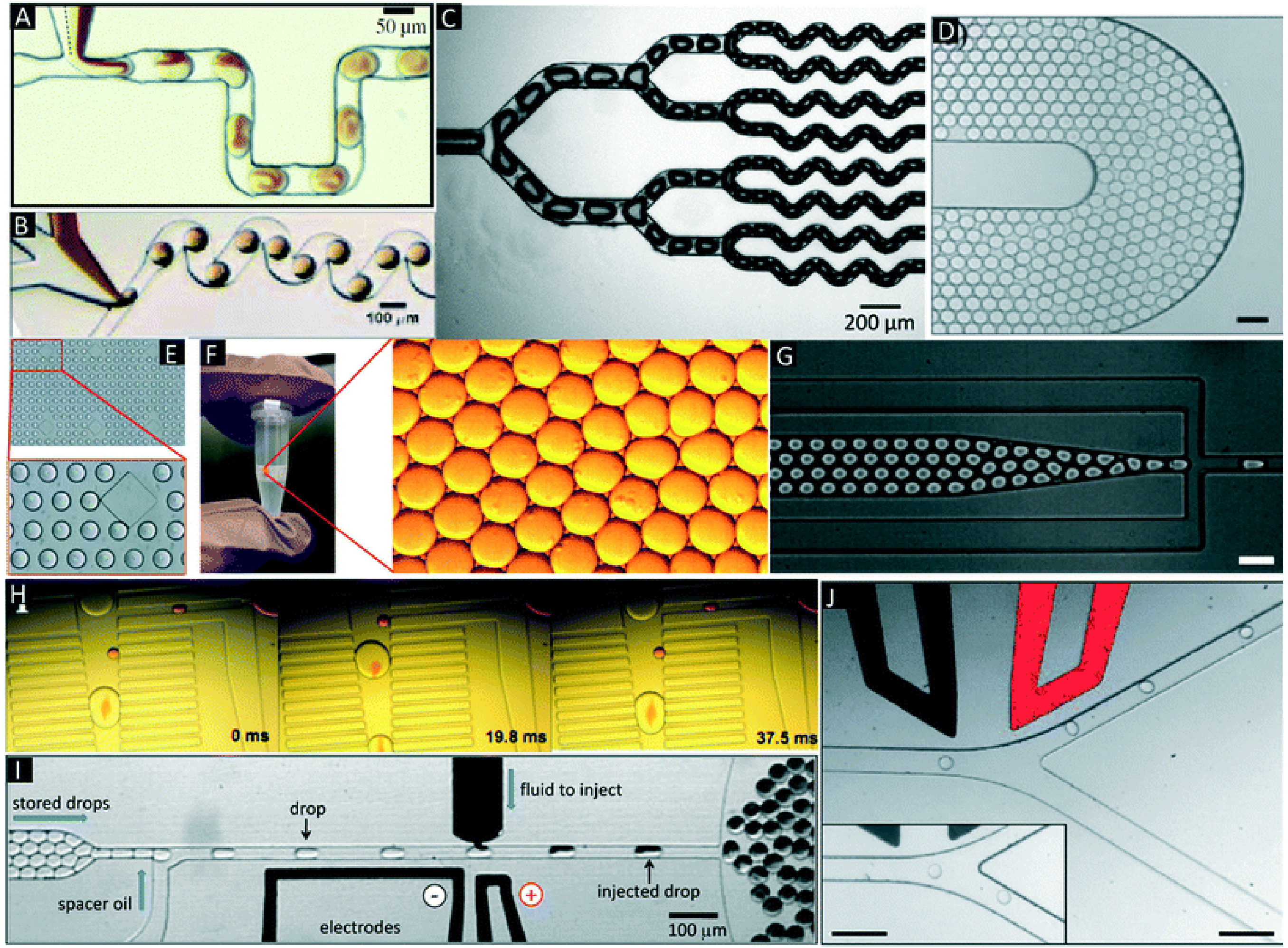
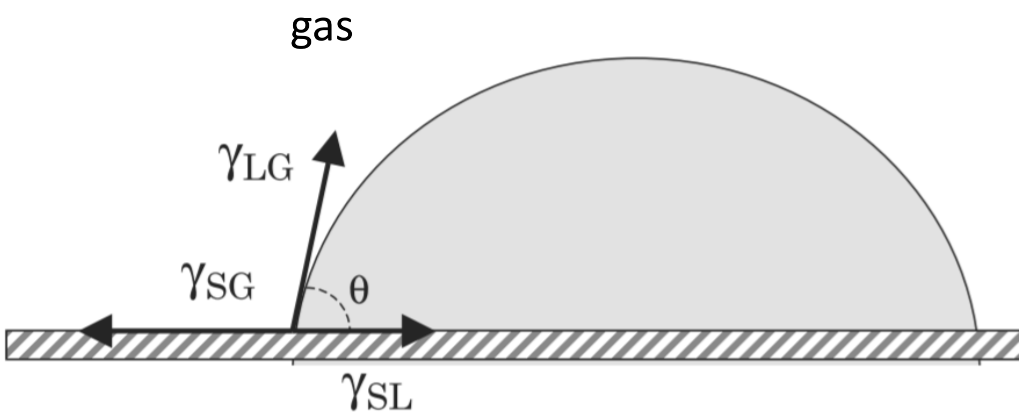
INTRODUCTION

- (+) Excellent **volume control**
- (+) **Chemical separation** between droplets
- (+) Ability to implement packed bed microreactors
- (+) **Storage** of droplets
- (-) Technologically challenging
  - Monodispersity
  - Generation rate
  - Droplet stability
  - Minimum droplet volume



YOUNG'S LAW

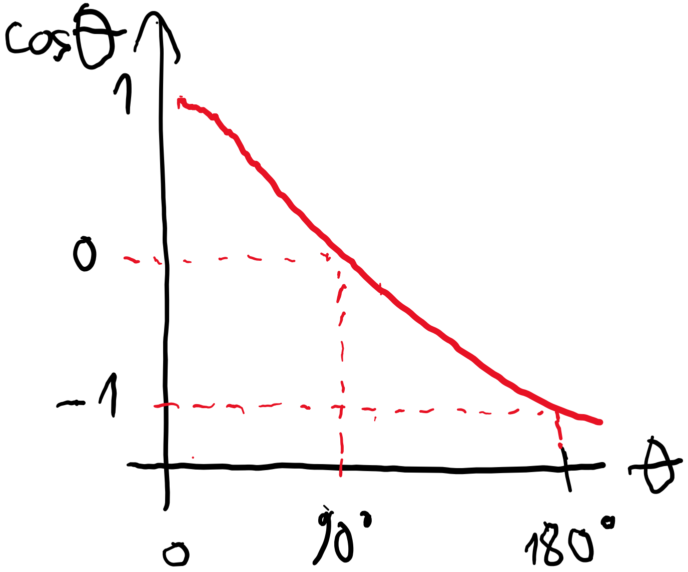


- Young's law:
  The contact angle that the liquid interface closes with the solid surface, is denoted θ

$$\gamma_{LG} \cdot \boldsymbol{cos\theta} = \gamma_{SG} - \gamma_{SL}$$

- Work of adhesion:

$$W_a = \gamma + \gamma_{SG} - \gamma_{SL} = \gamma(1 + \boldsymbol{cos\theta})$$



Superhydrophobic: $cos\theta \to -1; W_a \to 0$

Superhydrophilic: $cos\theta \to 1; W_a \to \infty$

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Berthier, J. (2012). *Micro-drops and digital microfluidics*. William Andrew.

LAPLACE LAW

- Pendant drop
  Starting from a pendant drop (hanging from a vertical pipe, like your tap at home)

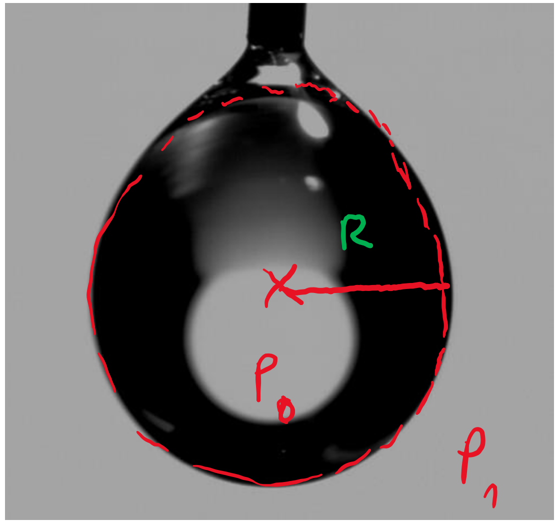
- Laplace law for a sphere:

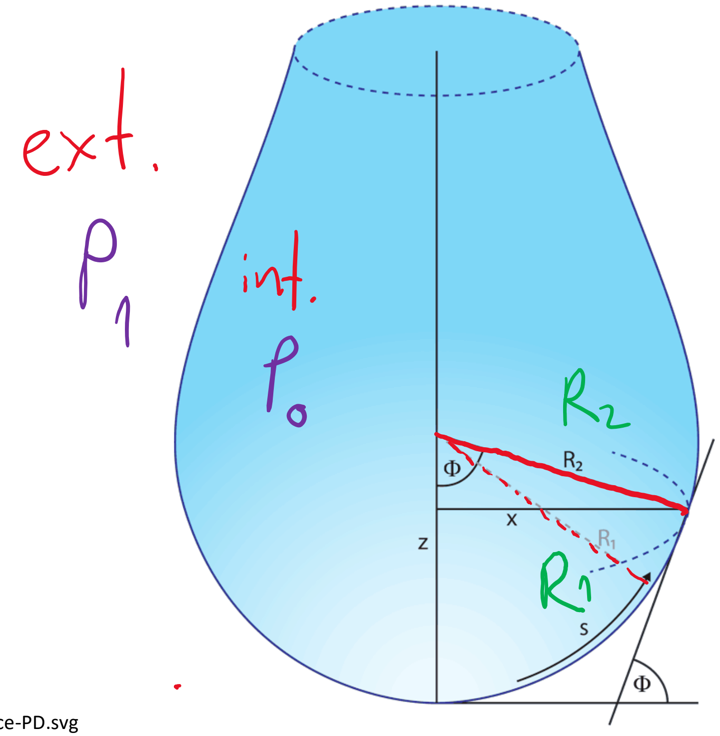
$$\Delta p = \mathbf{p_0} - \mathbf{p_1} = \frac{2\gamma}{R}$$

- Young-Laplace:

$$\Delta p = \rho g h - \gamma\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

(hydrostatic pressure <-> surface tension)
(this is how surface tension can be experimentally determined)

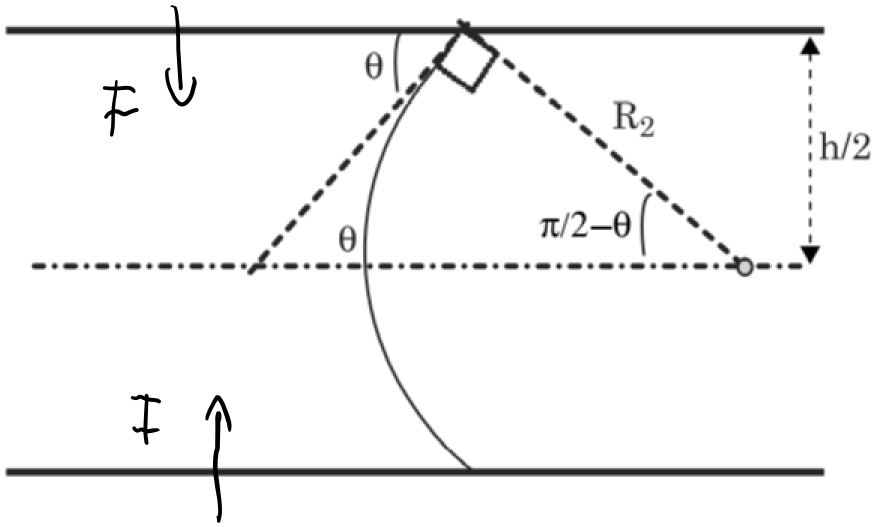




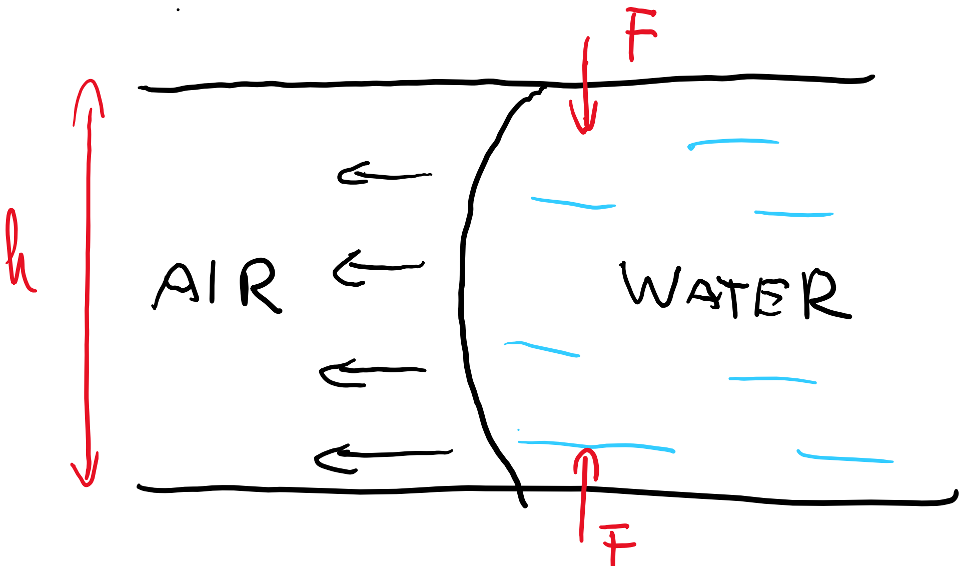
https://www.dataphysics-instruments.com/Images/Knowledge/UnderstandingInterfaces/Schematic-Young-Laplace-PD.svg

CAPILLARY FORCE BETWEEN PARALLEL PLATES

- Laplace law for the free interface:
  - $R_2 \sin\left(\frac{\pi}{2} - \theta\right) = \frac{h}{2} \rightarrow R_2 = \frac{h}{2\cos\theta}$
  - Laplace law: $\Delta P = \gamma\left(\frac{1}{R} - \frac{2\cos\theta}{h}\right)$
- Capillary pressure: $\Delta p \approx -\frac{2\gamma\cos\theta}{h}$
- Capillary force: $F \approx \frac{2\gamma\cos\theta}{h}\pi R^2$



Example:
$h = 10\mu m, R = 1cm, F \approx 2.5\ N$



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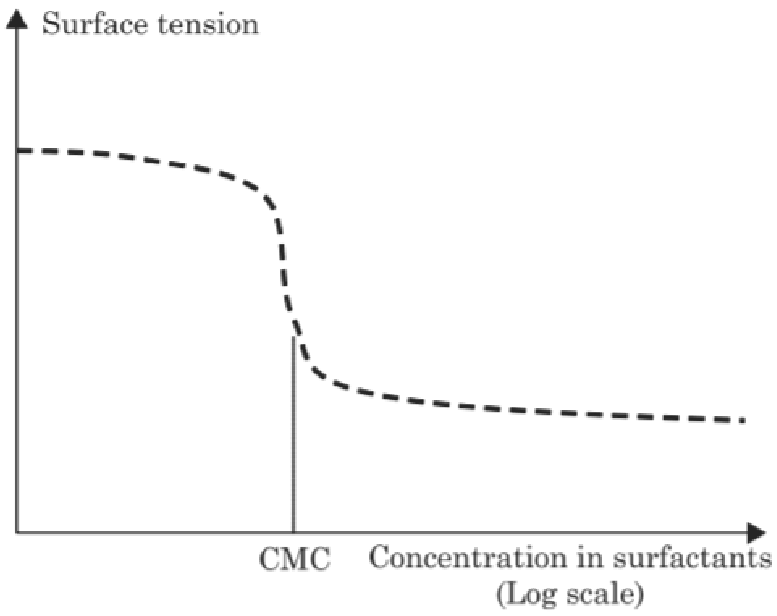
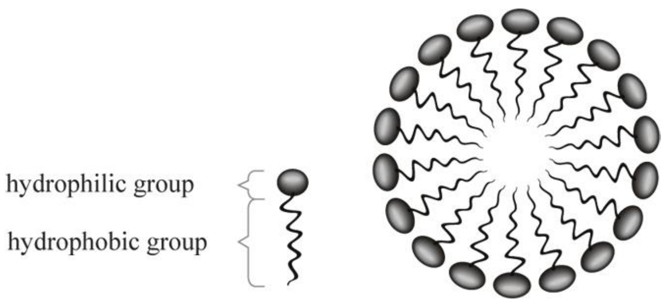
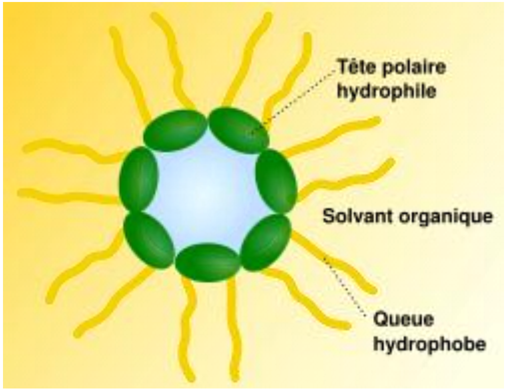
SURFACTANTS

- Surfactant = surface active agent
  - Hydrophilic head
  - Hydrophobic tail
- Surfactants decrease surface tension
  - CMC = critical micelle concentration ➔ "stable droplets"
  - Think of soap ☺
  - Example:
    - Water surface tension 72 mN/m
    - Tween 10 + water above CMC: 30 mN/m







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Berthier, J. (2012). *Micro-drops and digital microfluidics*. William Andrew.

INTRODUCTION TO DROPLETS

- Young's law
- Laplace law
- Capillary pressure
- Surfactants

13.01.2022

SUPERHYDROPHOBIC SURFACES

- Contact angle and wetting on micropatterned surfaces
- …on rectangular micropillars
- …in nature

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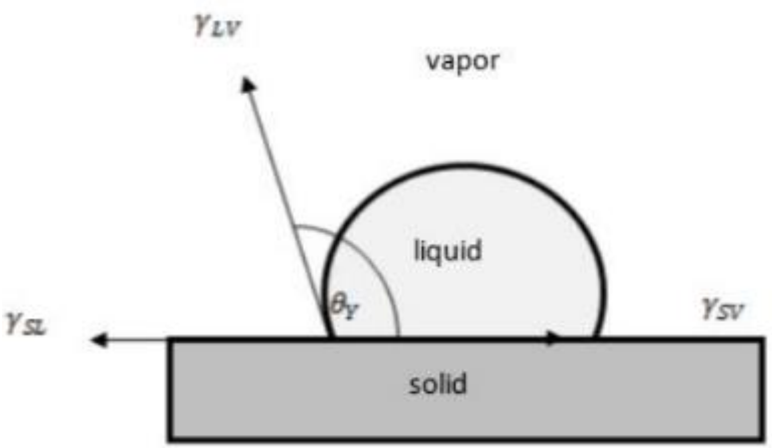
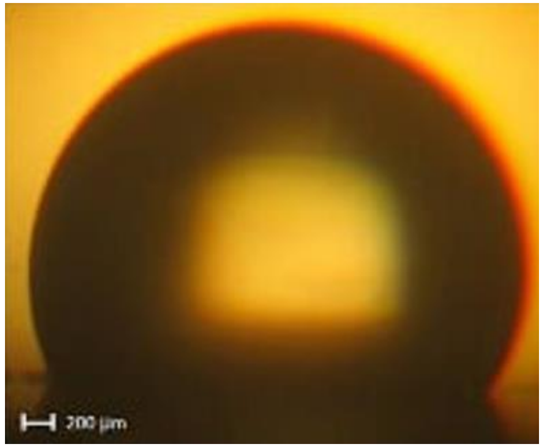
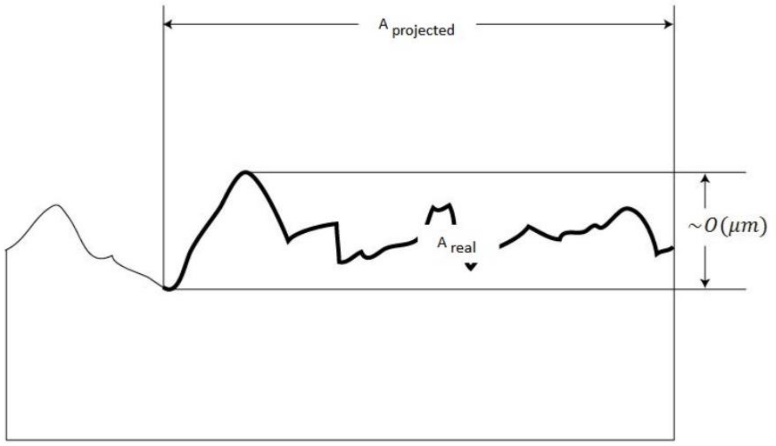
THEORY: DROPLETS





Equilibrium contact angle $$cos\theta_0 = \frac{\gamma_{SV} - \gamma_{SL}}{\gamma_{LV}}$$

DROPLETS ON MICROPATTERNED SURFACES



- **Roughness factor - micropatterned surface:**

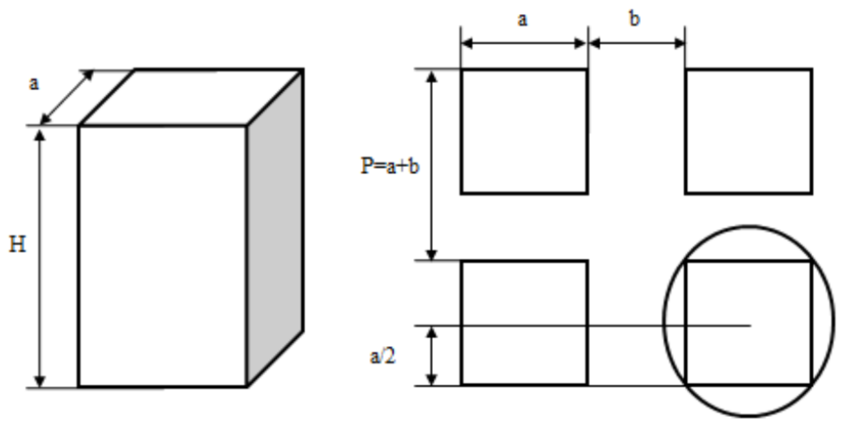
$$\boldsymbol{r = \frac{A_{real}}{A_{projected}}}$$

- For rectangular micropillars:
  - Roughness factor - homogeneous interface:

$$r = \frac{(a+b)^2 + 4aH}{(a+b)^2}$$

  - Roughness factor - inhomogeneous interface:

$$f = \frac{A_{top}}{A_{projected}} = \frac{a^2}{(a+b)^2} = A$$



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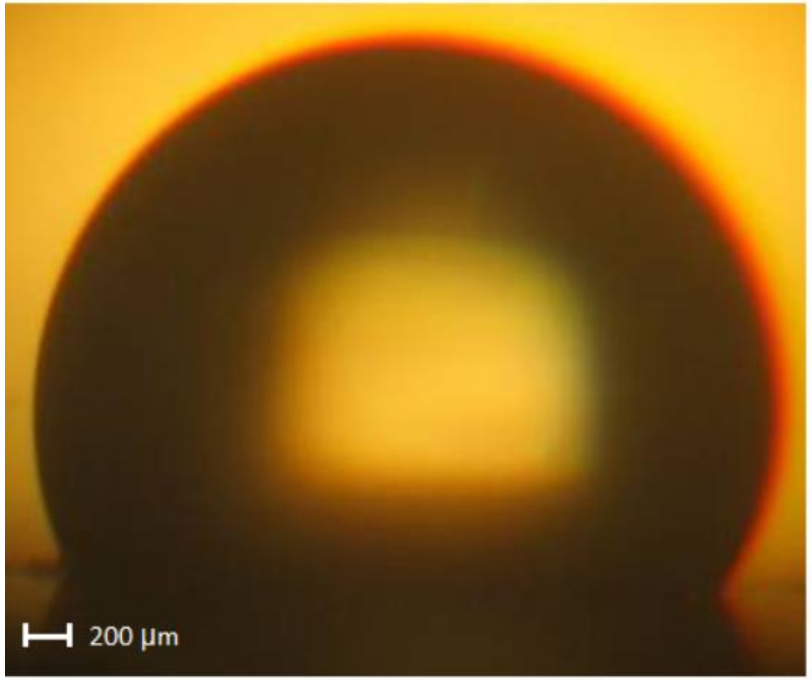
Wenzel state



Contact angle: $cos\theta_W = rcos\theta_0$

On rectangular micropillars:

$$cos\theta_W = \left[1 + \frac{4A}{\left(\frac{a}{H}\right)}\right] \cdot \cos\theta$$

DROPLETS ON MICROPATTERNED SURFACES

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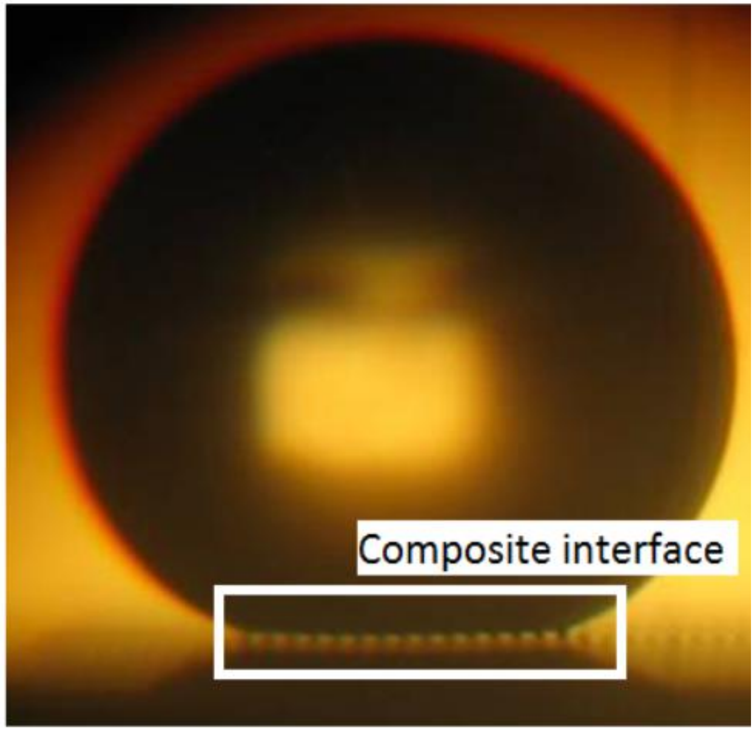
Cassie state



Contact angle: $cos\theta_C = f_1 cos\theta_1 + f_2 cos\theta_2$

On rectangular micropillars:

$$cos\theta_C = -1 + A(1 + cos\theta)$$

DROPLETS ON MICROPATTERNED SURFACES

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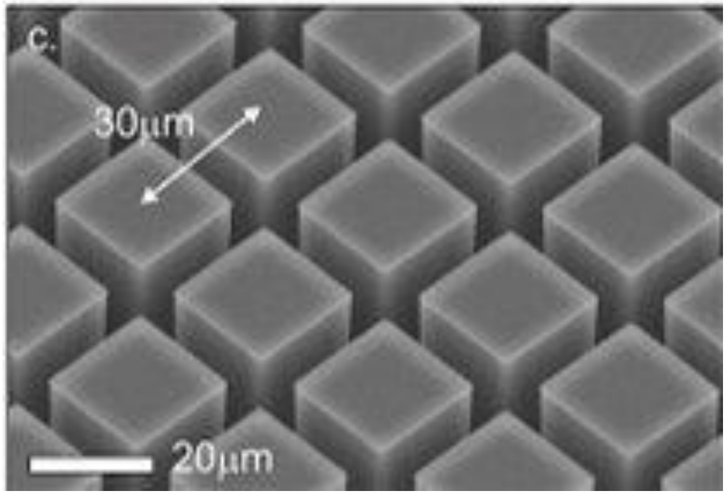

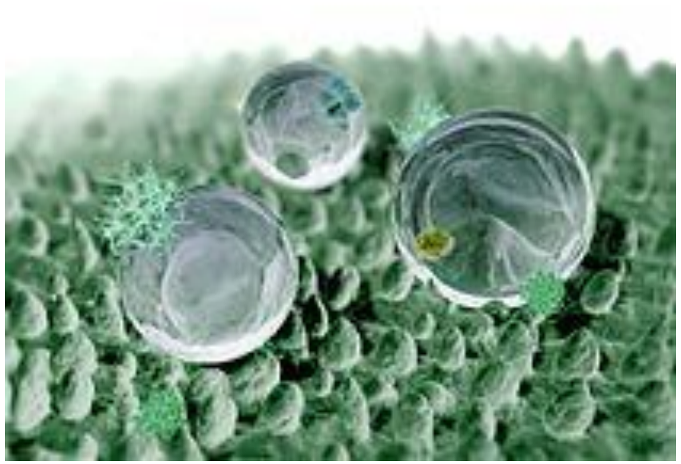

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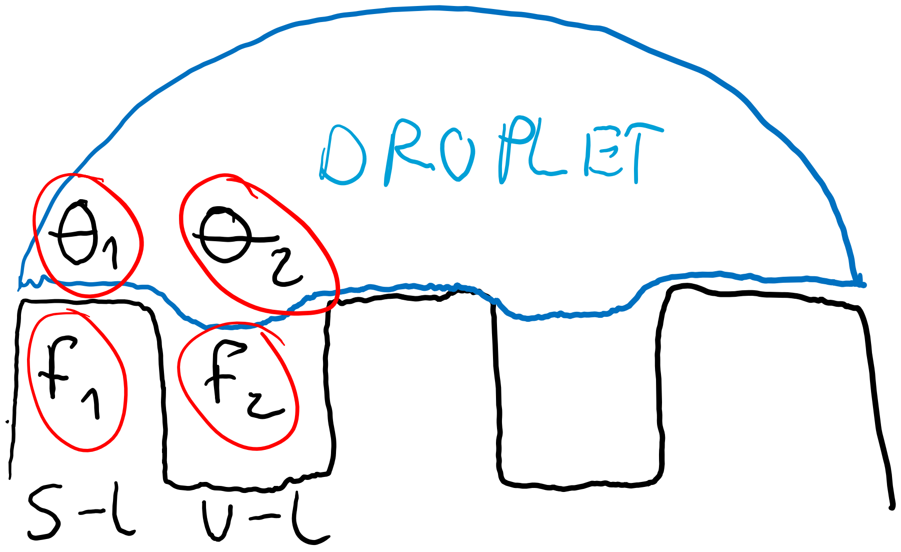

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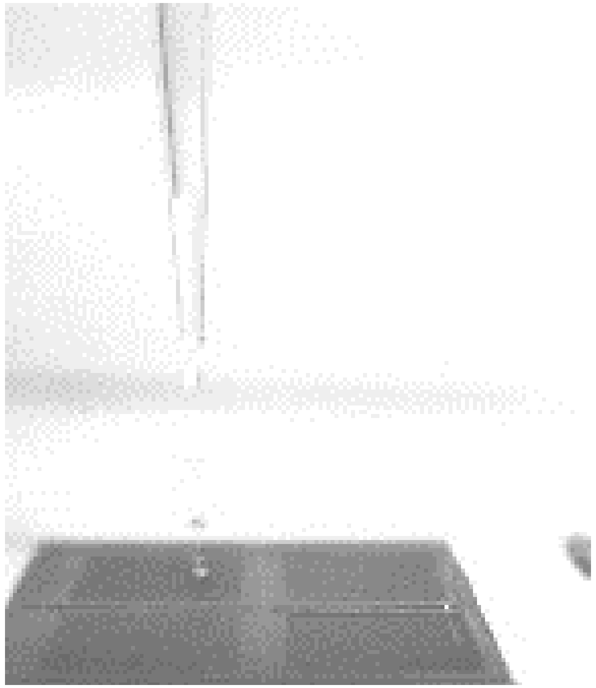
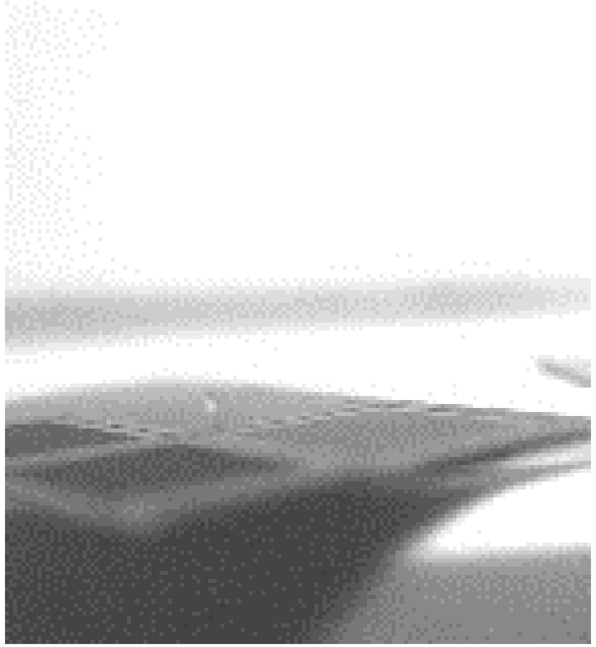
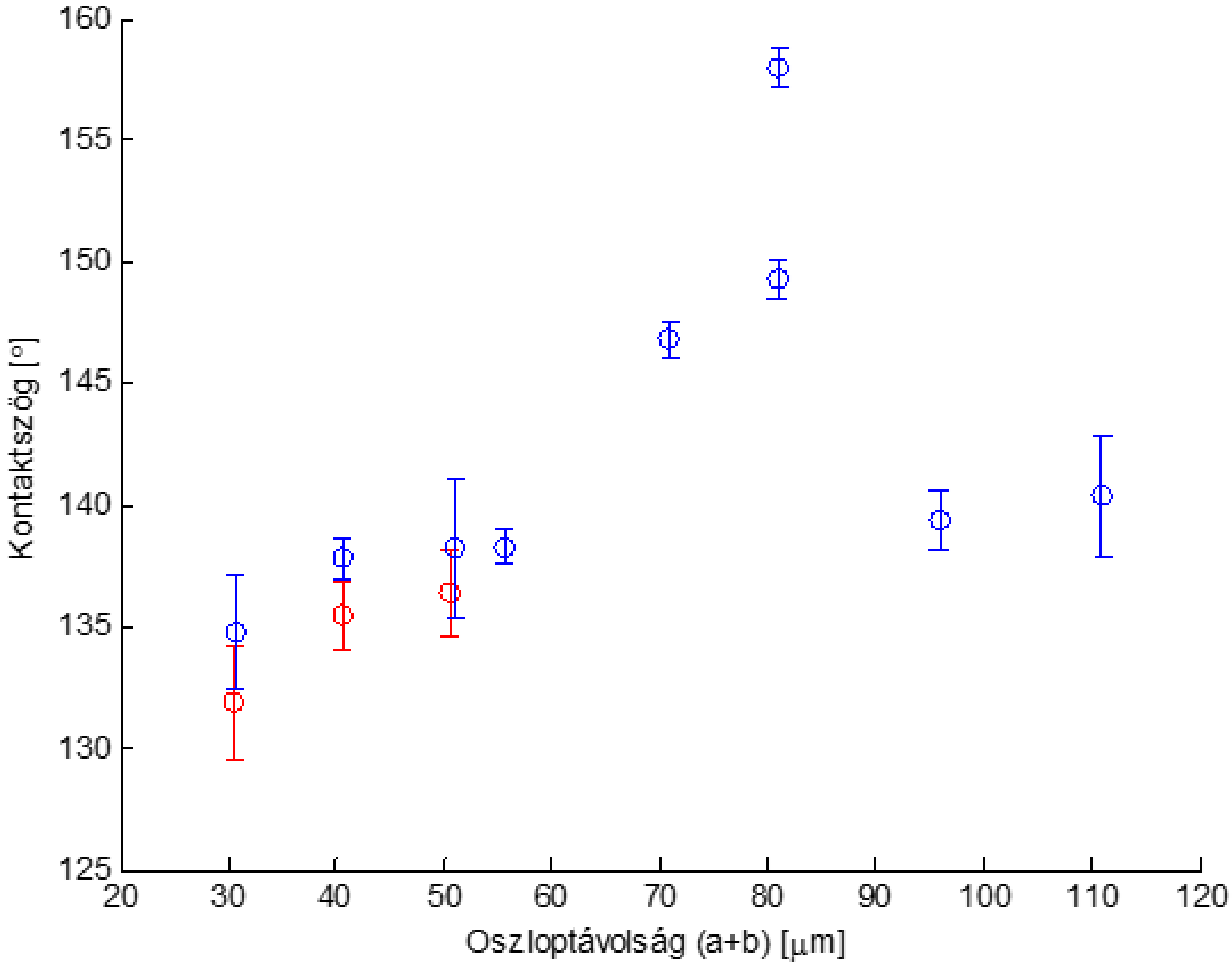
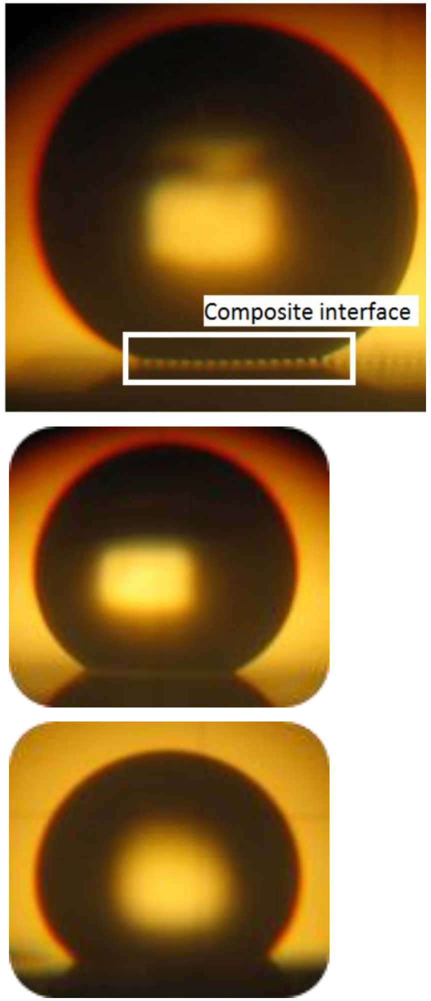




CONTACT ANGLE AND WETTING





SUPERHYDROPHOBIC SURFACES

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13.01.2022

PRESSURE AND FLOW RATE IN MICROCHANNELS

- Poisson equation
- Application to channels with…
- …circular cross section
- …rectangular cross section
- Flow control in microfluidics

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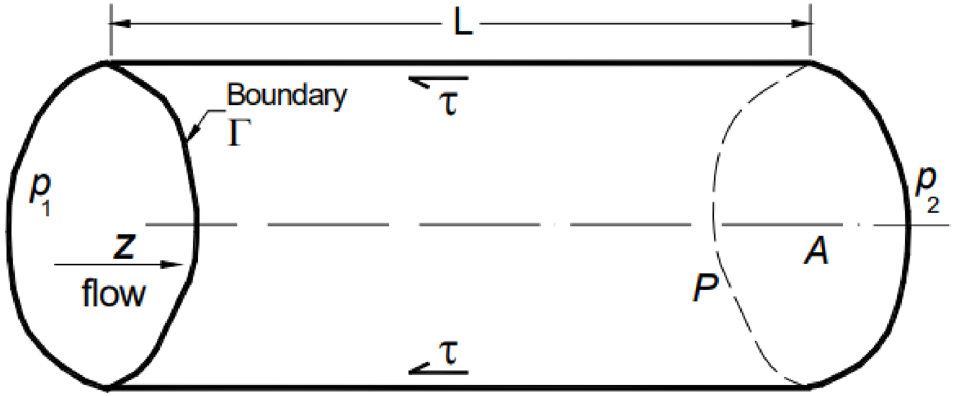
POISSON EQUATION

$$\nabla^2 w = \frac{1}{\mu}\frac{dp}{dz}$$



- Assuming
  - Constant velocity profile in longitudinal direction
  - Fully-developed laminar flow
  - Zero flow velocity on Γ boundary ($w = 0 \;\; on \;\; \Gamma$)

$w: flow\ velocity\ \left[\frac{m}{s}\right]$

$p: pressure\ [Pa]$

$\mu: dynamic\ viscosity [Pa \cdot s]$

Γ: boundary of duct (tube)
z: flow direction

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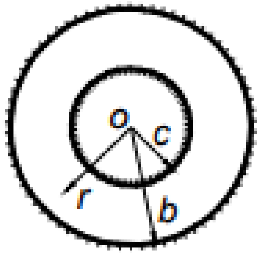
Pressure Drop of Fully-Developed, Laminar Flow in Microchannels of Arbitrary Cross-Section (uwaterloo.ca)

CIRCULAR MICROCHANNEL

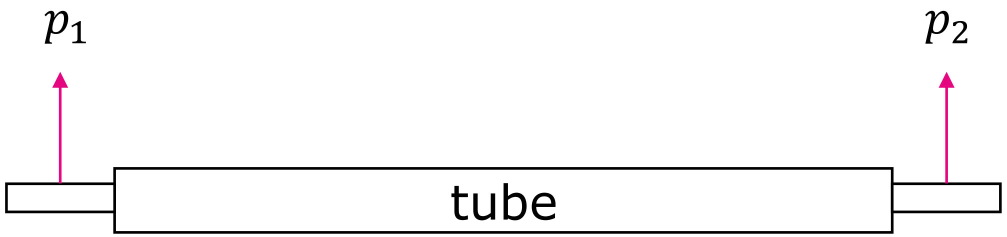
- Analytical solution exists for mean velocity $\bar{w}$ in:
  - Circular microchannel
  - Laminar flow



$$A = \pi\left(b^2 - c^2\right)$$

$$P = 2\pi\left(b + c\right)$$

$$\bar{w} = \frac{\Delta p b^2}{8\mu L}\left(\epsilon^2 - 1 + \frac{2\ln\left(\frac{1}{\epsilon}\right) + \epsilon^2 - 1}{\ln\left(\frac{1}{\epsilon}\right)}\right)$$



- Values:
  - Semi-axes b (major), c (minor), $b \geq c$
  - Δp pressure drop; $p_1 - p_2$ $(p_1 \geq p_2)$
  - L: channel length [m]
  - ϵ: aspect ratio; $\epsilon = \frac{c}{b}$

w: *flow velocity* $\left[\frac{m}{s}\right]$

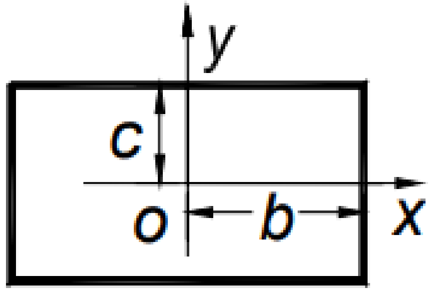
p: *pressure* $[Pa]$

μ: *dynamic viscosity* $[Pa \cdot s]$

Pressure Drop of Fully-Developed, Laminar Flow in Microchannels of Arbitrary Cross-Section (uwaterloo.ca)

RECTANGULAR MICROCHANNEL

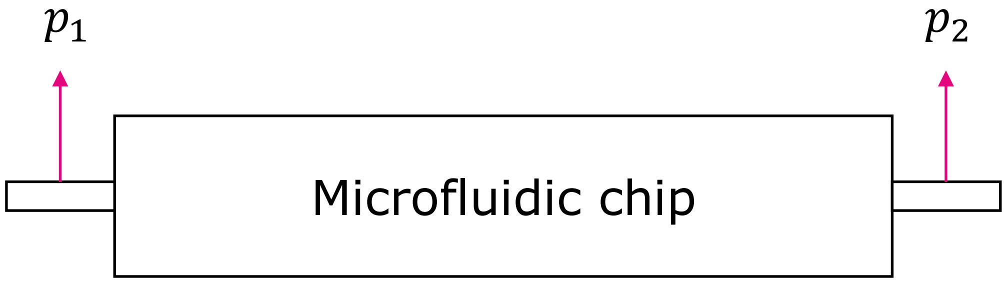
- Analytical solution exists for mean velocity $\bar{w}$ in:
  - Rectangular microchannel
  - Laminar flow



$$A = 4bc$$

$$P = 4\,(b + c)$$

$$\bar{w} = \frac{\Delta p c^2}{\mu L}\left[\frac{1}{3} - \frac{64}{\pi^5}\frac{c}{b}\tanh\left(\frac{\pi b}{2c}\right)\right]$$



- Values:
  - Semi-axes b (major), c (minor), $b \geq c$
  - Δp pressure drop; $p_1 - p_2$ $(p_1 \geq p_2)$
  - L: channel length [m]
  - ϵ: aspect ratio; $\epsilon = \frac{c}{b}$

w: $flow\ velocity$ $\left[\frac{m}{s}\right]$

p: $pressure$ $[Pa]$

μ: $dynamic\ viscosity [Pa \cdot s]$

FLOW CONTROL IN MICROFLUIDICS

- Flow rate: $Q = w \cdot A \quad unit: \left[\frac{m^3}{s}\right]$
- Liquid pumps (later lecture) are not perfect flow rate generators

- Back pressure Δp_{back} lowers flow rate Q:

$$Q = Q_{max} \cdot \frac{\Delta p_{max} - \Delta p_{back}}{\Delta p_{max}}$$

- Where:
  - Q_{max} is the flow rate when nothing's attached to the pump
  - Δp_{max} is the maximum backpressure the pump can take (find in specifications)

Flow control in microfluidic devices.pdf (elveflow.com)

PRESSURE AND FLOW RATE IN MICROCHANNELS

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[Dynamic Viscosity of some common Liquids (engineeringtoolbox.com)]
[Mineral oil dynamic viscosity data.... | Download Scientific Diagram (researchgate.net)]

EXAMPLE: DROPLET GENERATION

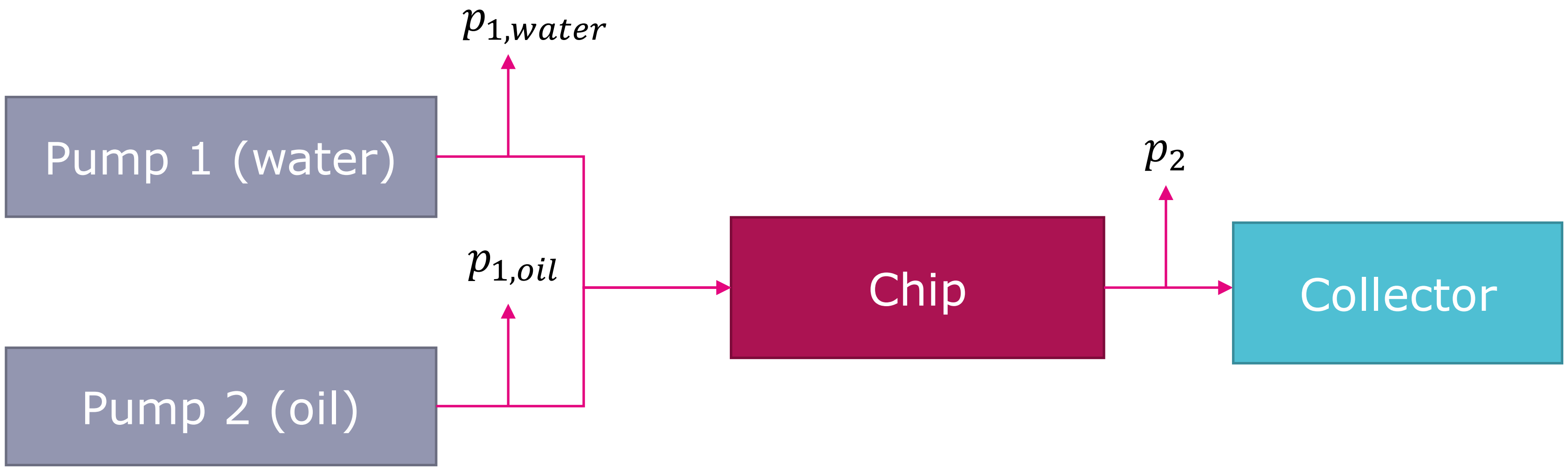
- 2 pumps
- 1 microfluidic chip (recangular channels)
- 3 pressure sensors are needed, 2 on each flow path before the chip and 1 on the output

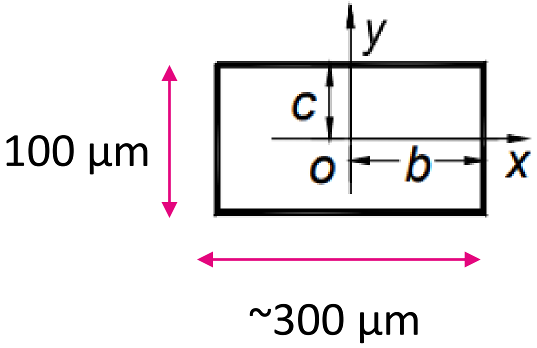
μ_{water}= 0.00089 $Pa \cdot s$

$\mu_{mineral\ oil}$= ~0.080 $Pa \cdot s$

$c = 50\mu m; b = 150\mu m; L = 1\ cm$

$\Delta p = 100\ Pa$





IEE1860 BIOMEMS

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self diagnostics

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