

# Conversion Factors :FPS to SI

	Multiply	By	To obtain
Length	ft	$3.048 \times 10^{-1}$	m
	mile	1.609	km
Area	ft <sup>2</sup>	$9.290 \times 10^{-2}$	m <sup>2</sup>
	acre	$4.047 \times 10^3$	m <sup>2</sup>
	mi <sup>2</sup>	2.590	km <sup>2</sup>
Volume	ft <sup>3</sup>	$2.832 \times 10^{-2}$	m <sup>3</sup>
	U.S. gal	$3.785 \times 10^{-3}$	m <sup>3</sup>
	U.K. gal	$4.546 \times 10^{-3}$	m <sup>3</sup>
	ft <sup>3</sup>	$2.832 \times 10$	ℓ
	U.S. gal	3.785	ℓ
	U.K. gal	4.546	ℓ
Velocity	ft/s	$3.048 \times 10^{-1}$	m/s
Acceleration	ft/s <sup>2</sup>	$3.048 \times 10^{-1}$	m/s <sup>2</sup>
Mass	lb <sub>m</sub>	$4.536 \times 10^{-1}$	kg
	ton	$1.016 \times 10^3$	kg
Force and weight	lb <sub>f</sub>	4.448	N
Pressure and stress	lb <sub>f</sub> /ft <sup>2</sup>	$4.788 \times 10$	Pa or N/m <sup>2</sup>
	psi	$6.895 \times 10^3$	Pa or N/m <sup>2</sup>
	atm	$1.013 \times 10^5$	Pa or N/m <sup>2</sup>
	atm	1.013	bar
Work and energy	ft-lb <sub>f</sub>	1.356	J
	calorie	4.187	J
Mass density	lb <sub>m</sub> /ft <sup>3</sup>	$1.602 \times 10$	kg/m <sup>3</sup>
Weight density	lb <sub>f</sub> /ft <sup>3</sup>	$1.571 \times 10^2$	N/m <sup>3</sup>
Discharge	ft <sup>3</sup> /s	$2.832 \times 10^{-2}$	m <sup>3</sup> /s
	ft <sup>3</sup> /s	$2.832 \times 10$	ℓ/s
	U.S. gal/min	$6.309 \times 10^{-5}$	m <sup>3</sup> /s
	U.S. gal/min	$6.309 \times 10^{-2}$	ℓ/s
	U.K. gal/min	$7.576 \times 10^{-5}$	m <sup>3</sup> /s
	U.K. gal/min	$7.576 \times 10^{-2}$	ℓ/s
Hydraulic conductivity	ft/s	$3.048 \times 10^{-1}$	m/s
	U.S. gal/day/ft <sup>2</sup>	$4.720 \times 10^{-7}$	m/s
Permeability	ft <sup>2</sup>	$9.290 \times 10^{-2}$	m <sup>2</sup>
	darcy	$9.870 \times 10^{-13}$	m <sup>2</sup>
Transmissivity	ft <sup>2</sup> /s	$9.290 \times 10^{-2}$	m <sup>2</sup> /s
	U.S. gal/day/ft	$1.438 \times 10^{-7}$	m <sup>2</sup> /s

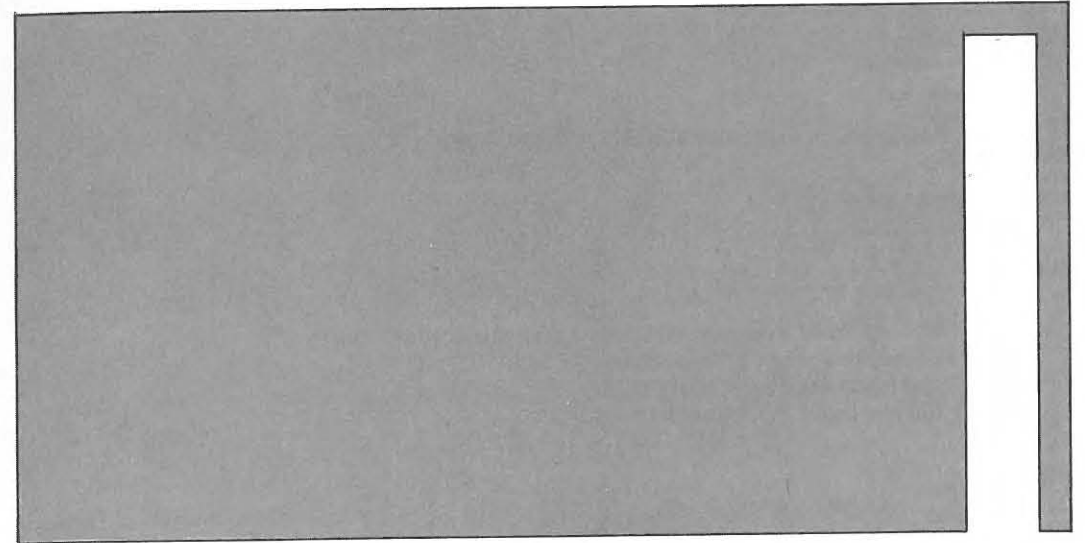
See also Table A1.3, Appendix I, and Table 2.3, Chapter 2.

William J. Harat  
May-1980

ANDRES,  
MARANDI

# GROUNDWATER





R. Allan Freeze

Department of Geological Sciences  
University of British Columbia  
Vancouver, British Columbia

John A. Cherry

Department of Earth Sciences  
University of Waterloo  
Waterloo, Ontario

# ***GROUNDWATER***

Prentice-Hall, Inc.  
Englewood Cliffs, New Jersey 07632

*Library of Congress Cataloging in Publication Data*

FREEZE, R. ALLAN.

Groundwater.

Bibliography: p.

Includes index.

I. Water, Underground. I. Cherry, John A., joint author. II. Title.

GB1003.2.F73 551.4'98 78-25796

ISBN 0-13-365312-9

Editorial/production supervision by Cathy Brenn/Kim McNeily

Interior design by Chris Gadekar

Manufacturing buyer: Harry Baisley

Chapter logos: Peter Russell

© 1979 by Prentice-Hall, Inc., Englewood Cliffs, N.J. 07632

All rights reserved. No part of this book  
may be reproduced in any form or  
by any means without permission in writing  
from the publisher.

Printed in the United States of America

10 9 8 7 6 5 4 3 2

PRENTICE-HALL INTERNATIONAL, INC., *London*

PRENTICE-HALL OF AUSTRALIA PTY. LIMITED, *Sydney*

PRENTICE-HALL OF CANADA, LTD., *Toronto*

PRENTICE-HALL OF INDIA PRIVATE LIMITED, *New Delhi*

PRENTICE-HALL OF JAPAN, INC., *Tokyo*

PRENTICE-HALL OF SOUTHEAST ASIA PTE. LTD., *Singapore*

WHITEHALL BOOKS LIMITED, *Wellington, New Zealand*

*This may well be the only book that either of us will ever write. We cannot save our dedications, as novelists do, to let them forth one by one. We recognize and appreciate the life-long influences of our parents, our wives, our families, our teachers, and our students. This book is dedicated to all of them.*

*This book is also dedicated to the taxpayers of Canada and the United States, few of whom will ever read it, but all of whom have contributed to its birth through scholarships in our student days and through research support and sabbatical periods in more recent years.*



# CONTENTS

Preface xv

## 1 Introduction 1

### 1.1 Groundwater, the Earth, and Man 2

*GROUNDWATER AND THE HYDROLOGIC CYCLE 3*

*GROUNDWATER AS A RESOURCE 6*

*GROUNDWATER CONTAMINATION 8*

*GROUNDWATER AS A GEOTECHNICAL PROBLEM 9*

*GROUNDWATER AND GEOLOGIC PROCESSES 10*

### 1.2 The Scientific Foundations for the Study of Groundwater 10

### 1.3 The Technical Foundations for the Development of Groundwater Resources 12

## 2 Physical Properties and Principles 14

### 2.1 Darcy's Law 15

### 2.2 Hydraulic Head and Fluid Potential 18

*HUBBERT'S ANALYSIS OF THE FLUID POTENTIAL 18*

*DIMENSIONS AND UNITS 22*

*PIEZOMETERS AND PIEZOMETER NESTS 23*

*COUPLED FLOW 25*

### 2.3 Hydraulic Conductivity and Permeability 26

### 2.4 Heterogeneity and Anisotropy of Hydraulic Conductivity 30

*HOMOGENEITY AND HETEROGENEITY 30*

*ISOTROPY AND ANISOTROPY 32*

*DARCY'S LAW IN THREE DIMENSIONS 34*

*HYDRAULIC CONDUCTIVITY ELLIPSOID 35*

- 2.5 Porosity and Void Ratio 36
- 2.6 Unsaturated Flow and the Water Table 38
  - MOISTURE CONTENT 39
  - WATER TABLE 39
  - NEGATIVE PRESSURE HEADS AND TENSIOMETERS 39
  - CHARACTERISTIC CURVES OF THE UNSATURATED HYDRAULIC PARAMETERS 41
  - SATURATED, UNSATURATED, AND TENSION-SATURATED ZONES 44
  - PERCHED AND INVERTED WATER TABLES 45
  - MULTIPHASE FLOW 45
- 2.7 Aquifers and Aquitards 47
  - AQUIFERS, AQUITARDS, AND AQUICLUDES 47
  - CONFINED AND UNCONFINED AQUIFERS 48
  - POTENTIOMETRIC SURFACE 49
- 2.8 Steady-State Flow and Transient Flow 49
- 2.9 Compressibility and Effective Stress 51
  - COMPRESSIBILITY OF WATER 51
  - EFFECTIVE STRESS 52
  - COMPRESSIBILITY OF A POROUS MEDIUM 54
  - AQUIFER COMPRESSIBILITY 56
  - EFFECTIVE STRESS IN THE UNSATURATED ZONE 57
- 2.10 Transmissivity and Storativity 58
  - SPECIFIC STORAGE 58
  - TRANSMISSIVITY AND STORATIVITY OF A CONFINED AQUIFER 59
  - TRANSMISSIVITY AND SPECIFIC YIELD IN UNCONFINED AQUIFERS 61
  - STORAGE IN THE UNSATURATED ZONE 62
- 2.11 Equations of Groundwater Flow 63
  - STEADY-STATE SATURATED FLOW 63
  - TRANSIENT SATURATED FLOW 64
  - TRANSIENT UNSATURATED FLOW 66
  - BOUNDARY-VALUE PROBLEMS 67
- 2.12 Limitations of the Darcian Approach 69
  - DARCIAN CONTINUUM AND REPRESENTATIVE ELEMENTARY VOLUME 69
  - SPECIFIC DISCHARGE, MACROSCOPIC VELOCITY, AND MICROSCOPIC VELOCITY 70
  - UPPER AND LOWER LIMITS OF DARCY'S LAW 72
  - FLOW IN FRACTURED ROCKS 73
- 2.13 Hydrodynamic Dispersion 75

### 3 Chemical Properties and Principles 80

- 3.1 Groundwater and Its Chemical Constituents 82
  - WATER AND ELECTROLYTES 82
  - ORGANIC CONSTITUENTS 86
  - DISSOLVED GASES 86
  - CONCENTRATION UNITS 87
- 3.2 Chemical Equilibrium 89
  - THE LAW OF MASS ACTION 89
  - ACTIVITY COEFFICIENTS 90

- EQUILIBRIUM AND FREE ENERGY 90
- DISSOLVED GASES 95
- 3.3 Association and Dissociation of Dissolved Species 96
  - THE ELECTRONEUTRALITY CONDITION 96
  - DISSOCIATION AND ACTIVITY OF WATER 97
  - POLYPROTIC ACIDS 98
  - ION COMPLEXES 100
  - CALCULATION OF DISSOLVED SPECIES 101
- 3.4 Effects of Concentration Gradients 103
- 3.5 Mineral Dissolution and Solubility 106
  - SOLUBILITY AND THE EQUILIBRIUM CONSTANT 106
  - EFFECT OF IONIC STRENGTH 107
  - THE CARBONATE SYSTEM 108
  - THE COMMON-ION EFFECT 112
  - DISEQUILIBRIUM AND THE SATURATION INDEX 112
- 3.6 Oxidation and Reduction Processes 114
  - OXIDATION STATES AND REDOX REACTIONS 114
  - CONSUMPTION OF OXYGEN AND ORGANIC MATTER 117
  - EQUILIBRIUM REDOX CONDITIONS 119
  - MICROBIOLOGICAL FACTORS 121
  - pE—pH DIAGRAMS 123
- 3.7 Ion Exchange and Adsorption 127
  - MECHANISMS 127
  - CATION EXCHANGE CAPACITY 128
  - MASS-ACTION EQUATIONS 129
- 3.8 Environmental Isotopes 134
  - CARBON-14 134
  - TRITIUM 136
  - OXYGEN AND DEUTERIUM 137
- 3.9 Field Measurement of Index Parameters 139

### 4 Groundwater Geology 144

- 4.1 Lithology, Stratigraphy, and Structure 145
- 4.2 Fluvial Deposits 147
- 4.3 Aeolian Deposits 149
- 4.4 Glacial Deposits 149
- 4.5 Sedimentary Rocks 152
  - SANDSTONE 152
  - CARBONATE ROCK 154
  - COAL 157
  - SHALE 158
- 4.6 Igneous and Metamorphic Rocks 158
- 4.7 Permafrost 163



## 5 Flow Nets 167

- 5.1 Flow Nets by Graphical Construction 168
  - HOMOGENEOUS, ISOTROPIC SYSTEMS 168*
  - HETEROGENEOUS SYSTEMS AND THE TANGENT LAW 172*
  - ANISOTROPIC SYSTEMS AND THE TRANSFORMED SECTION 174*
- 5.2 Flow Nets by Analog Simulation 178
  - CONDUCTIVE-PAPER ANALOGS 179*
  - RESISTANCE NETWORK ANALOGS 180*
- 5.3 Flow Nets by Numerical Simulation 181
- 5.4 Saturated-Unsaturated Flow Nets 185
- 5.5 The Seepage Face and Dupuit Flow 186
  - SEEPAGE FACE, EXIT POINT, AND FREE SURFACE 186*
  - DUPUIT-FORCHHEIMER THEORY OF FREE-SURFACE FLOW 188*

## 6 Groundwater and the Hydrologic Cycle 192

- 6.1 Steady-State Regional Groundwater Flow 193
  - RECHARGE AREAS, DISCHARGE AREAS, AND GROUNDWATER DIVIDES 193*
  - EFFECT OF TOPOGRAPHY ON REGIONAL FLOW SYSTEMS 195*
  - EFFECT OF GEOLOGY ON REGIONAL FLOW SYSTEMS 197*
  - FLOWING ARTESIAN WELLS 199*
  - FLOW-SYSTEM MAPPING 200*
- 6.2 Steady-State Hydrologic Budgets 203
  - QUANTITATIVE INTERPRETATION OF REGIONAL FLOW SYSTEMS 203*
  - GROUNDWATER RECHARGE AND DISCHARGE AS COMPONENTS OF A HYDROLOGIC BUDGET 205*
- 6.3 Transient Regional Groundwater Flow 208
- 6.4 Infiltration and Groundwater Recharge 211
  - THE THEORY OF INFILTRATION 211*
  - MEASUREMENTS OF FIELD SITES 215*
- 6.5 Hillslope Hydrology and Streamflow Generation 217
  - OVERLAND FLOW 218*
  - SUBSURFACE STORMFLOW 219*
  - CHEMICAL AND ISOTOPIC INDICATORS 221*
- 6.6 Baseflow Recession and Bank Storage 225
- 6.7 Groundwater-Lake Interactions 226
- 6.8 Fluctuations in Groundwater Levels 229
  - EVAPOTRANSPIRATION AND PHREATOPHYTIC CONSUMPTION 231*
  - AIR ENTRAPMENT DURING GROUNDWATER RECHARGE 231*
  - ATMOSPHERIC PRESSURE EFFECTS 233*
  - EXTERNAL LOADS 234*
  - TIME LAG IN PIEZOMETERS 234*

## 7 Chemical Evolution of Natural Groundwater 237

- 7.1 Hydrochemical Sequences and Facies 238
  - CHEMISTRY OF PRECIPITATION 238*
  - CARBON DIOXIDE IN THE SOIL ZONE 240*
  - MAJOR-ION EVOLUTION SEQUENCE 241*
  - ELECTROCHEMICAL EVOLUTION SEQUENCE 244*
- 7.2 Graphical Methods and Hydrochemical Facies 247
- 7.3 Groundwater in Carbonate Terrain 254
  - OPEN-SYSTEM DISSOLUTION 254*
  - CLOSED-SYSTEM CONDITIONS 256*
  - INCONGRUENT DISSOLUTION 257*
  - OTHER FACTORS 259*
  - INTERPRETATION OF CHEMICAL ANALYSES 261*
- 7.4 Groundwater in Crystalline Rocks 268
  - THEORETICAL CONSIDERATIONS 269*
  - LABORATORY EXPERIMENTS 273*
  - INTERPRETATION OF FIELD DATA 275*
- 7.5 Groundwater in Complex Sedimentary Systems 279
  - ORDER OF ENCOUNTER 280*
  - WATER COMPOSITION IN GLACIAL DEPOSITS 284*
  - GROUNDWATER IN STRATIFIED SEDIMENTARY ROCKS 286*
- 7.6 Geochemical Interpretation of  $^{14}\text{C}$  Dates 290
- 7.7 Membrane Effects in Deep Sedimentary Basins 292
- 7.8 Process Rates and Molecular Diffusion 295

## 8 Groundwater Resource Evaluation 303

- 8.1 Development of Groundwater Resources 304
  - EXPLORATION, EVALUATION, AND EXPLOITATION 304*
  - WELL YIELD, AQUIFER YIELD, AND BASIN YIELD 305*
- 8.2 Exploration for Aquifers 306
  - SURFACE GEOLOGICAL METHODS 306*
  - SUBSURFACE GEOLOGICAL METHODS 307*
  - SURFACE GEOPHYSICAL METHODS 308*
  - SUBSURFACE GEOPHYSICAL METHODS 309*
  - DRILLING AND INSTALLATION OF WELLS AND PIEZOMETERS 312*
- 8.3 The Response of Ideal Aquifers to Pumping 314
  - RADIAL FLOW TO A WELL 315*
  - THE THEIS SOLUTION 317*
  - LEAKY AQUIFERS 320*
  - UNCONFINED AQUIFERS 324*
  - MULTIPLE-WELL SYSTEMS, STEPPED PUMPING RATES, WELL RECOVERY, AND PARTIAL PENETRATION 327*
  - BOUNDED AQUIFERS 330*
  - THE RESPONSE OF IDEAL AQUITARDS 332*
  - THE REAL WORLD 334*



- 8.4 Measurement of Parameters: Laboratory Tests 335
  - HYDRAULIC CONDUCTIVITY 335
  - POROSITY 337
  - COMPRESSIBILITY 337
  - UNSATURATED CHARACTERISTIC CURVES 339
- 8.5 Measurement of Parameters: Piezometer Tests 339
- 8.6 Measurement of Parameters: Pumping Tests 343
  - LOG-LOG TYPE-CURVE MATCHING 343
  - SEMILOG PLOTS 347
  - ADVANTAGES AND DISADVANTAGES OF PUMPING TESTS 349
- 8.7 Estimation of Saturated Hydraulic Conductivity 350
- 8.8 Prediction of Aquifer Yield by Numerical Simulation 352
  - FINITE-DIFFERENCE METHODS 352
  - FINITE-ELEMENT METHODS 356
  - MODEL CALIBRATION AND THE INVERSE PROBLEM 356
- 8.9 Prediction of Aquifer Yield by Analog Simulation 359
  - ANALOGY BETWEEN ELECTRICAL FLOW AND GROUNDWATER FLOW 360
  - RESISTANCE-CAPACITANCE NETWORK 361
  - COMPARISON OF ANALOG AND DIGITAL SIMULATION 363
- 8.10 Basin Yield 364
  - SAFE YIELD AND OPTIMAL YIELD OF A GROUNDWATER BASIN 364
  - TRANSIENT HYDROLOGIC BUDGETS AND BASIN YIELD 365
- 8.11 Artificial Recharge and Induced Infiltration 367
- 8.12 Land Subsidence 370
  - MECHANISM OF LAND SUBSIDENCE 370
  - FIELD MEASUREMENT OF LAND SUBSIDENCE 373
- 8.13 Seawater Intrusion 375

## 9 Groundwater Contamination 383

- 9.1 Water Quality Standards 385
- 9.2 Transport Processes 388
  - NONREACTIVE CONSTITUENTS IN HOMOGENEOUS MEDIA 389
  - NONREACTIVE CONSTITUENTS IN HETEROGENEOUS MEDIA 397
  - TRANSPORT OF REACTIVE CONSTITUENTS 402
  - TRANSPORT IN FRACTURED MEDIA 408
- 9.3 Hydrochemical Behavior of Contaminants 413
  - NITROGEN 413
  - TRACE METALS 416
  - TRACE NONMETALS 420
  - ORGANIC SUBSTANCES 424
- 9.4 Measurement of Parameters 426
  - VELOCITY DETERMINATION 426
  - DISPERSIVITY 430
  - CHEMICAL PARTITIONING 432
- 9.5 Sources of Contamination 434
  - LAND DISPOSAL OF SOLID WASTES 434
  - SEWAGE DISPOSAL ON LAND 439

- AGRICULTURAL ACTIVITIES 442
- PETROLEUM LEAKAGE AND SPILLS 444
- DISPOSAL OF RADIOACTIVE WASTE 447
- DEEP-WELL DISPOSAL OF LIQUID WASTES 454
- OTHER SOURCES 456

## 10 Groundwater and Geotechnical Problems 463

- 10.1 Pore Pressures, Landslides, and Slope Stability 464
  - MOHR-COULOMB FAILURE THEORY 465
  - LIMIT EQUILIBRIUM METHODS OF SLOPE STABILITY ANALYSIS 467
  - EFFECT OF GROUNDWATER CONDITIONS ON SLOPE STABILITY IN SOILS 470
  - EFFECT OF GROUNDWATER CONDITIONS ON SLOPE STABILITY IN ROCK 472
- 10.2 Groundwater and Dams 475
  - TYPES OF DAMS AND DAM FAILURES 476
  - SEEPAGE UNDER CONCRETE DAMS 477
  - GROUTING AND DRAINAGE OF DAM FOUNDATIONS 479
  - STEADY-STATE SEEPAGE THROUGH EARTH DAMS 481
  - TRANSIENT SEEPAGE THROUGH EARTH DAMS 483
  - HYDROGEOLOGIC IMPACT OF RESERVOIRS 485
- 10.3 Groundwater Inflows Into Tunnels 487
  - A TUNNEL AS A STEADY-STATE OR TRANSIENT DRAIN 488
  - HYDROGEOLOGIC HAZARDS OF TUNNELING 489
  - PREDICTIVE ANALYSIS OF GROUNDWATER INFLOWS INTO TUNNELS 490
- 10.4 Groundwater Inflows Into Excavations 491
  - DRAINAGE AND DEWATERING OF EXCAVATIONS 492
  - PREDICTIVE ANALYSIS OF GROUNDWATER INFLOWS INTO EXCAVATIONS 494

## 11 Groundwater and Geologic Processes 497

- 11.1 Groundwater and Structural Geology 498
  - HUBBERT-RUBEY THEORY OF OVERTHRUST FAULTING 498
  - EARTHQUAKE PREDICTION AND CONTROL 500
- 11.2 Groundwater and Petroleum 502
  - MIGRATION AND ACCUMULATION OF PETROLEUM 503
  - HYDRODYNAMIC ENTRAPMENT OF PETROLEUM 504
  - REGIONAL FLOW SYSTEMS AND PETROLEUM ACCUMULATIONS 506
  - IMPLICATIONS FOR PETROLEUM EXPLORATION 507
- 11.3 Groundwater and Thermal Processes 507
  - THERMAL REGIMES IN NATURAL GROUNDWATER FLOW SYSTEMS 508
  - GEOTHERMAL SYSTEMS 510
  - PLUTON EMPLACEMENT 512
- 11.4 Groundwater and Geomorphology 513
  - KARST AND CAVES 513
  - NATURAL SLOPE DEVELOPMENT 515
  - FLUVIAL PROCESSES 516
  - GLACIAL PROCESSES 517

**11.5 Groundwater and Economic Mineralization 519***GENESIS OF ECONOMIC MINERAL DEPOSITS 519**IMPLICATIONS FOR GEOCHEMICAL EXPLORATION 521***Appendices 525**

- I Elements of Fluid Mechanics 526**
- II Equation of Flow for Transient Flow Through Deforming Saturated Media 531**
- III Example of an Analytical Solution to a Boundary-Value Problem 534**
- IV Debye-Hückel Equation and Kielland Table for Ion-Activity Coefficients 536**
- V Complementary Error Function (erfc) 539**
- VI Development of Finite-Difference Equation for Steady-State Flow in a Homogeneous, Isotropic Medium 540**
- VII Tóth's Analytical Solution for Regional Groundwater Flow 542**
- VIII Numerical Solution of the Boundary-Value Problem Representing One-Dimensional Infiltration Above a Recharging Groundwater Flow System 544**
- IX Development of Finite-Difference Equation for Transient Flow in a Heterogeneous, Anisotropic, Horizontal, Confined Aquifer 546**
- X Derivation of the Advection-Dispersion Equation for Solute Transport in Saturated Porous Media 549**

**References 555****Index 589**

# PREFACE

We perceive a trend in the study and practice of groundwater hydrology. We see a science that is emerging from its geological roots and its early hydraulic applications into a full-fledged environmental science. We see a science that is becoming more interdisciplinary in nature and of greater importance in the affairs of man.

This book is our response to these perceived trends. We have tried to provide a text that is suited to the study of groundwater during this period of emergence. We have made a conscious attempt to integrate geology and hydrology, physics and chemistry, and science and engineering to a greater degree than has been done in the past.

This book is designed for use as a text in introductory groundwater courses of the type normally taught in the junior or senior year of undergraduate geology, geological engineering, or civil engineering curricula. It has considerably more material than can be covered in a course of one-semester duration. Our intention is to provide a broad coverage of groundwater topics in a manner that will enable course instructors to use selected chapters or chapter segments as a framework for a semester-length treatment. The remaining material can serve as a basis for a follow-up undergraduate course with more specialization or as source material for an introductory course at the graduate level. We recognize that the interdisciplinary approach may create some difficulties for students grounded only in the earth sciences, but we are convinced that the benefits of the approach far outweigh the cost of the additional effort that is required.

The study of groundwater at the introductory level requires an understanding of many of the basic principles of geology, physics, chemistry, and mathematics. This text is designed for students who have a knowledge of these subjects at the level normally covered in freshman university courses. Additional background in these subjects is, of course, desirable. Elementary calculus is used frequently in several of the chapters. Although knowledge of topics of more advanced calculus is definitely an asset to students wishing to pursue specialized groundwater topics, we hope that for students without this background this text will serve as a pathway



to the understanding of the basic physical principles of groundwater flow. Differential equations have been used very sparingly, but are included where we view their use as essential. The physical meaning of the equations and their boundary conditions is held paramount. To avoid mathematical disruptions in continuity of presentation of physical concepts, detailed derivations and solution methods are restricted to the appendices.

Until recently, groundwater courses at the university level were normally viewed in terms of only the geologic and hydraulic aspects of the topic. In response to the increasing importance of natural groundwater quality and groundwater contamination by man, we have included three major chapters primarily chemical in emphasis. We assume that the reader is conversant with the usual chemical symbols and can write and balance equations for inorganic chemical reactions. On this basis, we describe the main principles of physical chemistry that are necessary for an introductory coverage of the geochemical aspects of the groundwater environment. Students wishing for a more advanced treatment of these topics would require training in thermodynamics at a level beyond the scope of this text.

Although we have attempted to provide a broad interdisciplinary coverage of groundwater principles, we have not been able to include detailed information on the technical aspects of such topics as well design and installation, operation of well pumps, groundwater sampling methods, procedures for chemical analysis of groundwater, and permeameter and consolidation tests. The principles of these practical and important techniques are discussed in the text but the operational aspects must be gleaned from the many manuals and technical papers cited throughout the text.

#### Acknowledgments

The manuscript for this text was reviewed in its entirety by Pat Domenico, Eugene Simpson, and Dave Stephenson. Their comments and suggestions aided us immeasurably in arriving at the final presentation. We are also indebted to Bill Back, Lee Clayton, Shlomo Neuman, Eric Reardon, and Leslie Smith, who provided valuable reviews of portions of the book. In addition, we requested and received help on individual sections from Bob Gillham, Gerry Grisak, Bill Mathews, Dave McCoy, Steve Moran, Nari Narasimhan, Frank Patton, John Pickens, Doug Piteau, Joe Poland, Dan Reynolds, and Warren Wood. In addition, we would be remiss not to recognize the vital influence of our long-time associations with Paul Witherspoon and Bob Farvolden.

We also owe a debt to the many graduate and undergraduate students in groundwater hydrology at U.B.C. and Waterloo who identified flaws in the presentation and who acted as guinea pigs on the problem sets.

R. ALLAN FREEZE  
*Vancouver, British Columbia*  
JOHN A. CHERRY  
*Waterloo, Ontario*

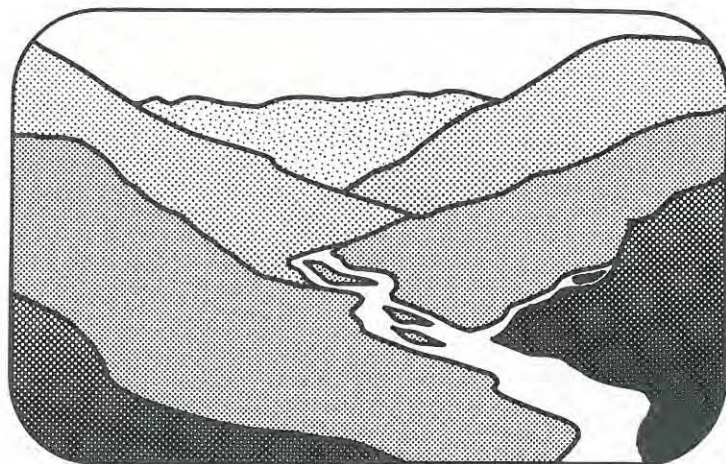
# GROUNDWATER



# CHAPTER

# 1

Introduction



## 1.1 Groundwater, the Earth, and Man

This book is about groundwater. It is about the geological environments that control the occurrence of groundwater. It is about the physical laws that describe the flow of groundwater. It is about the chemical evolution that accompanies flow. It is also about the influence of man on the natural groundwater regime; and the influence of the natural groundwater regime on man.

The term *groundwater* is usually reserved for the subsurface water that occurs beneath the water table in soils and geologic formations that are fully saturated. We shall retain this classical definition, but we do so in full recognition that the study of groundwater must rest on an understanding of the subsurface water regime in a broader sense. Our approach will be compatible with the traditional emphasis on shallow, saturated, groundwater flow; but it will also encompass the near-surface, unsaturated, soil-moisture regime that plays such an important role in the hydrologic cycle, and it will include the much deeper, saturated regimes that have an important influence on many geologic processes.

We view the study of groundwater as interdisciplinary in nature. There is a conscious attempt in this text to integrate chemistry and physics, geology and hydrology, and science and engineering to a greater degree than has been done in the past. The study of groundwater is germane to geologists, hydrologists, soil scientists, agricultural engineers, foresters, geographers, ecologists, geotechnical engineers, mining engineers, sanitary engineers, petroleum reservoir analysts, and probably others. We hope that our introductory treatment is in tune with these broad interdisciplinary needs.

If this book had been written a decade ago, it would have dealt almost entirely with groundwater as a resource. The needs of the time would have dictated that approach, and books written in that era reflected those needs. They emphasize the development of water supplies through wells and the calculation of aquifer yields. The groundwater problems viewed as such are those that threaten that yield. The

water supply aspects of groundwater are still important and they will be treated in this text with the deference they deserve. But groundwater is more than a resource. It is an important feature of the natural environment; it leads to environmental problems, and may in some cases offer a medium for environmental solutions. It is part of the hydrologic cycle, and an understanding of its role in this cycle is mandatory if integrated analyses are to be promoted in the consideration of watershed resources, and in the regional assessment of environmental contamination. In an engineering context, groundwater contributes to such geotechnical problems as slope stability and land subsidence. Groundwater is also a key to understanding a wide variety of geological processes, among them the generation of earthquakes, the migration and accumulation of petroleum, and the genesis of certain types of ore deposits, soil types, and landforms.

The first five chapters of this book lay the physical, chemical, and geologic foundations for the study of groundwater. The final six chapters apply these principles in the several spheres of interaction between groundwater, the earth, and man. The following paragraphs can be viewed as an introduction to each of the later chapters.

### Groundwater and the Hydrologic Cycle

The endless circulation of water between ocean, atmosphere, and land is called the *hydrologic cycle*. Our interest centers on the land-based portion of the cycle as it might be operative on an individual watershed. Figures 1.1 and 1.2 provide two schematic diagrams of the hydrologic cycle on a watershed. They are introduced here primarily to provide the reader with a diagrammatic introduction to hydro-

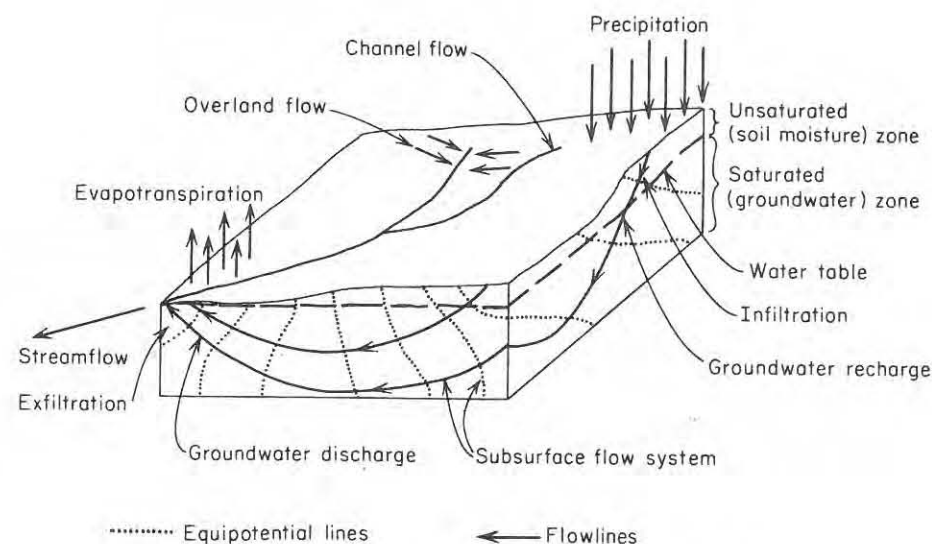


Figure 1.1 Schematic representation of the hydrologic cycle.



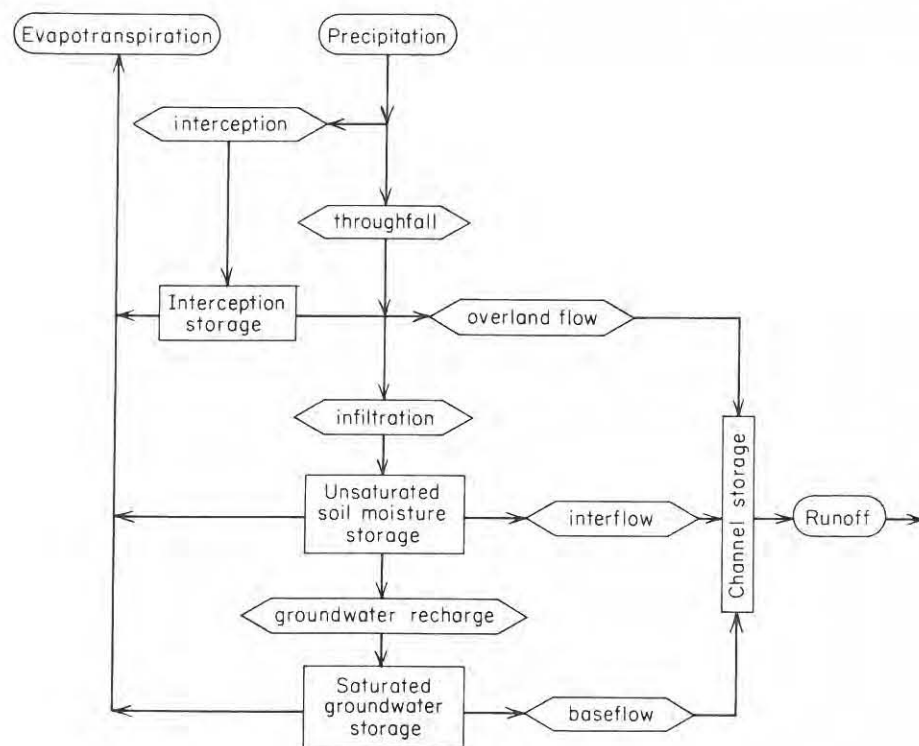


Figure 1.2 Systems representation of the hydrologic cycle.

logic terminology. Figure 1.1 is conceptually the better in that it emphasizes processes and illustrates the flow-system concept of the hydrologic cycle. The pot-and-pipeline representation of Figure 1.2 is often utilized in the systems approach to hydrologic modeling. It fails to reflect the dynamics of the situation, but it does differentiate clearly between those terms that involve rates of movement (in the hexagonal boxes) and those that involve storage (in the rectangular boxes).

Inflow to the hydrologic system arrives as *precipitation*, in the form of rainfall or snowmelt. Outflow takes place as *streamflow* (or runoff) and as *evapotranspiration*, a combination of evaporation from open bodies of water, evaporation from soil surfaces, and transpiration from the soil by plants. Precipitation is delivered to streams both on the land surface, as *overland flow* to tributary channels; and by subsurface flow routes, as *interflow* and *baseflow* following *infiltration* into the soil. Figure 1.1 makes it clear that a watershed must be envisaged as a combination of both the surface drainage area and the parcel of subsurface soils and geologic formations that underlie it. The subsurface hydrologic processes are just as important as the surface processes. In fact, one could argue that they are more important, for it is the nature of the subsurface materials that controls infiltration rates, and

the infiltration rates influence the timing and spatial distribution of surface runoff. In Chapter 6, we will examine the nature of regional groundwater flow patterns in some detail, and we will investigate the relations among infiltration, groundwater recharge, groundwater discharge, baseflow, and streamflow generation. In Chapter 7, we will look at the chemical evolution of groundwater that accompanies its passage through the subsurface portion of the hydrologic cycle.

Before closing this section, it is worth looking at some data that reflect the quantitative importance of groundwater relative to the other components of the hydrologic cycle. In recent years there has been considerable attention paid to the concept of the *world water balance* (Nace, 1971; Lvovitch, 1970; Sutcliffe, 1970), and the most recent estimates of these data emphasize the ubiquitous nature of groundwater in the hydrosphere. With reference to Table 1.1, if we remove from consideration the 94% of the earth's water that rests in the oceans and seas at high levels of salinity, then groundwater accounts for about two-thirds of the freshwater resources of the world. If we limit consideration to the utilizable freshwater resources (minus the icecaps and glaciers), groundwater accounts for almost the total volume. Even if we consider only the most "active" groundwater regimes, which Lvovitch (1970) estimates at  $4 \times 10^6 \text{ km}^3$  (rather than the  $60 \times 10^6 \text{ km}^3$  of Table 1.1), the freshwater breakdown comes to: groundwater, 95%; lakes, swamps, reservoirs, and river channels, 3.5%; and soil moisture, 1.5%.

Table 1.1 Estimate of the Water Balance of the World

Parameter	Surface area ( $\text{km}^2 \times 10^6$ )	Volume ( $\text{km}^3 \times 10^6$ )	Volume (%)	Equivalent depth (m)*	Residence time
Oceans and seas	361	1370	94	2500	~4000 years
Lakes and reservoirs	1.55	0.13	<0.01	0.25	~10 years
Swamps	<0.1	<0.01	<0.01	0.007	1-10 years
River channels	<0.1	<0.01	<0.01	0.003	~2 weeks
Soil moisture	130	0.07	<0.01	0.13	2 weeks-1 year
Groundwater	130	60	4	120	2 weeks-10,000 years
Icecaps and glaciers	17.8	30	2	60	10-1000 years
Atmospheric water	504	0.01	<0.01	0.025	~10 days
Biospheric water	<0.1	<0.01	<0.01	0.001	~1 week

SOURCE: Nace, 1971.

\*Computed as though storage were uniformly distributed over the entire surface of the earth.

This volumetric superiority, however, is tempered by the average residence times. River water has a turnover time on the order of 2 weeks. Groundwater, on the other hand, moves slowly, and residence times in the 10's, 100's, and even 1000's of years are not uncommon. The principles laid out in Chapter 2 and the regional flow considerations of Chapter 6 will clarify the hydrogeologic controls on the large-scale movement of groundwater.

Most hydrology texts contain detailed discussions of the hydrologic cycle and of the global water balance. Wisler and Brater (1959) and Linsley, Kohler, and



Paulhus (1975) are widely used introductory hydrology texts. A recent text by Eagleson (1970) updates the science at a more advanced level. The massive *Handbook of Applied Hydrology*, edited by Chow (1964a), is a valuable reference.

The history of development of hydrological thought is an interesting study. Chow (1964b) provides a concise discussion; Biswas' (1970) booklength study provides a wealth of detail, from the contributions of the early Egyptians and the Greek and Roman philosophers, right up to and through the birth of scientific hydrology in western Europe in the eighteenth and nineteenth centuries.

### Groundwater as a Resource

The primary motivation for the study of groundwater has traditionally been its importance as a resource. For the United States, the significance of the role of groundwater as a component of national water use can be gleaned from the statistical studies of the U.S. Geological Survey as reported most recently for the year 1970 by Murray and Reeves (1972) and summarized by Murray (1973).

Table 1.2 documents the growth in water utilization in the United States during the period 1950–1970. In 1970 the nation used  $1400 \times 10^6 \text{ m}^3/\text{day}$ . Of this, 57% went for industrial use and 35% for irrigation. Surface water provided 81% of the total, groundwater 19%. Figure 1.3 graphically illustrates the role of groundwater relative to surface water in the four major areas of use for the 1950–1970 period. Groundwater is less important in industrial usage, but it provides a significant percentage of the supply for domestic use, both rural and urban, and for irrigation.

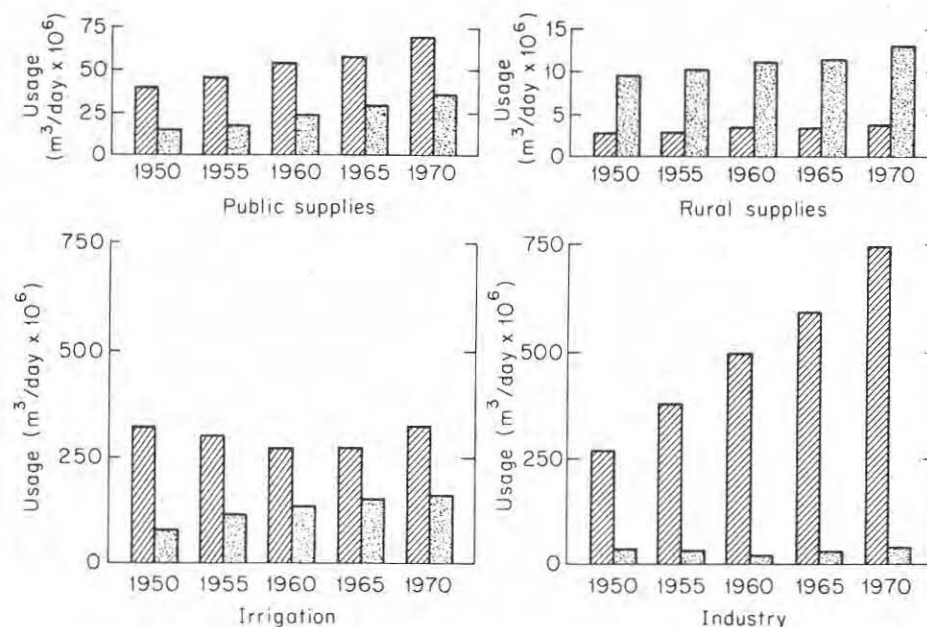
The data of Table 1.2 and Figure 1.3 obscure some striking regional variations. About 80% of the total irrigation use occurs in the 17 western states, whereas 84% of the industrial use is in the 31 eastern states. Groundwater is more widely used in the west, where it accounts for 46% of the public supply and 44% of the industrial use (as opposed to 29% and 16%, respectively, in the east).

**Table 1.2 Water Use in the United States, 1950–1970**

	Cubic meters/day $\times 10^6$ *					Percent of 1970 use
	1950	1955	1960	1965	1970	
Total water withdrawals	758	910	1023	1175	1400	100
Use						
Public supplies	53	64	80	91	102	7
Rural supplies	14	14	14	15	17	1
Irrigation	420	420	420	455	495	35
Industrial	292	420	560	667	822	57
Source						
Groundwater	130	182	190	227	262	19
Surface water	644	750	838	960	1150	81

SOURCE: Murray, 1973.

\* $1 \text{ m}^3 = 10^3 \ell = 264 \text{ U.S. gal.}$



**Figure 1.3** Surface water (hatched) and groundwater (stippled) use in the United States, 1950–1970 (after Murray, 1973).

In Canada, rural and municipal groundwater use was estimated by Meyboom (1968) at  $1.71 \times 10^6 \text{ m}^3/\text{day}$ , or 20% of the total rural and municipal water consumption. This level of groundwater use is considerably lower than that of the United States, even when one considers the population ratio between the two countries. A more detailed look at the figures shows that rural groundwater development in Canada is relatively on a par with rural development in the United States, but municipal groundwater use is significantly smaller. The most striking differences lie in irrigation and industrial use, where the relative total water consumption in Canada is much less than in the United States and the groundwater component of this use is extremely small.

McGuinness (1963), quoting a U.S. Senate committee study, has provided predictions of future U.S. national water requirements. It is suggested that water needs will reach  $1700 \times 10^6 \text{ m}^3/\text{day}$  by 1980 and  $3360 \times 10^6 \text{ m}^3/\text{day}$  by the year 2000. The attainment of these levels of production would represent a significant acceleration in the rate of increase in water use outlined in Table 1.2. The figure for the year 2000 begins to approach the total water resource potential of the nation, which is estimated to be about  $4550 \times 10^6 \text{ m}^3/\text{day}$ . If the requirements are to be met, it is widely accepted that groundwater resources will have to provide a greater proportion of the total supply. McGuinness notes that for the predictions above, if the percent groundwater contribution is to increase from 19% to 33%, groundwater usage would have to increase from its current  $262 \times 10^6 \text{ m}^3/\text{day}$  to  $705 \times$



$10^6 \text{ m}^3/\text{day}$  in 1980 and  $1120 \times 10^6 \text{ m}^3/\text{day}$  in the year 2000. He notes that the desirable properties of groundwater, such as its clarity, bacterial purity, consistent temperature, and chemical quality, may encourage the needed large-scale development, but he warns that groundwater, especially when large quantities are sought, is inherently more difficult and expensive to locate, to evaluate, to develop, and to manage than surface water. He notes, as we have, that groundwater is an integral phase of the hydrologic cycle. The days when groundwater and surface water could be regarded as two separate resources are past. Resource planning must be carried out with the realization that groundwater and surface water have the same origin.

In Chapter 8, we will discuss the techniques of groundwater resource evaluation: from the geologic problems of aquifer exploration, through the field and laboratory methods of parameter measurement and estimation, to the simulation of well performance, aquifer yield, and basin-wide groundwater exploitation.

### Groundwater Contamination

If groundwater is to continue to play an important role in the development of the world's water-resource potential, then it will have to be protected from the increasing threat of subsurface contamination. The growth of population and of industrial and agricultural production since the second world war, coupled with the resulting increased requirements for energy development, has for the first time in man's history begun to produce quantities of waste that are greater than that which the environment can easily absorb. The choice of a waste-disposal method has become a case of choosing the least objectionable course from a set of objectionable alternatives. As shown schematically on Figure 1.4, there are no currently-feasible, large-scale waste disposal methods that do not have the potential for serious pollution of some part of our natural environment. While there has been a growing concern over air- and surface-water pollution, this activism has not yet encompassed the subsurface environment. In fact, the pressures to reduce surface pollution are in part responsible for the fact that those in the waste management field are beginning to covet the subsurface environment for waste disposal. Two of the disposal techniques that are now being used and that are viewed most optimistically for the future are deep-well injection of liquid wastes and sanitary landfill for solid wastes. Both these techniques can lead to subsurface pollution. In addition, subsurface pollution can be caused by leakage from ponds and lagoons which are widely used as components of larger waste-disposal systems, and by leaching of animal wastes, fertilizers, and pesticides from agricultural soils.

In Chapter 9 we will consider the analysis of groundwater contamination. We will treat the principles and processes that allow us to analyze the general problems of municipal and industrial waste disposal, as well as some more specialized problems associated with agricultural activities, petroleum spills, mining activities, and radioactive waste. We will also discuss contamination of coastal groundwater supplies by salt-water intrusion. In all of these problems, physical considerations of groundwater flow must be coupled with the chemical properties and principles

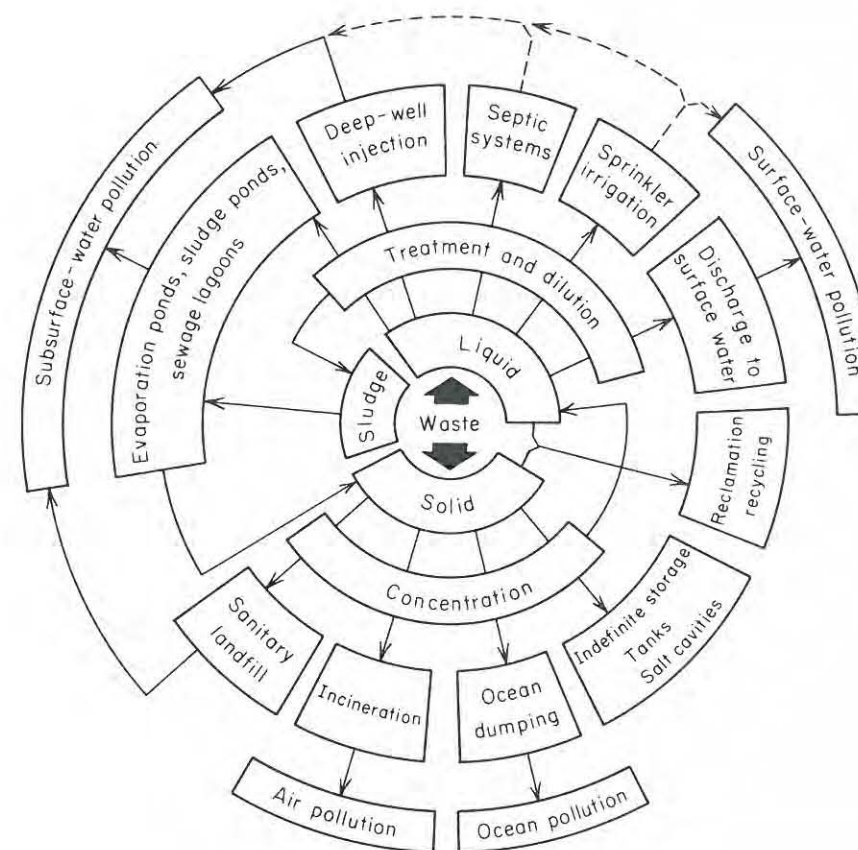


Figure 1.4 Spectrum of waste disposal alternatives.

introduced in Chapter 3; and the coupling must be carried out in light of the concepts of natural geochemical evolution discussed in Chapter 7.

### Groundwater as a Geotechnical Problem

Groundwater is not always a blessing. During the construction of the San Jacinto tunnel in California, tunnel driving on this multi-million-dollar water aqueduct was held up for many months as a result of massive unexpected inflows of groundwater from a system of highly fractured fault zones.

In Mexico City during the period 1938–1970, parts of the city subsided as much as 8.5 m. Differential settlements still provide severe problems for engineering design. The primary cause of the subsidence is now recognized to be excessive groundwater withdrawals from subsurface aquifers.

At the Jerome dam in Idaho, the dam “failed,” not through any structural weakness in the dam itself, but for the simple reason that the dam would not hold



water. The groundwater flow systems set up in the rock formations adjacent to the reservoir provided leakage routes of such efficiency that the dam had to be abandoned.

At the proposed Revelstoke dam in British Columbia, several years of exploratory geological investigation were carried out on an ancient landslide that was identified on the reservoir bank several miles above the damsite. The fear lay in the possibility that increased groundwater pressures in the slide caused by the impoundment of the reservoir could retrigger slope instability. An event of this type took almost 2500 lives in 1963 in the infamous Vaiont reservoir disaster in Italy. At the Revelstoke site, a massive drainage program was carried out to ensure that the Vaiont experience would not be repeated.

In Chapter 10 we will explore the application of the principles of groundwater flow to these types of geotechnical problems and to others. Some of the problems, such as leakage at dams and inflows to tunnels and open pit mines, arise as a consequence of excessive rates and quantities of groundwater flow. For others, such as land subsidence and slope instability, the influence arises from the presence of excessive fluid pressures in the groundwater rather than from the rate of flow itself. In both cases, flow-net construction, which is introduced in Chapter 5, is a powerful analytic tool.

#### *Groundwater and Geologic Processes*

There are very few geologic processes that do not take place in the presence of groundwater. For example, there is a close interrelationship between groundwater flow systems and the geomorphological development of landforms, whether by fluvial processes and glacial processes, or by natural slope development. Groundwater is the most important control on the development of karst environments.

Groundwater plays a role in the concentration of certain economic mineral deposits, and in the migration and accumulation of petroleum.

Perhaps the most spectacular geologic role played by groundwater lies in the control that fluid pressures exert on the mechanisms of faulting and thrusting. One exciting recent outgrowth of this interaction is the suggestion that it may be possible to control earthquakes on active faults by manipulating the natural fluid pressures in the fault zones.

In Chapter 11, we will delve more deeply into the role of groundwater as an agent in various geologic processes.

## **1.2 The Scientific Foundations for the Study of Groundwater**

The study of groundwater requires knowledge of many of the basic principles of geology, physics, chemistry and mathematics. For example, the flow of groundwater in the natural environment is strongly dependent on the three-dimensional configuration of geologic deposits through which flow takes place. The groundwater

hydrologist or geologist must therefore have some background in the interpretation of geologic evidence, and some flair for the visualization of geologic environments. He should have training in sedimentation and stratigraphy, and an understanding of the processes that lead to the emplacement of volcanic and intrusive igneous rocks. He should be familiar with the basic concepts of structural geology and be able to recognize and predict the influence of faulting and folding on geologic systems. Of particular importance to the student of groundwater is an understanding of the nature of surficial deposits and landforms. A large proportion of groundwater flow and a significant percentage of groundwater resource development takes place in the unconsolidated surficial deposits created by fluvial, lacustrine, glacial, deltaic, and aeolian geologic processes. In the northern two-thirds of North America an understanding of the occurrence and flow of groundwater rests almost entirely on an understanding of the glacial geology of the Pleistocene deposits.

Geology provides us with a qualitative knowledge of the framework of flow, but it is physics and chemistry that provide the tools for quantitative analysis. Groundwater flow exists as a *field* just as heat and electricity do, and previous exposure to these more classic fields provides good experience for the analysis of groundwater flow. The body of laws that controls the flow of groundwater is a special case of that branch of physics known as fluid mechanics. Some understanding of the basic mechanical properties of fluids and solids, and a dexterity with their dimensions and units, will aid the student in grasping the laws of groundwater flow. Appendix I provides a review of the elements of fluid mechanics. Any reader who does not feel facile with such concepts as density, pressure, energy, work, stress, and head would be well advised to peruse the appendix before attacking Chapter 2. If a more detailed treatment of fluid mechanics is desired, Streeter (1962) and Vennard (1961) are standard texts; Albertson and Simons (1964) provide a useful short review. For the specific topic of flow through porous media, a more advanced treatment of the physics than is attempted in this text can be found in Scheidegger (1960) and Collins (1961), and especially in Bear (1972).

The analysis of the natural chemical evolution of groundwater and of the behavior of contaminants in groundwater requires use of some of the principles of inorganic and physical chemistry. These principles have long been part of the methodology of geochemists and have in recent decades come into common use in groundwater studies. Principles and techniques from the field of nuclear chemistry are now contributing to our increased understanding of the groundwater environment. Naturally-occurring stable and radioactive isotopes, for example, are being used to determine the age of water in subsurface systems.

Groundwater hydrology is a quantitative science, so it should come as no surprise to find that mathematics is its language, or at the very least one of its principal dialects. It would be almost impossible, and quite foolish, to ignore the powerful tools of the groundwater trade that rest on an understanding of mathematics. The mathematical methods upon which classical studies of groundwater flow are based were borrowed by the early researchers in the field from areas of applied mathematics originally developed for the treatment of problems of heat



flow, electricity, and magnetism. With the advent of the digital computer and its widespread availability, many of the important recent advances in the analysis of groundwater systems have been based on much different mathematical approaches generally known as numerical methods. Although in this text neither the classical analytical methods nor numerical methods are pursued in any detail, our intention has been to include sufficient introductory material to illustrate some of the more important concepts.

Our text is certainly not the first to be written on groundwater. There is much material of interest in several earlier texts. Todd (1959) has for many years been the standard introductory engineering text in groundwater hydrology. Davis and De Wiest (1966) place a much heavier emphasis on the geology. For a text totally committed to the resource evaluation aspects of groundwater, there are none better than Walton (1970), and Kruseman and De Ridder (1970). A more recent text by Domenico (1972) differs from its predecessors in that it presents the basic theory in the context of hydrologic systems modeling. Among the best texts from abroad are those of Schoeller (1962), Bear, Zaslavsky, and Irmay (1968), Custodio and Llamas (1974), and the advanced Russian treatise of Polubarinova-Kochina (1962).

There are several other applied earth sciences that involve the flow of fluids through porous media. There is a close kinship between groundwater hydrology, soil physics, soil mechanics, rock mechanics, and petroleum reservoir engineering. Students of groundwater will find much of interest in textbooks from these fields such as Baver, Gardner, and Gardner (1972), Kirkham and Powers (1972), Scott (1963), Jaeger and Cook (1969), and Pirson (1958).

### 1.3 The Technical Foundations for the Development of Groundwater Resources

The first two sections of this chapter provide an introduction to the topics we plan to cover in this text. It is equally important that we set down what we do not intend to cover. Like most applied sciences, the study of groundwater can be broken into three broad aspects: science, engineering, and technology. This textbook places heavy emphasis on the scientific principles; it includes much in the way of engineering analysis; it is *not* in any sense a handbook on the technology.

Among the technical subjects that are not discussed in any detail are: methods of drilling; the design, construction, and maintenance of wells; and geophysical logging and sampling. All are required knowledge for the complete groundwater specialist, but all are treated well elsewhere, and all are learned best by experience rather than rote.

There are several books (Briggs and Fiedler, 1966; Gibson and Singer, 1971; Campbell and Lehr, 1973; U.S. Environmental Protection Agency, 1973a, 1976) that provide technical descriptions of the various types of water well drilling equipment. They also contain information on the design and setting of well screens, the selection and installation of pumps, and the construction and maintenance of wells.

On the subject of geophysical logging of boreholes, the standard reference in the petroleum industry, where most of the techniques arose, is Pirson (1963). Patten and Bennett (1963) discuss the various techniques with specific reference to groundwater exploration. We will give brief mention to subsurface drilling and borehole logging in Section 8.2, but the reader who wants to see examples in greater number in the context of case histories of groundwater resource evaluation is directed to Walton (1970).

There is one other aspect of groundwater that is technical, but in a different sense, that is not considered in this text. We refer to the subject of groundwater law. The development and management of groundwater resources must take place within the framework of water rights set down by existing legislation. Such legislation is generally established at the state or provincial level, and the result in North America is a patchwork quilt of varying traditions, rights, and statutes. Piper (1960) and Dewsnut et al. (1973) have assessed the situation in the United States. Thomas (1958) has drawn attention to some of the paradoxes that arise out of conflicts between hydrology and the law.

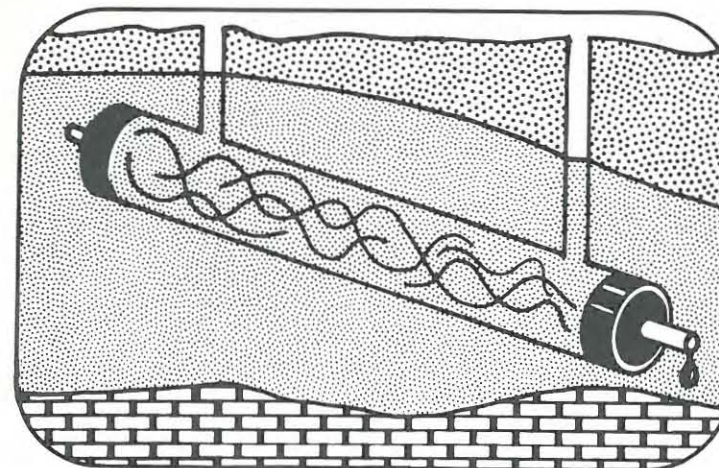
### Suggested Readings

- CHOW, V. T. 1964. Hydrology and its development. *Handbook of Applied Hydrology*, ed. V. T. Chow. McGraw-Hill, New York, pp. 1.1-1.22.
- MCGUINNESS, C. L. 1963. The role of groundwater in the national water situation. *U.S. Geol. Surv. Water-Supply Paper 1800*.
- MURRAY, C. R. 1973. Water use, consumption, and outlook in the U.S. in 1970. *J. Amer. Water Works Assoc.*, 65, pp. 302-308.
- NACE, R. L., ed. 1971. Scientific framework of world water balance. *UNESCO Tech. Papers Hydrol.*, 7, 27 pp.



# 2 CHAPTER

## Physical Properties and Principles



### 2.1 Darcy's Law

The birth of groundwater hydrology as a quantitative science can be traced to the year 1856. It was in that year that a French hydraulic engineer named Henry Darcy published his report on the water supply of the city of Dijon, France. In the report Darcy described a laboratory experiment that he had carried out to analyze the flow of water through sands. The results of his experiment can be generalized into the empirical law that now bears his name.

Consider an experimental apparatus like that shown in Figure 2.1. A circular cylinder of cross section  $A$  is filled with sand, stoppered at each end, and outfitted with inflow and outflow tubes and a pair of manometers. Water is introduced into the cylinder and allowed to flow through it until such time as all the pores are filled with water and the inflow rate  $Q$  is equal to the outflow rate. If we set an arbitrary datum at elevation  $z = 0$ , the elevations of the manometer intakes are

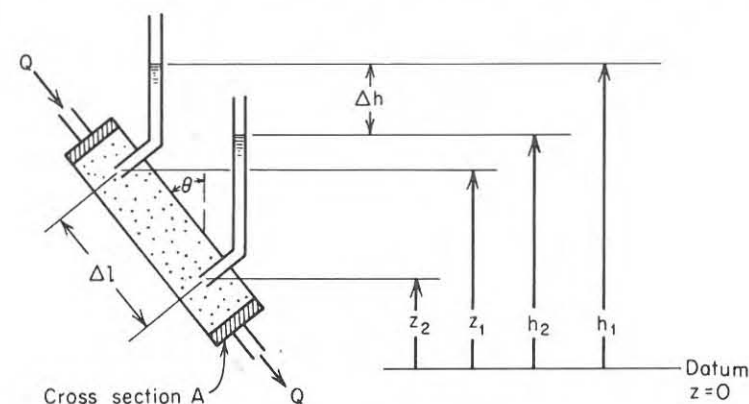


Figure 2.1 Experimental apparatus for the illustration of Darcy's law.



$z_1$  and  $z_2$  and the elevations of the fluid levels are  $h_1$  and  $h_2$ . The distance between the manometer intakes is  $\Delta l$ .

We will define  $v$ , the *specific discharge* through the cylinder, as

$$v = \frac{Q}{A} \quad (2.1)$$

If the dimensions of  $Q$  are  $[L^3/T]$  and those of  $A$  are  $[L^2]$ ,  $v$  has the dimensions of a velocity  $[L/T]$ .

The experiments carried out by Darcy showed that  $v$  is directly proportional to  $h_1 - h_2$  when  $\Delta l$  is held constant, and inversely proportional to  $\Delta l$  when  $h_1 - h_2$  is held constant. If we define  $\Delta h = h_2 - h_1$  (an arbitrary sign convention that will stand us in good stead in later developments), we have  $v \propto -\Delta h$  and  $v \propto 1/\Delta l$ . Darcy's law can now be written as

$$v = -K \frac{\Delta h}{\Delta l} \quad (2.2)$$

or, in differential form,

$$v = -K \frac{dh}{dl} \quad (2.3)$$

In Eq. (2.3),  $h$  is called the *hydraulic head* and  $dh/dl$  is the *hydraulic gradient*.  $K$  is a constant of proportionality. It must be a property of the soil in the cylinder, for were we to hold the hydraulic gradient constant, the specific discharge would surely be larger for some soils than for others. In other words, if  $dh/dl$  is held constant,  $v \propto K$ . The parameter  $K$  is known as the *hydraulic conductivity*. It has high values for sand and gravel and low values for clay and most rocks. Since  $\Delta h$  and  $\Delta l$  both have units of length  $[L]$ , a quick dimensional analysis of Eq. (2.2) shows that  $K$  has the dimensions of a velocity  $[L/T]$ . In Section 2.3, we will show that  $K$  is a function not only of the media, but also of the fluid flowing through it.

An alternative form of Darcy's law can be obtained by substituting Eq. (2.1) in Eq. (2.3) to yield

$$Q = -K \frac{dh}{dl} A \quad (2.4)$$

This is sometimes compacted even further into the form

$$Q = -KiA \quad (2.5)$$

where  $i$  is the hydraulic gradient.

Darcy's law is valid for groundwater flow in any direction in space. With regard to Figure 2.1 and Eq. (2.3), if the hydraulic gradient  $dh/dl$  and the hydraulic conductivity  $K$  are held constant,  $v$  is independent of the angle  $\theta$ . This is true even for  $\theta$  values greater than  $90^\circ$  when the flow is being forced up through the cylinder against gravity.

We have noted that the specific discharge  $v$  has the dimensions of a velocity, or flux. For this reason it is sometimes known as the *Darcy velocity* or *Darcy flux*. The specific discharge is a macroscopic concept and it is easily measured. It must be clearly differentiated from the microscopic velocities associated with the actual paths of individual particles of water as they wind their way through the grains of sand (Figure 2.2). The microscopic velocities are real, but they are probably impossible to measure. In the remainder of the chapter we will work exclusively with concepts of flow on a macroscopic scale. Despite its dimensions we will not refer to  $v$  as a velocity; rather we will utilize the more correct term, *specific discharge*.



Figure 2.2 Macroscopic and microscopic concepts of groundwater flow.

This last paragraph may appear innocuous, but it announces a decision of fundamental importance. When we decide to analyze groundwater flow with the Darcian approach, it means, in effect, that we are going to replace the actual ensemble of sand grains (or clay particles or rock fragments) that make up the porous medium by a representative continuum for which we can define macroscopic parameters, such as the hydraulic conductivity, and utilize macroscopic laws, such as Darcy's law, to provide macroscopically averaged descriptions of the microscopic behavior. This is a conceptually simple and logical step to take, but it rests on some knotty theoretical foundations. Bear (1972), in his advanced text on porous-media flow, discusses these foundations in detail. In Section 2.12, we will further explore the interrelationships between the microscopic and macroscopic descriptions of groundwater flow.

Darcy's law is an empirical law. It rests only on experimental evidence. Many attempts have been made to derive Darcy's law from more fundamental physical laws, and Bear (1972) also reviews these studies in some detail. The most successful approaches attempt to apply the Navier-Stokes equations, which are widely known in the study of fluid mechanics, to the flow of water through the pore channels of idealized conceptual models of porous media. Hubbert (1956) and Irmay (1958) were apparently the earliest to attempt this exercise.

This text will provide ample evidence of the fundamental importance of Darcy's law in the analysis of groundwater flow, but it is worth noting here that it is equally important in many other applications of porous-media flow. It describes the flow of soil moisture and is used by soil physicists, agricultural engineers, and soil mechanics specialists. It describes the flow of oil and gas in deep geological formations and is used by petroleum reservoir analysts. It is used in the design of filters by chemical engineers and in the design of porous ceramics by materials scientists. It has even been used by bioscientists to describe the flow of bodily fluids across porous membranes in the body.



Darcy's law is a powerful empirical law and its components deserve our more careful attention. The next two sections provide a more detailed look at the physical significance of the hydraulic head  $h$  and the hydraulic conductivity  $K$ .

## 2.2 Hydraulic Head and Fluid Potential

The analysis of a physical process that involves flow usually requires the recognition of a potential gradient. For example, it is known that heat flows through solids from higher temperatures toward lower and that electrical current flows through electrical circuits from higher voltages toward lower. For these processes, the temperature and the voltage are potential quantities, and the rates of flow of heat and electricity are proportional to these potential gradients. Our task is to determine the potential gradient that controls the flow of water through porous media.

Fortunately, this question has been carefully considered by Hubbert in his classical treatise on groundwater flow (Hubbert, 1940). In the first part of this section we will review his concepts and derivations.

### Hubbert's Analysis of the Fluid Potential

Hubbert (1940) defines *potential* as "a physical quantity, capable of measurement at every point in a flow system, whose properties are such that flow always occurs from regions in which the quantity has higher values to those in which it has lower, regardless of the direction in space" (p. 794). In the Darcy experiment (Figure 2.1) the hydraulic head  $h$ , indicated by the water levels in the manometers, would appear to satisfy the definition, but as Hubbert points out, "to adopt it empirically without further investigation would be like reading the length of the mercury column of a thermometer without knowing that temperature was the physical quantity being indicated" (p. 795).

Two obvious possibilities for the potential quantity are elevation and fluid pressure. If the Darcy apparatus (Figure 2.1) were set up with the cylinder vertical ( $\theta = 0$ ), flow would certainly occur down through the cylinder (from high elevation to low) in response to gravity. On the other hand, if the cylinder were placed in a horizontal position ( $\theta = 90^\circ$ ) so that gravity played no role, flow could presumably be induced by increasing the pressure at one end and decreasing it at the other. Individually, neither elevation nor pressure are adequate potentials, but we certainly have reason to expect them to be components of the total potential quantity.

It will come as no surprise to those who have been exposed to potential concepts in elementary physics or fluid mechanics that the best way to search out our quarry is to examine the energy relationships during the flow process. In fact, the classical definition of potential as it is usually presented by mathematicians and physicists is in terms of the work done during the flow process; and the work done in moving a unit mass of fluid between any two points in a flow system is a measure of the energy loss of the unit mass.

Fluid flow through porous media is a mechanical process. The forces driving the fluid forward must overcome the frictional forces set up between the moving

fluid and the grains of the porous medium. The flow is therefore accompanied by an irreversible transformation of mechanical energy to thermal energy through the mechanism of frictional resistance. The direction of flow in space must therefore be away from regions in which the mechanical energy per unit mass of fluid is higher and toward regions in which it is lower. In that the mechanical energy per unit mass at any point in a flow system can be defined as the work required to move a unit mass of fluid from an arbitrarily chosen standard state to the point in question, it is clear that we have uncovered a physical quantity that satisfies both Hubbert's definition of a potential (in terms of the direction of flow) and the classical definition (in terms of the work done). The *fluid potential* for flow through porous media is therefore the *mechanical energy per unit mass of fluid*.

It now remains to relate this quantity to the elevation and pressure terms that we anticipated earlier. Consider an arbitrary standard state (Figure 2.3) at elevation  $z = 0$  and pressure  $p = p_0$ , where  $p_0$  is atmospheric. A unit mass of fluid of density  $\rho_0$  will occupy a volume  $V_0$ , where  $V_0 = 1/\rho_0$ . We wish to calculate the work required to lift the unit mass of fluid from the standard state to some point  $P$  in the flow system which is at elevation  $z$  and where the fluid pressure is  $p$ . Here, a unit mass of the fluid may have density  $\rho$  and will occupy a volume  $V = 1/\rho$ . In addition, we will consider the fluid to have velocity  $v = 0$  at the standard state and velocity  $v$  at the point  $P$ .

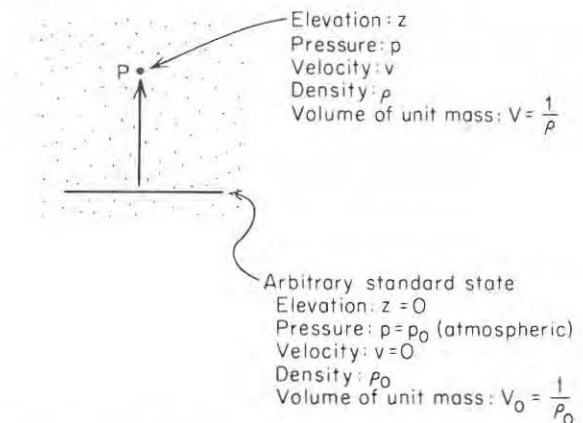


Figure 2.3 Data for calculation of mechanical energy of unit mass of fluid.

There are three components to the work calculation. First, there is the work required to lift the mass from elevation  $z = 0$  to elevation  $z$ :

$$w_1 = mgz \quad (2.6)$$

Second, there is the work required to accelerate the fluid from velocity  $v = 0$  to velocity  $v$ :

$$w_2 = \frac{mv^2}{2} \quad (2.7)$$



Third, there is the work done on the fluid in raising the fluid pressure from  $p = p_0$  to  $p$ :

$$w_3 = m \int_{p_0}^p \frac{V}{m} dp = m \int_{p_0}^p \frac{dp}{\rho} \quad (2.8)$$

If the fluid were to flow from point  $P$  to a point at the standard state, Eq. (2.6) represents the loss in potential energy, Eq. (2.7) is the loss in kinetic energy, and Eq. (2.8) is the loss in elastic energy, or  $p$ - $V$  work.

The fluid potential  $\Phi$  (the mechanical energy per unit mass) is the sum of  $w_1$ ,  $w_2$ , and  $w_3$ . For a unit mass of fluid,  $m = 1$  in Eqs. (2.6), (2.7), and (2.8), and we have

$$\Phi = gz + \frac{v^2}{2} + \int_{p_0}^p \frac{dp}{\rho} \quad (2.9)$$

For porous-media flow, velocities are extremely low, so the second term can almost always be neglected. For incompressible fluids (fluids with a constant density, so that  $\rho$  is not a function of  $p$ ), Eq. (2.9) can be simplified further to give

$$\Phi = gz + \frac{p - p_0}{\rho} \quad (2.10)$$

Our earlier premonitions as to the likely components of the fluid potential are now seen to be correct. The first term of Eq. (2.10) involves the elevation  $z$  and the second term involves the fluid pressure  $p$ .

But how do these terms relate to the hydraulic head  $h$ ? Let us return to the Darcy manometer (Figure 2.4). At  $P$ , the fluid pressure  $p$  is given by

$$p = \rho g \psi + p_0 \quad (2.11)$$

where  $\psi$  is the height of the liquid column above  $P$  and  $p_0$  is atmospheric pressure or pressure at the standard state. It is clear from Figure 2.4 and Eq. (2.11) that

$$p = \rho g(h - z) + p_0 \quad (2.12)$$

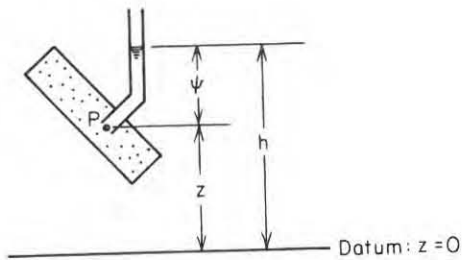


Figure 2.4 Hydraulic head  $h$ , pressure head  $\psi$ , and elevation head  $z$  for a laboratory manometer.

Substituting Eq. (2.12) in Eq. (2.10) yields

$$\Phi = gz + \frac{[\rho g(h - z) + p_0] - p_0}{\rho} \quad (2.13)$$

or, canceling terms,

$$\Phi = gh \quad (2.14)$$

Our long exercise has led us to the simplest of conclusions. The fluid potential  $\Phi$  at any point  $P$  in a porous medium is simply the hydraulic head at the point multiplied by the acceleration due to gravity. Since  $g$  is very nearly constant in the vicinity of the earth's surface,  $\Phi$  and  $h$  are almost perfectly correlated. To know one is to know the other. The hydraulic head  $h$  is therefore just as suitable a potential as is  $\Phi$ . To recall Hubbert's definition: it is a physical quantity, it is capable of measurement, and flow always occurs from regions where  $h$  has higher values to regions where it has lower. In fact reference to Eq. (2.14) shows that, if  $\Phi$  is energy per unit mass, then  $h$  is energy per unit weight.

It is common in groundwater hydrology to set the atmospheric pressure  $p_0$  equal to zero and work in gage pressures (i.e., pressures above atmospheric). In this case Eqs. (2.10) and (2.14) become

$$\Phi = gz + \frac{p}{\rho} = gh \quad (2.15)$$

Dividing through by  $g$ , we obtain

$$h = z + \frac{p}{\rho g} \quad (2.16)$$

Putting Eq. (2.11) in terms of gage pressures yields

$$p = \rho g \psi \quad (2.17)$$

and Eq. (2.16) becomes

$$h = z + \psi \quad (2.18)$$

The hydraulic head  $h$  is thus seen to be the sum of two components: the elevation of the point of measurement, or *elevation head*,  $z$ , and the *pressure head*  $\psi$ . This fundamental head relationship is basic to an understanding of groundwater flow. Figure 2.4 displays the relationship for the Darcy manometer, Figure 2.5 for a field measurement site.

Those who are familiar with elementary fluid mechanics may already have recognized Eq. (2.9) as the *Bernoulli equation*, the classical formulation of energy loss during fluid flow. Some authors (Todd, 1959; Domenico, 1972) use the Bernoulli equation as the starting point for their development of the concepts of fluid



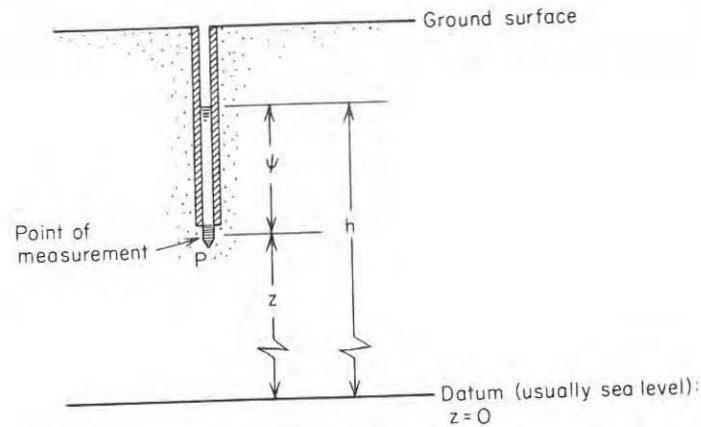


Figure 2.5 Hydraulic head  $h$ , pressure head  $\psi$ , and elevation head  $z$  for a field piezometer.

potential and hydraulic head. If we put Eq. (2.9) in terms of head and use a simplified notation, it becomes

$$h_T = h_z + h_p + h_v \quad (2.19)$$

where  $h_z$  is the elevation head,  $h_p$  the pressure head, and  $h_v$  the velocity head. In our earlier notation:  $h_z = z$ ,  $h_p = \psi$ , and  $h_v = v^2/2g$ . The term  $h_T$  is called the *total head*, and for the special case where  $h_v = 0$ , it is equal to the hydraulic head  $h$ , and Eq. (2.18) holds.

### Dimensions and Units

The dimensions of the head terms  $h$ ,  $\psi$ , and  $z$  are those of length  $[L]$ . They are usually expressed as “meters of water” or “feet of water.” The specification “of water” emphasizes that head measurements are dependent on fluid density through the relationship of Eq. (2.17). Given the same fluid pressure  $p$  at point  $P$  in Figure 2.5, the hydraulic head  $h$  and pressure head  $\psi$  would have different values if the fluid in the pores of the geological formation were oil rather than water. In this text, where we will always be dealing with water, the adjectival phrase will usually be dropped and we will record heads in meters.

As for the other terms introduced in this section; in the SI system, with its  $[M][L][T]$  base, pressure has dimensions  $[M/LT^2]$ , mass density has dimensions  $[M/L^3]$ , and the fluid potential, from its definition, is energy per unit mass with dimensions  $[L^2/T^2]$ . Table 2.1 clarifies the dimensions and common units for all the important parameters introduced thus far. Reference to Appendix I should resolve any confusion. In this text, we will use SI metric units as our basic system of units, but Table 2.1 includes the FPS equivalents. Table A1.3 in Appendix I provides conversion factors.

Note in Table 2.1 that the weight density of water,  $\gamma$ , defined by

$$\gamma = \rho g \quad (2.20)$$

is a more suitable parameter than the mass density  $\rho$  for the FPS system of units, which has force as one of its fundamental dimensions.

Table 2.1 Dimensions and Common Units for Basic Groundwater Parameters\*

Parameter	Symbol	Système International† SI		Foot-pound-second system,‡ FPS	
		Dimension	Units	Dimension	Units
Hydraulic head	$h$	$[L]$	m	$[L]$	ft
Pressure head	$\psi$	$[L]$	m	$[L]$	ft
Elevation head	$z$	$[L]$	m	$[L]$	ft
Fluid pressure	$p$	$[M/LT^2]$	N/m <sup>2</sup> or Pa	$[F/L^2]$	lb/ft <sup>2</sup>
Fluid potential	$\Phi$	$[L^2/T^2]$	m <sup>2</sup> /s <sup>2</sup>	$[L^2/T^2]$	ft <sup>2</sup> /s <sup>2</sup>
Mass density	$\rho$	$[M/L^3]$	kg/m <sup>3</sup>	—	—
Weight density	$\gamma$	—	—	$[F/L^3]$	lb/ft <sup>3</sup>
Specific discharge	$v$	$[L/T]$	m/s	$[L/T]$	ft/s
Hydraulic conductivity	$K$	$[L/T]$	m/s	$[L/T]$	ft/s

\*See also Tables A1.1, A1.2, and A1.3, Appendix I.

†Basic dimensions are length  $[L]$ , mass  $[M]$ , and time  $[T]$ .

‡Basic dimensions are length  $[L]$ , force  $[F]$ , and time  $[T]$ .

### Piezometers and Piezometer Nests

The basic device for the measurement of hydraulic head is a tube or pipe in which the elevation of a water level can be determined. In the laboratory (Figure 2.4) the tube is a *manometer*; in the field (Figure 2.5) the pipe is called a *piezometer*. A piezometer must be sealed along its length. It must be open to water flow at the bottom and be open to the atmosphere at the top. The intake is usually a section of slotted pipe or a commercially available *well point*. In either case the intake must be designed to allow the inflow of water but not of the sand grains or clay particles that make up the geologic formation. It must be emphasized that the point of measurement in a piezometer is at its base, not at the level of the fluid surface. One can view the functioning of a piezometer much like that of a thermometer. It is simply the instrument, if such it can be called, used to determine the value of  $h$  at some point  $P$  in a groundwater reservoir. In recent years, the simple standpipe piezometer has been replaced in some applications by more complex designs utilizing pressure transducers, pneumatic devices, and electronic components.

Piezometers are usually installed in groups so that they can be used to determine directions of groundwater flow. In Figure 2.6(a) three piezometers tap a water-bearing geological formation. It is a worthwhile exercise to remove the instruments of measurement from the diagram [Figure 2.6(b)] and consider only



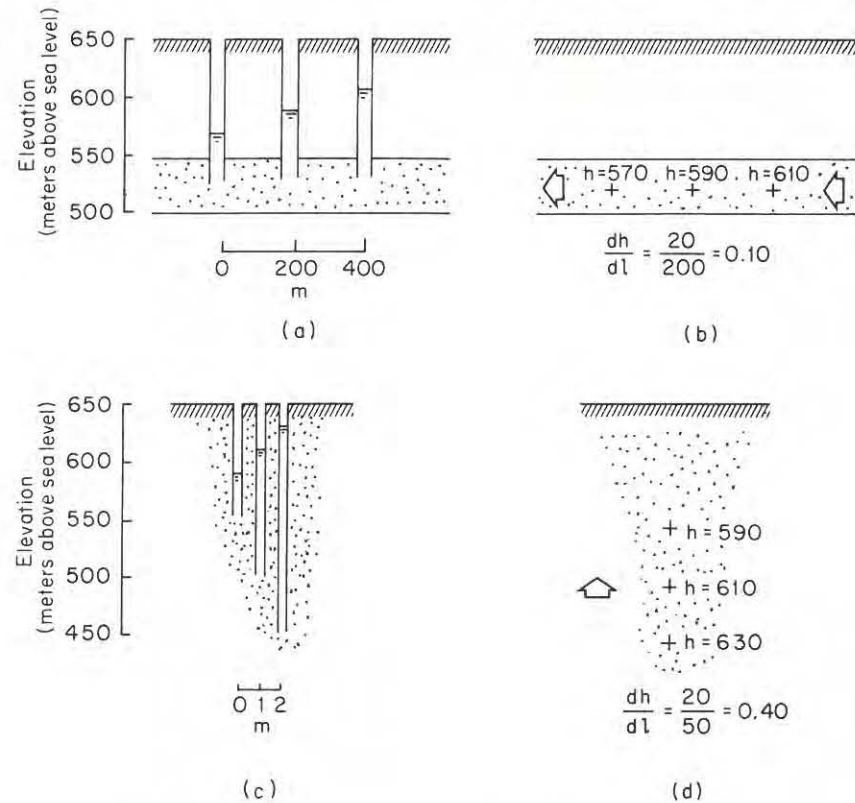


Figure 2.6 Determination of hydraulic gradients from piezometer installations.

the measured values. Flow is from higher  $h$  to lower, in this case from right to left. If the distance between the piezometers were known, the hydraulic gradient  $dh/dl$  could be calculated; and if the hydraulic conductivity  $K$  of the geological formation were known, Darcy's law could be used to calculate the specific discharge (or volume rate of flow through any cross-sectional area perpendicular to the flow direction).

Sometimes it is the vertical potential gradient that is of interest. In such cases a *piezometer nest* is utilized, with two or more piezometers installed side by side at the same location (or possibly in the same hole), each bottoming at a different depth and possibly in a different geological formation. Figure 2.6(c) and (d) shows a piezometer nest in a region of upward groundwater flow.

The distribution of hydraulic heads in a groundwater system is three-dimensional through space. The piezometer groupings shown in Figure 2.6 only prove the existence of *components* of flow in the directions indicated. If a large number of piezometers could be distributed throughout the three-dimensional hydrogeologic system, it would be possible to contour the positions of equal hydraulic head.

In three dimensions the locus of such points forms an *equipotential surface*. In any two-dimensional cross section, be it horizontal, vertical or otherwise, the traces of the equipotential surfaces on the section are called *equipotential lines*. If the pattern of hydraulic heads is known in a cross section, *flowlines* can be constructed perpendicular to the equipotential lines (in the direction of the maximum potential gradient). The resulting set of intersecting equipotential lines and flowlines is known as a *flow net*. Chapter 5 will provide detailed instructions on the construction of flow nets, and Chapter 6 will prove their usefulness in the interpretation of regional groundwater flow.

### Coupled Flow

There is now a large body of experimental and theoretical evidence to show that water can be induced to flow through porous media under the influence of gradients other than that of hydraulic head. For example, the presence of a *temperature gradient* can cause groundwater flow (as well as heat flow) even when hydraulic gradients do not exist (Gurr et al., 1952; Philip and de Vries, 1957). This component becomes important in the formation of frost wedges in soil (Hoekstra, 1966; Harlan, 1973).

An *electrical gradient* can create a flow of water from high voltage to low when earth currents are set up in a soil. The mechanism of flow involves an interaction between charged ions in the water and the electrical charge associated with clay minerals in the soil (Casagrande, 1952). The principle is used in soil mechanics in the *electrokinetic* approach to soil drainage (Terzaghi and Peck, 1967).

*Chemical gradients* can cause the flow of water (as well as the movement of chemical constituents *through* the water) from regions where water has higher salinity to regions where it has lower salinity, even in the absence of other gradients. The role of chemical gradients in the production of water flow is relatively unimportant, but their direct influence on the movement of chemical constituents is of major importance in the analysis of groundwater contamination. These concepts will come to the fore in Chapters 3, 7, and 9.

If each of these gradients plays a role in producing flow, it follows that a more general flow law than Eq. (2.3) can be written in the form

$$v = -L_1 \frac{dh}{dl} - L_2 \frac{dT}{dl} - L_3 \frac{dc}{dl} \quad (2.21)$$

where  $h$  is hydraulic head,  $T$  is temperature, and  $c$  is chemical concentration;  $L_1$ ,  $L_2$ , and  $L_3$  are constants of proportionality. For the purposes of discussion, let us set  $dc/dl = 0$ . We are left with a situation where fluid flow is occurring in response to both a hydraulic head gradient and a temperature gradient:

$$v = -L_1 \frac{dh}{dl} - L_2 \frac{dT}{dl} \quad (2.22)$$

In general,  $L_1 dh/dl \gg L_2 dT/dl$ .



If a temperature gradient can cause fluid flow as well as heat flow in a porous medium, it should come as no surprise to find that a hydraulic gradient can cause heat flow as well as fluid flow. This mutual interdependency is a reflection of the well-known thermodynamic concept of *coupled flow*. If we set  $dh/dl = i_1$  and  $dT/dl = i_2$ , we can write a pair of equations patterned after Eq. (2.22):

$$v_1 = -L_{11}i_1 - L_{12}i_2 \quad (2.23)$$

$$v_2 = -L_{21}i_1 - L_{22}i_2 \quad (2.24)$$

where  $v_1$  is the specific discharge of *fluid* through the medium and  $v_2$  is the specific discharge of *heat* through the medium. The  $L$ 's are known as *phenomenological coefficients*. If  $L_{12} = 0$  in Eq. (2.23), we are left with Darcy's law of groundwater flow and  $L_{11}$  is the hydraulic conductivity. If  $L_{21} = 0$  in Eq. (2.24), we are left with Fourier's law of heat flow and  $L_{22}$  is the thermal conductivity.

It is possible to write a complete set of coupled equations. The set of equations would have the form of Eq. (2.23) but would involve all the gradients of Eq. (2.21) and perhaps others. The development of the theory of coupled flows in porous media was pioneered by Taylor and Cary (1964). Olsen (1969) has carried out significant experimental research. Bear (1972) provides a more detailed development of the concepts than can be attempted here. The thermodynamic description of the physics of porous media flow is conceptually powerful, but in practice there are very few data on the nature of the off-diagonal coefficients in the matrix of phenomenological coefficients  $L_{ij}$ . In this text we will assume that groundwater flow is fully described by Darcy's law [Eq. (2.3)]; that the hydraulic head [Eq. (2.18)], with its elevation and pressure components, is a suitable representation of the total head; and that the hydraulic conductivity is the only important phenomenological coefficient in Eq. (2.21).

### 2.3 Hydraulic Conductivity and Permeability

As Hubbert (1956) has pointed out, the constant of proportionality in Darcy's law, which has been christened the hydraulic conductivity, is a function not only of the porous medium but also of the fluid. Consider once again the experimental apparatus of Figure 2.1. If  $\Delta h$  and  $\Delta l$  are held constant for two runs using the same sand, but water is the fluid in the first run and molasses in the second, it would come as no surprise to find the specific discharge  $v$  much lower in the second run than in the first. In light of such an observation, it would be instructive to search for a parameter that can describe the conductive properties of a porous medium independently from the fluid flowing through it.

To this end experiments have been carried out with ideal porous media consisting of uniform glass beads of diameter  $d$ . When various fluids of density  $\rho$  and dynamic viscosity  $\mu$  are run through the apparatus under a constant hydraulic

gradient  $dh/dl$ , the following proportionality relationships are observed:

$$v \propto d^2$$

$$v \propto \rho g$$

$$v \propto \frac{1}{\mu}$$

Together with Darcy's original observation that  $v \propto -dh/dl$ , these three relationships lead to a new version of Darcy's law:

$$v = -\frac{Cd^2\rho g}{\mu} \frac{dh}{dl} \quad (2.25)$$

The parameter  $C$  is yet another constant of proportionality. For real soils it must include the influence of other media properties that affect flow, apart from the mean grain diameter: for example, the distribution of grain sizes, the sphericity and roundness of the grains, and the nature of their packing.

Comparison of Eq. (2.25) with the original Darcy equation [Eq. (2.3)] shows that

$$K = \frac{Cd^2\rho g}{\mu} \quad (2.26)$$

In this equation,  $\rho$  and  $\mu$  are functions of the fluid alone and  $Cd^2$  is a function of the medium alone. If we define

$$k = Cd^2 \quad (2.27)$$

then

$$K = \frac{k\rho g}{\mu} \quad (2.28)$$

The parameter  $k$  is known as the *specific* or *intrinsic permeability*. If  $K$  is always called hydraulic conductivity, it is safe to drop the adjectives and refer to  $k$  as simply the permeability. That is the convention that will be followed in this text, but it can lead to some confusion, especially when dealing with older texts and reports where the hydraulic conductivity  $K$  is sometimes called the *coefficient of permeability*.

Hubbert (1940) developed Eqs. (2.25) through (2.28) from fundamental principles by considering the relationships between driving and resisting forces on a microscopic scale during flow through porous media. The dimensional considerations inherent in his analysis provided us with the foresight to include the constant  $g$  in the proportionality relationship leading to Eq. (2.25). In this way  $C$  emerges as a dimensionless constant.

The permeability  $k$  is a function only of the medium and has dimensions  $[L^2]$ . The term is widely used in the petroleum industry, where the existence of gas,



oil, and water in multiphase flow systems makes the use of a fluid-free conductance parameter attractive. When measured in  $\text{m}^2$  or  $\text{cm}^2$ ,  $k$  is very small, so petroleum engineers have defined the *darcy* as a unit of permeability. If Eq. (2.28) is substituted in Eq. (2.3), Darcy's law becomes

$$v = \frac{-k \rho g}{\mu} \frac{dh}{dl} \quad (2.29)$$

Referring to this equation, 1 darcy is defined as the permeability that will lead to a specific discharge of 1 cm/s for a fluid with a viscosity of 1 cp under a hydraulic gradient that makes the term  $\rho g dh/dl$  equal to 1 atm/cm. One darcy is approximately equal to  $10^{-8} \text{ cm}^2$ .

In the water well industry, the unit gal/day/ft<sup>2</sup> is widely used for hydraulic conductivity. Its relevance is clearest when Darcy's law is couched in terms of Eq. (2.4):

$$Q = -K \frac{dh}{dl} A$$

The early definitions provided by the U.S. Geological Survey with regard to this unit differentiate between a laboratory coefficient and a field coefficient. However, a recent updating of these definitions (Lohman, 1972) has discarded this formal differentiation. It is sufficient to note that differences in the temperature of measurement between the field environment and the laboratory environment can influence hydraulic conductivity values through the viscosity term in Eq. (2.28). The effect is usually small, so correction factors are seldom introduced. It still makes good sense to report whether hydraulic conductivity measurements have been carried out in the laboratory or in the field, because the methods of measurement are very different and the interpretations placed on the values may be dependent on the type of measurement. However, this information is of practical rather than conceptual importance.

Table 2.2 indicates the range of values of hydraulic conductivity and permeability in five different systems of units for a wide range of geological materials. The table is based in part on the data summarized in Davis' (1969) review. The primary conclusion that can be drawn from the data is that hydraulic conductivity varies over a very wide range. There are very few physical parameters that take on values over 13 orders of magnitude. In practical terms, this property implies that an order-of-magnitude knowledge of hydraulic conductivity can be very useful. Conversely, the third decimal place in a reported conductivity value probably has little significance.

Table 2.3 provides a set of conversion factors for the various common units of  $k$  and  $K$ . As an example of its use, note that a  $k$  value in  $\text{cm}^2$  can be converted to one in  $\text{ft}^2$  by multiplying by  $1.08 \times 10^{-3}$ . For the reverse conversion from  $\text{ft}^2$  to  $\text{cm}^2$ , multiply by  $9.29 \times 10^2$ .

Table 2.2 Range of Values of Hydraulic Conductivity and Permeability

	Rocks	Unconsolidated deposits	$k$ (darcy)	$k$ (cm <sup>2</sup> )	$K$ (cm/s)	$K$ (m/s)	$K$ (gal/day/ft <sup>2</sup> )
			$10^5$	$10^{-3}$	$10^2$	1	
			$10^4$	$10^{-4}$	10	$10^{-1}$	$10^6$
			$10^3$	$10^{-5}$	1	$10^{-2}$	$10^5$
			$10^2$	$10^{-6}$	$10^{-1}$	$10^{-3}$	$10^4$
			10	$10^{-7}$	$10^{-2}$	$10^{-4}$	$10^3$
			1	$10^{-8}$	$10^{-3}$	$10^{-5}$	$10^2$
			$10^{-1}$	$10^{-9}$	$10^{-4}$	$10^{-6}$	10
			$10^{-2}$	$10^{-10}$	$10^{-5}$	$10^{-7}$	1
			$10^{-3}$	$10^{-11}$	$10^{-6}$	$10^{-8}$	$10^{-1}$
			$10^{-4}$	$10^{-12}$	$10^{-7}$	$10^{-9}$	$10^{-2}$
			$10^{-5}$	$10^{-13}$	$10^{-8}$	$10^{-10}$	$10^{-3}$
			$10^{-6}$	$10^{-14}$	$10^{-9}$	$10^{-11}$	$10^{-4}$
			$10^{-7}$	$10^{-15}$	$10^{-10}$	$10^{-12}$	$10^{-5}$
			$10^{-8}$	$10^{-16}$	$10^{-11}$	$10^{-13}$	$10^{-6}$
							$10^{-7}$

Table 2.3 Conversion Factors for Permeability and Hydraulic Conductivity Units

	Permeability, $k^*$			Hydraulic conductivity, $K$		
	cm <sup>2</sup>	ft <sup>2</sup>	darcy	m/s	ft/s	gal/day/ft <sup>2</sup>
cm <sup>2</sup>	1	$1.08 \times 10^{-3}$	$1.01 \times 10^8$	$9.80 \times 10^2$	$3.22 \times 10^3$	$1.85 \times 10^9$
ft <sup>2</sup>	$9.29 \times 10^2$	1	$9.42 \times 10^{10}$	$9.11 \times 10^5$	$2.99 \times 10^6$	$1.71 \times 10^{12}$
darcy	$9.87 \times 10^{-9}$	$1.06 \times 10^{-11}$	1	$9.66 \times 10^{-6}$	$3.17 \times 10^{-5}$	$1.82 \times 10^1$
m/s	$1.02 \times 10^{-3}$	$1.10 \times 10^{-6}$	$1.04 \times 10^5$	1	3.28	$2.12 \times 10^6$
ft/s	$3.11 \times 10^{-4}$	$3.35 \times 10^{-7}$	$3.15 \times 10^4$	$3.05 \times 10^{-1}$	1	$5.74 \times 10^5$
gal/day/ft <sup>2</sup>	$5.42 \times 10^{-10}$	$5.83 \times 10^{-13}$	$5.49 \times 10^{-2}$	$4.72 \times 10^{-7}$	$1.74 \times 10^{-6}$	1

\*To obtain  $k$  in ft<sup>2</sup>, multiply  $k$  in cm<sup>2</sup> by  $1.08 \times 10^{-3}$ .



The various approaches to the measurement of hydraulic conductivity in the laboratory and in the field are described in Sections 8.4 through 8.6.

## 2.4 Heterogeneity and Anisotropy of Hydraulic Conductivity

Hydraulic conductivity values usually show variations through space within a geologic formation. They may also show variations with the direction of measurement at any given point in a geologic formation. The first property is termed *heterogeneity* and the second *anisotropy*. The evidence that these properties are commonplace is to be found in the spread of measurements that arises in most field sampling programs. The geological reasoning that accounts for their prevalence lies in an understanding of the geologic processes that produce the various geological environments.

### Homogeneity and Heterogeneity

If the hydraulic conductivity  $K$  is *independent* of position within a geologic formation, the formation is *homogeneous*. If the hydraulic conductivity  $K$  is *dependent* on position within a geologic formation, the formation is *heterogeneous*. If we set up an  $xyz$  coordinate system in a homogeneous formation,  $K(x, y, z) = C$ ,  $C$  being a constant; whereas in a heterogeneous formation,  $K(x, y, z) \neq C$ .

There are probably as many types of heterogeneous configurations as there are geological environments, but it may be instructive to draw attention to three broad classes. Figure 2.7(a) is a vertical cross section that shows an example of *layered heterogeneity*, common in sedimentary rocks and unconsolidated lacustrine and marine deposits. Here, the individual beds making up the formation each have a homogeneous conductivity value  $K_1, K_2, \dots$ , but the entire system can be thought of as heterogeneous. Layered heterogeneity can result in  $K$  contrasts of almost the full 13-order range (Table 2.2), as, for example, in interlayered deposits of clay and sand. Equally large contrasts can arise in cases of *discontinuous heterogeneity* caused by the presence of faults or large-scale stratigraphic features. Perhaps the most ubiquitous discontinuous feature is the overburden-bedrock contact. Figure 2.7(b) is a map that shows a case of *trending heterogeneity*. Trends are possible in any type of geological formation, but they are particularly common in response to the sedimentation processes that create deltas, alluvial fans, and glacial outwash plains. The A, B, and C soil horizons often show vertical trends in hydraulic conductivity, as do rock types whose conductivity is primarily dependent on joint and fracture concentration. Trending heterogeneity in large consolidated or unconsolidated sedimentary formations can attain gradients of 2–3 orders of magnitude in a few miles.

Many hydrogeologists and petroleum geologists have used statistical distributions to provide a quantitative description of the degree of heterogeneity in a geological formation. There is now a large body of direct evidence to support the

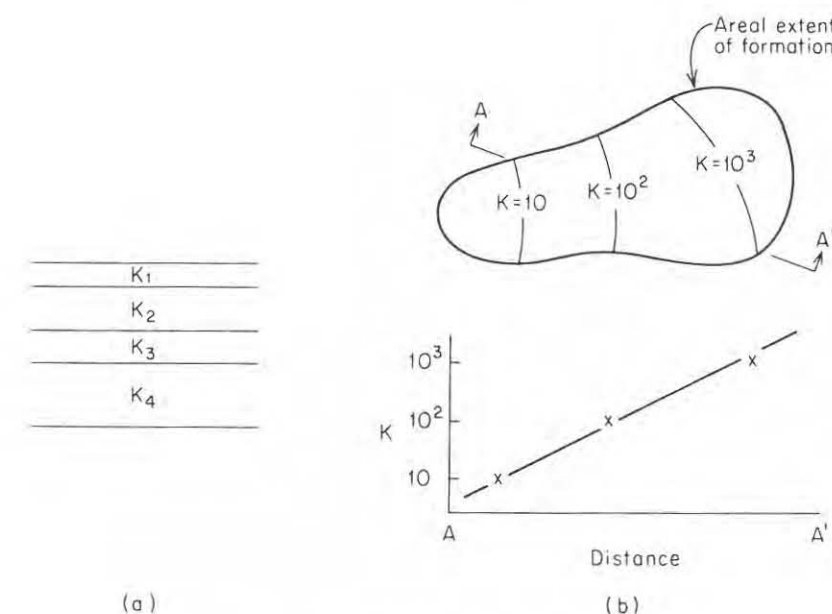


Figure 2.7 Layered heterogeneity and trending heterogeneity.

statement that the probability density function for hydraulic conductivity is log-normal. Warren and Price (1961) and Bennion and Griffiths (1966) found this to be the case in oilfield reservoir rocks, and Willardson and Hurst (1965) and Davis (1969) support the conclusion for unconsolidated water-bearing formations. A log-normal distribution for  $K$  is one for which a parameter  $Y$ , defined as  $Y = \log K$ , shows a normal distribution. Freeze (1975) provides a table, based on the references above, that shows the standard deviation on  $Y$  (which is independent of the units of measurement) is usually in the range 0.5–1.5. This means that  $K$  values in most geological formations show internal heterogeneous variations of 1–2 orders of magnitude. Trending heterogeneity within a geological formation can be thought of as a trend in the *mean value* of the probability distribution. The same standard deviation may be evident in measurements at different positions in the formation, but the trending means lead to an increase in the overall observed range for the formation.

Greenkorn and Kessler (1969) have provided a set of definitions of heterogeneity that are consistent with the statistical observations. In effect, they argue that if all geologic formations display spatial variations in  $K$ , then under the classical definitions, there is no such thing as a homogeneous formation. They redefine a homogeneous formation as one in which the probability density function of hydraulic conductivity is monomodal. That is, it shows variations in  $K$ , but maintains a constant mean  $K$  through space. A heterogeneous formation is defined as one in which the probability density function is multimodal. To describe a porous



medium that satisfies the classical definition of homogeneity ( $K$  constant everywhere, such as in experimental glass beads of diameter  $d$ ) they use the term *uniform*. If we wish to adapt the classical definitions given at the start of this section to this more rational set of concepts, we can do so by adding the adjective "mean" and couching the original definitions in terms of *mean hydraulic conductivity*.

### Isotropy and Anisotropy

If the hydraulic conductivity  $K$  is *independent* of the direction of measurement at a point in a geologic formation, the formation is *isotropic* at that point. If the hydraulic conductivity  $K$  *varies* with the direction of measurement at a point in a geologic formation, the formation is *anisotropic* at that point.

Consider a two-dimensional vertical section through an anisotropic formation. If we let  $\theta$  be the angle between the horizontal and the direction of measurement of a  $K$  value at some point in the formation, then  $K = K(\theta)$ . The directions in space corresponding to the  $\theta$  angle at which  $K$  attains its maximum and minimum values are known as the *principal directions of anisotropy*. They are always perpendicular to one another. In three dimensions, if a plane is taken perpendicular to one of the principal directions, the other two principal directions are the directions of maximum and minimum  $K$  in that plane.

If an  $xyz$  coordinate system is set up in such a way that the coordinate directions coincide with the principal directions of anisotropy, the hydraulic conductivity values in the principal directions can be specified as  $K_x$ ,  $K_y$ , and  $K_z$ . At any point  $(x, y, z)$ , an isotropic formation will have  $K_x = K_y = K_z$ , whereas an anisotropic formation will have  $K_x \neq K_y \neq K_z$ . If  $K_x = K_y \neq K_z$ , as is common in horizontally bedded sedimentary deposits, the formation is said to be *transversely isotropic*.

To fully describe the nature of the hydraulic conductivity in a geologic formation, it is necessary to use two adjectives, one dealing with heterogeneity and one with anisotropy. For example, for a homogeneous, isotropic system in two dimensions:  $K_x(x, z) = K_z(x, z) = C$  for all  $(x, z)$ , where  $C$  is a constant. For a homogeneous, anisotropic system,  $K_x(x, z) = C_1$  for all  $(x, z)$  and  $K_z(x, z) = C_2$  for all  $(x, z)$  but  $C_1 \neq C_2$ . Figure 2.8 attempts to further clarify the four possible combinations. The length of the arrow vectors is proportional to the  $K_x$  and  $K_z$  values at the two points  $(x_1, z_1)$  and  $(x_2, z_2)$ .

The primary cause of anisotropy on a small scale is the orientation of clay minerals in sedimentary rocks and unconsolidated sediments. Core samples of clays and shales seldom show horizontal to vertical anisotropy greater than 10:1, and it is usually less than 3:1.

On a larger scale, it can be shown (Maasland, 1957; Marcus and Evenson, 1961) that there is a relation between layered heterogeneity and anisotropy. Consider the layered formation shown in Figure 2.9. Each layer is homogeneous and isotropic with hydraulic conductivity values  $K_1, K_2, \dots, K_n$ . We will show that the system as a whole acts like a single homogeneous, anisotropic layer. First, consider flow perpendicular to the layering. The specific discharge  $v$  must be the

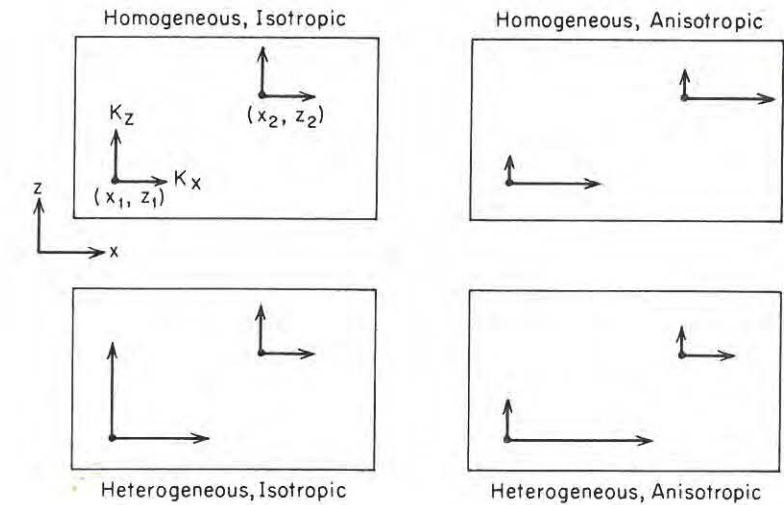


Figure 2.8 Four possible combinations of heterogeneity and anisotropy.

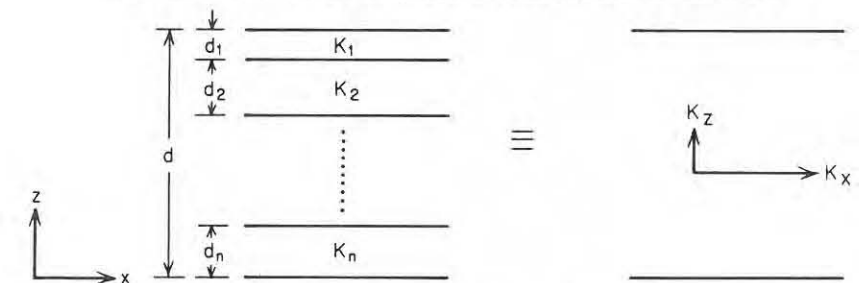


Figure 2.9 Relation between layered heterogeneity and anisotropy.

same entering the system as it is leaving; in fact, it must be constant throughout the system. Let  $\Delta h_1$  be the head loss across the first layer,  $\Delta h_2$  across the second layer, and so on. The total head loss is then  $\Delta h = \Delta h_1 + \Delta h_2 + \dots + \Delta h_n$ , and from Darcy's law,

$$v = \frac{K_1 \Delta h_1}{d_1} = \frac{K_2 \Delta h_2}{d_2} = \dots = \frac{K_n \Delta h_n}{d_n} = \frac{K_z \Delta h}{d} \quad (2.30)$$

where  $K_z$  is an equivalent vertical hydraulic conductivity for the system of layers. Solving the outside relationship of Eq. (2.30) for  $K_z$  and using the inside relationships for  $\Delta h_1, \Delta h_2, \dots$ , we obtain

$$\begin{aligned} K_z &= \frac{vd}{\Delta h} = \frac{vd}{\Delta h_1 + \Delta h_2 + \dots + \Delta h_n} \\ &= \frac{vd}{vd_1/K_1 + vd_2/K_2 + \dots + vd_n/K_n} \end{aligned}$$



which leads to

$$K_x = \frac{d}{\sum_{i=1}^n d_i/K_i} \quad (2.31)$$

Now consider flow parallel to the layering. Let  $\Delta h$  be the head loss over a horizontal distance  $l$ . The discharge  $Q$  through a unit thickness of the system is the sum of the discharges through the layers. The specific discharge  $v = Q/d$  is therefore given by

$$v = \sum_{i=1}^n \frac{K_i d_i}{d} \frac{\Delta h}{l} = K_x \frac{\Delta h}{l}$$

where  $K_x$  is an equivalent horizontal hydraulic conductivity. Simplification yields

$$K_x = \sum_{i=1}^n \frac{K_i d_i}{d} \quad (2.32)$$

Equations (2.31) and (2.32) provide the  $K_x$  and  $K_z$  values for a single homogeneous but anisotropic formation that is hydraulically equivalent to the layered system of homogeneous, isotropic geologic formations of Figure 2.9. With some algebraic manipulation of these two equations it is possible to show that  $K_x > K_z$  for all possible sets of values of  $K_1, K_2, \dots, K_n$ . In fact, if we consider a set of cyclic couplets  $K_1, K_2, K_1, K_2, \dots$  with  $K_1 = 10^4$  and  $K_2 = 10^2$ , then  $K_x/K_z = 25$ . For  $K_1 = 10^4$  and  $K_2 = 1$ ,  $K_x/K_z = 2500$ . In the field, it is not uncommon for layered heterogeneity to lead to regional anisotropy values on the order of 100:1 or even larger.

Snow (1969) showed that fractured rocks also behave anisotropically because of the directional variations in joint aperture and spacing. In this case, it is quite common for  $K_z > K_x$ .

#### Darcy's Law in Three Dimensions

For three-dimensional flow, in a medium that may be anisotropic, it is necessary to generalize the one-dimensional form of Darcy's law [Eq. (2.3)] presented earlier. In three dimensions the velocity  $v$  is a vector with components  $v_x$ ,  $v_y$ , and  $v_z$ , and the simplest generalization would be

$$\begin{aligned} v_x &= -K_x \frac{\partial h}{\partial x} \\ v_y &= -K_y \frac{\partial h}{\partial y} \\ v_z &= -K_z \frac{\partial h}{\partial z} \end{aligned} \quad (2.33)$$

where  $K_x$ ,  $K_y$ , and  $K_z$  are the hydraulic conductivity values in the  $x$ ,  $y$ , and  $z$  direction. Since  $h$  is now a function of  $x$ ,  $y$ , and  $z$ , the derivatives must be partial.

In this text we will assume this simple generalization to be an adequate description of three-dimensional flow, but it is worth noting that a more generalized set of equations could be written in the form

$$\begin{aligned} v_x &= -K_{xx} \frac{\partial h}{\partial x} - K_{xy} \frac{\partial h}{\partial y} - K_{xz} \frac{\partial h}{\partial z} \\ v_y &= -K_{yx} \frac{\partial h}{\partial x} - K_{yy} \frac{\partial h}{\partial y} - K_{yz} \frac{\partial h}{\partial z} \\ v_z &= -K_{zx} \frac{\partial h}{\partial x} - K_{zy} \frac{\partial h}{\partial y} - K_{zz} \frac{\partial h}{\partial z} \end{aligned} \quad (2.34)$$

This set of equations exposes the fact that there are actually nine components of hydraulic conductivity in the most general case. If these components are put in matrix form, they form a second-rank symmetric tensor known as the *hydraulic conductivity tensor* (Bear, 1972). For the special case  $K_{xy} = K_{xz} = K_{yx} = K_{yz} = K_{zx} = K_{zy} = 0$ , the nine components reduce to three and Eq. (2.33) is a suitable generalization of Darcy's law. The necessary and sufficient condition that allows use of Eq. (2.33) rather than Eq. (2.34) is that the principal directions of anisotropy coincide with the  $x$ ,  $y$ , and  $z$  coordinate axes. In most cases it is possible to choose a coordinate system that satisfies this requirement, but one can conceive of heterogeneous anisotropic systems in which the principal directions of anisotropy vary from one formation to another, and in such systems the choice of suitable axes would be impossible.

#### Hydraulic Conductivity Ellipsoid

Consider an arbitrary flowline in the  $xz$  plane in a homogeneous, anisotropic medium with principal hydraulic conductivities  $K_x$  and  $K_z$  [Figure 2.10(a)]. Along

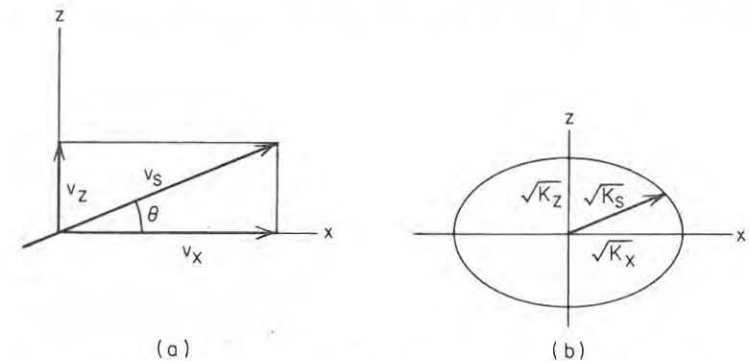


Figure 2.10 (a) Specific discharge  $v_s$  in an arbitrary direction of flow. (b) Hydraulic conductivity ellipse.



the flowline

$$v_s = -K_s \frac{\partial h}{\partial s} \quad (2.35)$$

where  $K_s$  is unknown, although it presumably lies in the range  $K_x - K_z$ . We can separate  $v_s$  into its components  $v_x$  and  $v_z$ , where

$$v_x = -K_s \frac{\partial h}{\partial x} = v_s \cos \theta \quad (2.36)$$

$$v_z = -K_s \frac{\partial h}{\partial z} = v_s \sin \theta$$

Now, since  $h = h(x, z)$ ,

$$\frac{\partial h}{\partial s} = \frac{\partial h}{\partial x} \cdot \frac{\partial x}{\partial s} + \frac{\partial h}{\partial z} \cdot \frac{\partial z}{\partial s} \quad (2.37)$$

Geometrically,  $\partial x/\partial s = \cos \theta$  and  $\partial z/\partial s = \sin \theta$ . Substituting these relationships together with Eqs. (2.35) and (2.36) in Eq. (2.37) and simplifying yields

$$\frac{1}{K_s} = \frac{\cos^2 \theta}{K_x} + \frac{\sin^2 \theta}{K_z} \quad (2.38)$$

This equation relates the principal conductivity components  $K_x$  and  $K_z$  to the resultant  $K_s$  in any angular direction  $\theta$ . If we put Eq. (2.38) into rectangular coordinates by setting  $x = r \cos \theta$  and  $z = r \sin \theta$ , we get

$$\frac{r^2}{K_s} = \frac{x^2}{K_x} + \frac{z^2}{K_z} \quad (2.39)$$

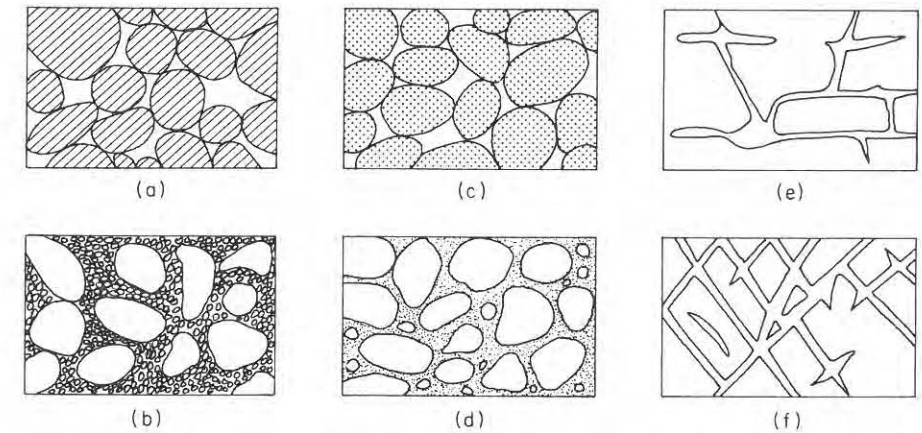
which is the equation of an ellipse with major axes  $\sqrt{K_x}$  and  $\sqrt{K_z}$  [Figure 2.10(b)]. In three dimensions, it becomes an ellipsoid with major axes  $\sqrt{K_x}$ ,  $\sqrt{K_y}$ , and  $\sqrt{K_z}$ , and it is known as the *hydraulic conductivity ellipsoid*. In Figure 2.10(b), the conductivity value  $K_s$  for any direction of flow in an anisotropic medium can be determined graphically if  $K_x$  and  $K_z$  are known.

In Section 5.1, the construction of flow nets in anisotropic media will be discussed, and it will be shown there that, in contrast to isotropic media, flowlines are not perpendicular to equipotential lines in anisotropic media.

## 2.5 Porosity and Void Ratio

If the total unit volume  $V_T$  of a soil or rock is divided into the volume of the solid portion  $V_s$  and the volume of the voids  $V_v$ , the *porosity*  $n$  is defined as  $n = V_v/V_T$ . It is usually reported as a decimal fraction or a percent.

Figure 2.11 shows the relation between various rock and soil textures and porosity. It is worth distinguishing between *primary porosity*, which is due to the



**Figure 2.11** Relation between texture and porosity. (a) Well-sorted sedimentary deposit having high porosity; (b) poorly sorted sedimentary deposit having low porosity; (c) well-sorted sedimentary deposit consisting of pebbles that are themselves porous, so that the deposit as a whole has a very high porosity; (d) well-sorted sedimentary deposit whose porosity has been diminished by the deposition of mineral matter in the interstices; (e) rock rendered porous by solution; (f) rock rendered porous by fracturing (after Meinzer, 1923).

soil or rock matrix [Figure 2.11(a), (b), (c), and (d)], and *secondary porosity*, which may be due to such phenomena as secondary solution [Figure 2.11(e)] or structurally controlled regional fracturing [Figure 2.11(f)].

Table 2.4, based in part on data summarized by Davis (1969), lists representative porosity ranges for various geologic materials. In general, rocks have lower porosities than soils; gravels, sands, and silts, which are made up of angular and

**Table 2.4** Range of Values of Porosity

	$n(\%)$
Unconsolidated deposits	
Gravel	25-40
Sand	25-50
Silt	35-50
Clay	40-70
Rocks	
Fractured basalt	5-50
Karst limestone	5-50
Sandstone	5-30
Limestone, dolomite	0-20
Shale	0-10
Fractured crystalline rock	0-10
Dense crystalline rock	0-5



rounded particles, have lower porosities than soils rich in platy clay minerals; and poorly sorted deposits [Figure 2.11(b)] have lower porosities than well-sorted deposits [Figure 2.11(a)].

The porosity  $n$  can be an important controlling influence on hydraulic conductivity  $K$ . In sampling programs carried out within deposits of well-sorted sand or in fractured rock formations, samples with higher  $n$  generally also have higher  $K$ . Unfortunately, the relationship does not hold on a regional basis across the spectrum of possible rock and soil types. Clay-rich soils, for example, usually have higher porosities than sandy or gravelly soils but lower hydraulic conductivities. In Section 8.7 techniques will be presented for the estimation of hydraulic conductivity from porosity and from grain-size analyses.

The porosity  $n$  is closely related to the *void ratio*  $e$ , which is widely used in soil mechanics. The void ratio is defined as  $e = V_v/V_s$ , and  $e$  is related to  $n$  by

$$e = \frac{n}{1-n} \quad \text{or} \quad n = \frac{e}{1+e} \quad (2.40)$$

Values of  $e$  usually fall in the range 0–3.

The measurement of porosity on soil samples in the laboratory will be treated in Section 8.4.

## 2.6 Unsaturated Flow and the Water Table

Up until this point, Darcy's law and the concepts of hydraulic head and hydraulic conductivity have been developed with respect to a *saturated* porous medium, that is, one in which all the voids are filled with water. It is clear that some soils, especially those near the ground surface, are seldom saturated. Their voids are usually only partially filled with water, the remainder of the pore space being taken up by air. The flow of water under such conditions is termed *unsaturated* or *partially saturated*. Historically, the study of unsaturated flow has been the domain of soil physicists and agricultural engineers, but recently both soil scientists and groundwater hydrologists have recognized the need to pool their talents in the development of an integrated approach to the study of subsurface flow, both saturated and unsaturated.

Our emphasis in this section will be on the hydraulics of *liquid-phase* transport of water in the unsaturated zone. We will not discuss *vapor-phase* transport, nor will we consider *soil water-plant interactions*. These latter topics are of particular interest in the agricultural sciences and they play an important role in the interpretation of soil geochemistry. More detailed consideration of the physics and chemistry of moisture transfer in unsaturated soils can be found at an introductory level in Baver et al. (1972) and at a more advanced level in Kirkham and Powers (1972) and Childs (1969).

### Moisture Content

If the total unit volume  $V_T$  of a soil or rock is divided into the volume of the solid portion  $V_s$ , the volume of the water  $V_w$ , and the volume of the air  $V_a$ , the *volumetric moisture content*  $\theta$  is defined as  $\theta = V_w/V_T$ . Like the porosity  $n$ , it is usually reported as a decimal fraction or a percent. For saturated flow,  $\theta = n$ ; for unsaturated flow,  $\theta < n$ .

### Water Table

The simplest hydrologic configuration of saturated and unsaturated conditions is that of an *unsaturated zone* at the surface and a *saturated zone* at depth [Figure 2.12(a)]. We commonly think of the water table as being the boundary between the two, yet we are aware that a saturated capillary fringe often exists above the water table. With this type of complication lurking in the background, we must take care to set up a consistent set of definitions for the various saturated-unsaturated concepts.

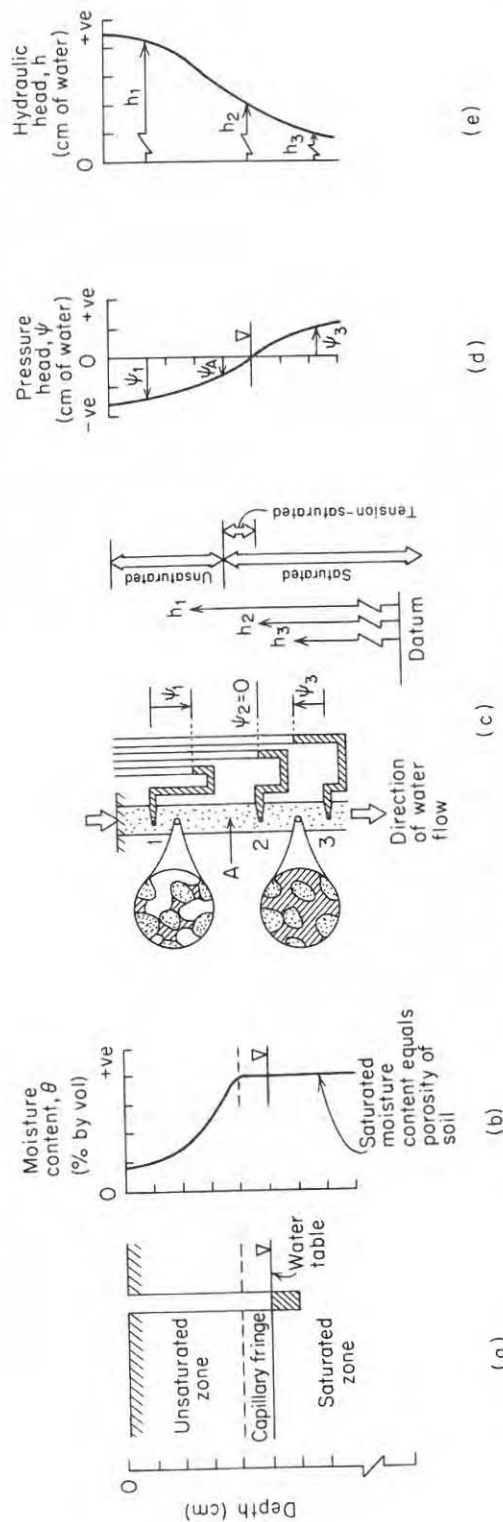
The *water table* is best defined as the surface on which the fluid pressure  $p$  in the pores of a porous medium is exactly atmospheric. The location of this surface is revealed by the level at which water stands in a shallow well open along its length and penetrating the surficial deposits just deeply enough to encounter standing water in the bottom. If  $p$  is measured in gage pressure, then on the water table,  $p = 0$ . This implies  $\psi = 0$ , and since  $h = \psi + z$ , the hydraulic head at any point on the water table must be equal to the elevation  $z$  of the water table at that point. On figures we will often indicate the position of the water table by means of a small inverted triangle, as in Figure 2.12(a).

### Negative Pressure Heads and Tensiometers

We have seen that  $\psi > 0$  (as indicated by piezometer measurements) in the saturated zone and that  $\psi = 0$  on the water table. It follows that  $\psi < 0$  in the unsaturated zone. This reflects the fact that water in the unsaturated zone is held in the soil pores under surface-tension forces. A microscopic inspection would reveal a concave meniscus extending from grain to grain across each pore channel [as shown in the upper circular inset on Figure 2.12(c)]. The radius of curvature on each meniscus reflects the surface tension on that individual, microscopic air-water interface. In reference to this physical mechanism of water retention, soil physicists often call the pressure head  $\psi$ , when  $\psi < 0$ , the *tension head* or *suction head*. In this text, on the grounds that one concept deserves only one name, we will use the term *pressure head* to refer to both positive and negative  $\psi$ .

Regardless of the sign of  $\psi$ , the hydraulic head  $h$  is still equal to the algebraic sum of  $\psi$  and  $z$ . However, above the water table, where  $\psi < 0$ , piezometers are no longer a suitable instrument for the measurement of  $h$ . Instead,  $h$  must be obtained indirectly from measurements of  $\psi$  determined with *tensiometers*. Kirkham (1964) and S. J. Richards (1965) provide detailed descriptions of the design





**Figure 2.12** Groundwater conditions near the ground surface. (a) Saturated and unsaturated zones; (b) profile of moisture content versus depth; (c) pressure-head and hydraulic-head relationships; insets: water retention under pressure heads less than (top) and greater than (bottom) atmospheric; (d) profile of pressure head versus depth; (e) profile of hydraulic head versus depth.

and use of these instruments. Very briefly, a tensiometer consists of a porous cup attached to an airtight, water-filled tube. The porous cup is inserted into the soil at the desired depth, where it comes into contact with the soil water and reaches hydraulic equilibrium. The equilibration process involves the passage of water through the porous cup from the tube into the soil. The vacuum created at the top of the airtight tube is a measure of the pressure head in the soil. It is usually measured by a vacuum gage attached to the tube above the ground surface, but it can be thought of as acting like the inverted manometer shown for point 1 in the soil profile of Figure 2.12(c). To obtain the hydraulic head  $h$ , the negative  $\psi$  value indicated by the vacuum gage on a tensiometer must be added algebraically to the elevation  $z$  of the point of measurement. In Figure 2.12(c) the instrument at point 1 is a tensiometer; the one at point 3 is a piezometer. The diagram is, of course, schematic. In practice, the tensiometer would be a tube with a gage and a porous cup at the base; the piezometer would be an open pipe with a well point at the base.

### Characteristic Curves of the Unsaturated Hydraulic Parameters

There is a further complication to the analysis of flow in the unsaturated zone. Both the moisture content  $\theta$  and the hydraulic conductivity  $K$  are functions of the pressure head  $\psi$ . On reflection, the first of these conditions should come as no great surprise. In that soil moisture is held between the soil grains under surface-tension forces that are reflected in the radius of curvature of each meniscus, we might expect that higher moisture contents would lead to larger radii of curvature, lower surface-tension forces, and lower tension heads (i.e., less-negative pressure heads). Further, it has been observed experimentally that the  $\theta$ - $\psi$  relationship is hysteretic; it has a different shape when soils are wetting than when they are drying. Figure 2.13(a) shows the hysteretic functional relationship between  $\theta$  and  $\psi$  for a naturally occurring sand soil (after Liakopoulos, 1965a). If a sample of this soil were saturated at a pressure head greater than zero and the pressure was then lowered step by step until it reached levels much less than atmospheric ( $\psi \ll 0$ ), the moisture contents at each step would follow the *drying curve* (or *drainage curve*) on Figure 2.13(a). If water were then added to the dry soil in small steps, the pressure heads would take the return route along the *wetting curve* (or *imbibition curve*). The internal lines are called *scanning curves*. They show the course that  $\theta$  and  $\psi$  would follow if the soil were only partially wetted, then dried, or vice versa.

One would expect, on the basis of what has been presented thus far, that the moisture content  $\theta$  would equal the porosity  $n$  for all  $\psi > 0$ . For coarse-grained soils this is the case, but for fine-grained soils this relationship holds over a slightly larger range  $\psi > \psi_a$ , where  $\psi_a$  is a small negative pressure head [Figure 2.13(a)] known as the *air entry pressure head*. The corresponding pressure  $p_a$  is called the *air entry pressure* or the *bubbling pressure*.

Figure 2.13(b) displays the hysteretic curves relating the hydraulic conductivity  $K$  to the pressure head  $\psi$  for the same soil. For  $\psi > \psi_a$ ,  $K = K_0$ , where  $K_0$



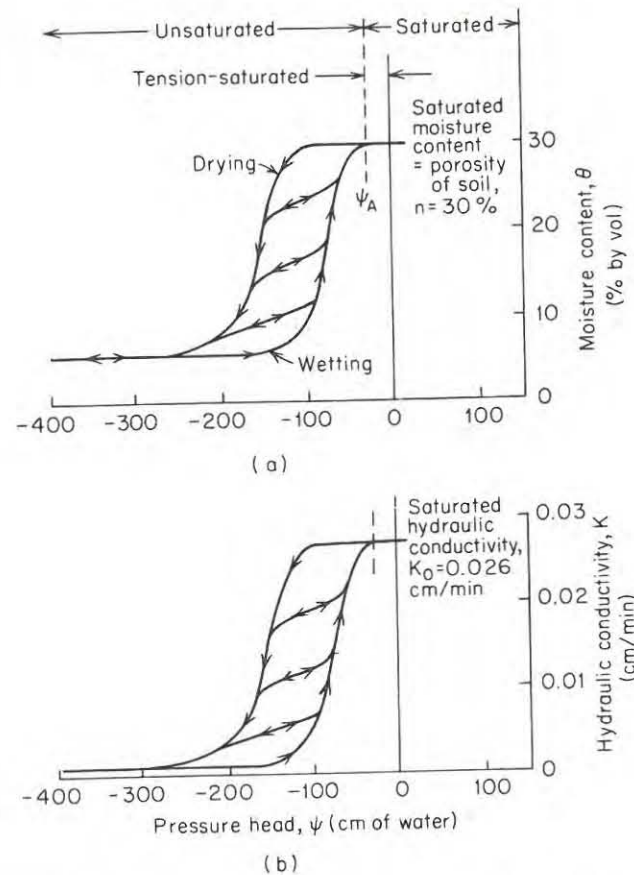


Figure 2.13 Characteristic curves relating hydraulic conductivity and moisture content to pressure head for a naturally occurring sand soil (after Liakopoulos, 1965a).

is now known as the *saturated hydraulic conductivity*. Since  $K = K(\psi)$  and  $\theta = \theta(\psi)$ , it is also true that  $K = K(\theta)$ . The curves of Figure 2.13(b) reflect the fact that the hydraulic conductivity of an unsaturated soil increases with increasing moisture content. If we write Darcy's law for unsaturated flow in the  $x$  direction in an isotropic soil as

$$v_x = -K(\psi) \frac{\partial h}{\partial x} \quad (2.41)$$

we see that the existence of the  $K(\psi)$  relationship implies that, given a constant hydraulic gradient, the specific discharge  $v$  increases with increasing moisture content.

In actual fact, it would be impossible to hold the hydraulic gradient constant while increasing the moisture content. Since  $h = \psi + z$  and  $\theta = \theta(\psi)$ , the hydrau-

lic head  $h$  is also affected by the moisture content. In other words, a hydraulic-head gradient infers a pressure-head gradient (except in pure gravity flow), and this in turn infers a moisture-content gradient. In Figure 2.12, the vertical profiles for these three variables are shown schematically for a hypothetical case of downward infiltration from the surface. Flow must be downward because the hydraulic heads displayed in Figure 2.12(e) decrease in that direction. The large positive values of  $h$  infer that  $|z| \gg |\psi|$ . In other words, the  $z = 0$  datum lies at some depth. For a real case, these three profiles would be quantitatively interlinked through the  $\theta(\psi)$  and  $K(\psi)$  curves for the soil at the site. For example, if the  $\theta(\psi)$  curve were known for the soil and the  $\theta(z)$  profile measured in the field, then the  $\psi(z)$  profile, and hence the  $h(z)$  profile, could be calculated.

The pair of curves  $\theta(\psi)$  and  $K(\psi)$  shown in Figure 2.13 are characteristic for any given soil. Sets of measurements carried out on separate samples from the same homogeneous soil would show only the usual statistical variations associated with spatially separated sampling points. The curves are often called the *characteristic curves*. In the saturated zone we have the two fundamental hydraulic parameters  $K_0$  and  $n$ ; in the unsaturated zone these become the functional relationships  $K(\psi)$  and  $\theta(\psi)$ . More succinctly,

$$\theta = \theta(\psi) \quad \psi < \psi_a \quad (2.42)$$

$$\theta = n \quad \psi \geq \psi_a$$

$$K = K(\psi) \quad \psi < \psi_a \quad (2.43)$$

$$K = K_0 \quad \psi \geq \psi_a$$

Figure 2.14 shows some hypothetical single-valued characteristic curves (i.e., without hysteresis) that are designed to show the effect of soil texture on the shape of the curves. For a more complete description of the physics of moisture retention in unsaturated soils, the reader is directed to White et al. (1971).

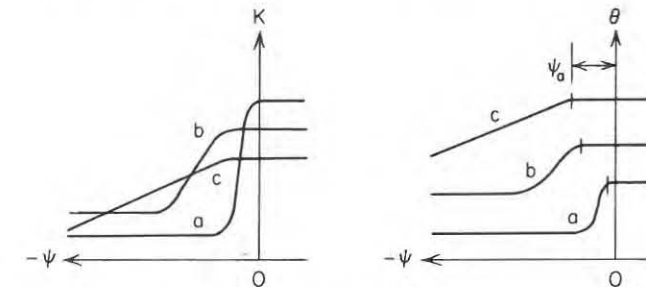


Figure 2.14 Single-valued characteristic curves for three hypothetical soils. (a) Uniform sand; (b) silty sand; (c) silty clay.



### Saturated, Unsaturated, and Tension-Saturated Zones

It is worthwhile at this point to summarize the properties of the saturated and unsaturated zones as they have been unveiled thus far. For the *saturated zone*, we can state that:

1. It occurs below the water table.
2. The soil pores are filled with water, and the moisture content  $\theta$  equals the porosity  $n$ .
3. The fluid pressure  $p$  is greater than atmospheric, so the pressure head  $\psi$  (measured as gage pressure) is greater than zero.
4. The hydraulic head  $h$  must be measured with a piezometer.
5. The hydraulic conductivity  $K$  is a constant; it is not a function of the pressure head  $\psi$ .

For the *unsaturated zone* (or, as it is sometimes called, the *zone of aeration* or the *vadose zone*):

1. It occurs above the water table and above the capillary fringe.
2. The soil pores are only partially filled with water; the moisture content  $\theta$  is less than the porosity  $n$ .
3. The fluid pressure  $p$  is less than atmospheric; the pressure head  $\psi$  is less than zero.
4. The hydraulic head  $h$  must be measured with a tensiometer.
5. The hydraulic conductivity  $K$  and the moisture content  $\theta$  are both functions of the pressure head  $\psi$ .

In short, for saturated flow,  $\psi > 0$ ,  $\theta = n$ , and  $K = K_0$ ; for unsaturated flow,  $\psi < 0$ ,  $\theta = \theta(\psi)$ , and  $K = K(\psi)$ .

The capillary fringe fits into neither of the groupings above. The pores there are saturated, but the pressure heads are less than atmospheric. A more descriptive name that is now gaining acceptance is the *tension-saturated zone*. An explanation of its seemingly anomalous properties can be discovered in Figure 2.13. It is the existence of the air entry pressure head  $\psi_a < 0$  on the characteristic curves that is responsible for the existence of a capillary fringe.  $\psi_a$  is the value of  $\psi$  that will exist at the top of the tension-saturated zone, as shown by  $\psi_a$  for point A in Figure 2.12(d). Since  $\psi_a$  has greater negative values in clay soils than it does in sands, these fine-grained soils develop thicker tension-saturated zones than do coarse-grained soils.

Some authors consider the tension-saturated zone as part of the saturated zone, but in that case the water table is no longer the boundary between the two zones. From a physical standpoint it is probably best to retain all three zones—saturated, tension-saturated, and unsaturated—in one's conception of the complete hydrologic system.

A point that follows directly from the foregoing discussion in this section may warrant a specific statement. In that fluid pressures are less than atmospheric, there

can be no natural outflow to the atmosphere from an unsaturated or tension-saturated face. Water can be transferred from the unsaturated zone to the atmosphere by evaporation and transpiration, but natural outflows, such as springs on streambanks or inflows to well bores, must come from the saturated zone. The concept of a saturated seepage face is introduced in Section 5.5 and its importance in relation to hillslope hydrology is emphasized in Section 6.5.

### Perched and Inverted Water Tables

The simple hydrologic configuration that we have considered thus far, with a single unsaturated zone overlying the main saturated groundwater body, is a common one. It is the rule where homogeneous geologic deposits extend to some depth. Complex geological environments, on the other hand, can lead to more complex saturated-unsaturated conditions. The existence of a low-permeability clay layer in a high-permeability sand formation, for example, can lead to the formation of a discontinuous saturated lense, with unsaturated conditions existing both above and below. If we consider the line *ABCD* in Figure 2.15 to be the  $\psi = 0$  isobar, we can refer to the *ABC* portion as a *perched water table* and *ADC* as an *inverted water table*. *EF* is the true water table.

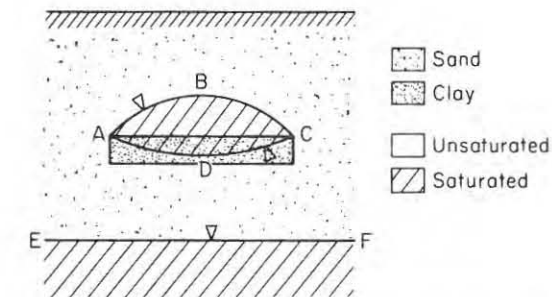


Figure 2.15 Perched water table *ABC*, inverted water table *ADC*, and true water table *EF*.

Saturated conditions can be discontinuous in time as well as space. Heavy rainfall can lead to the formation of a temporary saturated zone at the ground surface, its lower boundary being an inverted water table underlain by unsaturated conditions. Saturated zones of this type dissipate with time under the influence of downward percolation and evaporation from the surface. In Chapter 6 we will examine the interactions of rainfall and infiltration in saturated-unsaturated systems in greater detail.

### Multiphase Flow

The approach to unsaturated flow outlined in this section is the one used almost universally by soil physicists, but it is, at root, an approximate method. Unsaturated flow is actually a special case of *multiphase flow* through porous media, with



two phases, air and water, coexisting in the pore channels. Let  $\theta_w$  be the volumetric moisture content (previously denoted by  $\theta$ ) and  $\theta_a$  be the volumetric air content, defined analogously to  $\theta_w$ . There are now two fluid pressures to consider:  $p_w$  for the water phase and  $p_a$  for the air phase; and two pressure heads,  $\psi_w$  and  $\psi_a$ . Each soil now possesses two characteristic curves of fluid content versus pressure head, one for the water,  $\theta_w(\psi_w)$ , and one for the air,  $\theta_a(\psi_a)$ .

When it comes to conductivity relationships, it makes sense to work with the permeability  $k$  [Eq. (2.28)] rather than the hydraulic conductivity  $K$ , since  $k$  is independent of the fluid and  $K$  is not. The flow parameters  $k_w$  and  $k_a$  are called the *effective permeabilities* of the medium to water and air. Each soil has two characteristic curves of effective permeability versus pressure head, one for water,  $k_w(\psi_w)$ , and one for air,  $k_a(\psi_a)$ .

The single-phase approach to unsaturated flow leads to techniques of analysis that are accurate enough for almost all practical purposes, but there are some unsaturated flow problems where the multiphase flow of air and water must be considered. These commonly involve cases where a buildup in air pressure in the entrapped air ahead of a wetting front influences the rate of propagation of the front through a soil. Wilson and Luthin (1963) encountered the effects experimentally, Youngs and Peck (1964) provide a theoretical discussion, and McWhorter (1971) presents a complete analysis. As will be shown in Section 6.8, air entrapment also influences water-table fluctuations. Bianchi and Haskell (1966) discuss air entrapment problems in a field context, and Green et al. (1970) describe a field application of the multiphase approach to the analysis of a subsurface flow system.

Much of the original research on multiphase flow through porous media was carried out in the petroleum industry. Petroleum reservoir engineering involves the analysis of three-phase flow of oil, gas, and water. Pirson (1958) and Amyx et al. (1960) are standard references in the field. Stallman (1964) provides an interpretive review of the petroleum multiphase contributions as they pertain to groundwater hydrology.

The two-phase analysis of unsaturated flow is an example of *immiscible displacement*; that is, the fluids displace each other without mixing, and there is a distinct fluid-fluid interface within each pore. The simultaneous flow of two fluids that are soluble in each other is termed *miscible displacement*, and in such cases a distinct fluid-fluid interface does not exist. Bear (1972) provides an advanced theoretical treatment of both miscible and immiscible displacement in porous media. In this text, the only examples of immiscible displacement are those that have been discussed in this subsection. In the rest of the text, unsaturated flow will be treated as a single-phase problem using the concepts and approach of the first part of this section. The most common occurrences of miscible displacement in groundwater hydrology involve the mixing of two waters with different chemistry (such as seawater and fresh-water, or pure water and contaminated water). The transport processes associated with miscible displacement and the techniques of analysis of groundwater contamination will be discussed in Chapter 9.

## 2.7 Aquifers and Aquitards

Of all the words in the hydrologic vocabulary, there are probably none with more shades of meaning than the term *aquifer*. It means different things to different people, and perhaps different things to the same person at different times. It is used to refer to individual geologic layers, to complete geologic formations, and even to groups of geologic formations. The term must always be viewed in terms of the scale and context of its usage.

### *Aquifers, Aquitards, and Aquicludes*

An *aquifer* is best defined as a saturated permeable geologic unit that can transmit significant quantities of water under ordinary hydraulic gradients. An *aquiclude* is defined as a saturated geologic unit that is incapable of transmitting significant quantities of water under ordinary hydraulic gradients.

An alternative pair of definitions that are widely used in the water-well industry states that an aquifer is permeable enough to yield economic quantities of water to wells, whereas aquicludes are not.

In recent years the term *aquitard* has been coined to describe the less-permeable beds in a stratigraphic sequence. These beds may be permeable enough to transmit water in quantities that are significant in the study of regional groundwater flow, but their permeability is not sufficient to allow the completion of production wells within them. Most geologic strata are classified as either aquifers or aquitards; very few formations fit the classical definition of an aquiclude. As a result, there is a trend toward the use of the first two of these terms at the expense of the third.

The most common aquifers are those geologic formations that have hydraulic conductivity values in the upper half of the observed range (Table 2.2): unconsolidated sands and gravels, permeable sedimentary rocks such as sandstones and limestones, and heavily fractured volcanic and crystalline rocks. The most common aquitards are clays, shales, and dense crystalline rocks. In Chapter 4, the principal aquifer and aquitard types will be examined more fully within the context of a discussion on geological controls on groundwater occurrence.

The definitions of aquifer and aquitard are purposely imprecise with respect to hydraulic conductivity. This leaves open the possibility of using the terms in a relative sense. For example, in an interlayered sand-silt sequence, the silts may be considered aquitards, whereas in a silt-clay system, they are aquifers.

Aquifers are often called by their stratigraphic names. The Dakota Sandstone, for example, owes its geological fame largely to Meinzer's (1923) assessment of its properties as an aquifer. Two other well-known North American aquifers are the St. Peter Sandstone in Illinois and the Ocala Limestone in Florida. A summary of the principal aquifer systems in the United States can be found in McGuinness (1963) and Maxey (1964), who build on the earlier compilations of Meinzer (1923), Tolman (1937), and Thomas (1951). Brown (1967) provides information on Canada's major aquifers.



In the ideal world of analysis where many of the expository sections of this book must reside, aquifers tend to appear as homogeneous, isotropic formations of constant thickness and simple geometry. We hope the reader will bear in mind that the real world is somewhat different. The hydrogeologist constantly faces complex aquifer-aquitard systems of heterogeneous and anisotropic formations rather than the idealized cases pictured in texts. It will often seem that the geological processes have maliciously conspired to maximize the interpretive and analytical difficulties.

### Confined and Unconfined Aquifers

A *confined aquifer* is an aquifer that is confined between two aquitards. An *unconfined aquifer*, or *water-table aquifer*, is an aquifer in which the water table forms the upper boundary. Confined aquifers occur at depth, unconfined aquifers near the ground surface (Figure 2.16). A saturated lense that is bounded by a perched water table (Figure 2.15) is a special case of an unconfined aquifer. Such lenses are sometimes called *perched aquifers*.

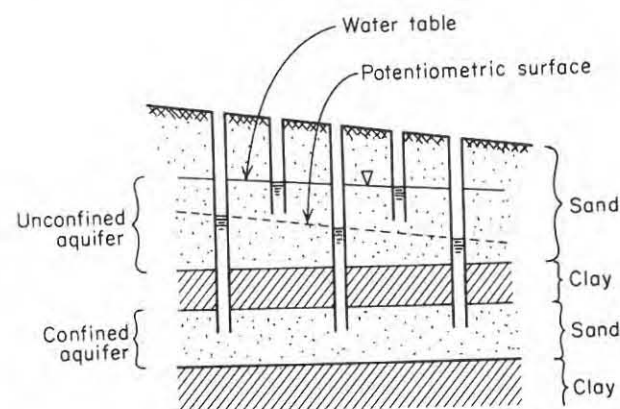


Figure 2.16 Unconfined aquifer and its water table; confined aquifer and its potentiometric surface.

In a confined aquifer, the water level in a well usually rises above the top of the aquifer. If it does, the well is called an *artesian well* and the aquifer is said to exist under *artesian conditions*. In some cases the water level may rise above the ground surface, in which case the well is known as a *flowing artesian well* and the aquifer is said to exist under *flowing artesian conditions*. In Section 6.1, we will examine the topographic and geologic circumstances that lead to flowing artesian conditions. The water level in a well in an unconfined aquifer rests at the water table.

### Potentiometric Surface

For confined aquifers, which are extensively tapped by wells for water supply, there has grown up a traditional concept that is not particularly sound but which is firmly entrenched in usage. If the water-level elevations in wells tapping a confined aquifer are plotted on a map and contoured, the resulting surface, which is actually a map of the hydraulic head in the aquifer, is called a *potentiometric surface*. A potentiometric map of an aquifer provides an indication of the directions of groundwater flow in the aquifer.

The concept of a potentiometric surface is only rigorously valid for horizontal flow in horizontal aquifers. The condition of horizontal flow is met only in aquifers with hydraulic conductivities that are much higher than those in the associated confining beds. Some hydrogeological reports contain potentiometric surface maps based on water-level data from sets of wells that bottom near the same elevation but that are not associated with a specific well-defined confined aquifer. This type of potentiometric surface is essentially a map of hydraulic head contours on a two-dimensional horizontal cross section taken through the three-dimensional hydraulic head pattern that exists in the subsurface in that area. If there are vertical components of flow, as there usually are, calculations and interpretations based on this type of potentiometric surface can be grossly misleading.

It is also possible to confuse a potentiometric surface with the water table in areas where both confined and unconfined aquifers exist. Figure 2.16 schematically distinguishes between the two. In general, as we shall see from the flow nets in Chapter 6, the two do not coincide.

## 2.8 Steady-State Flow and Transient Flow

*Steady-state flow* occurs when at any point in a flow field the magnitude and direction of the flow velocity are constant with time. *Transient flow* (or *unsteady flow*, or *nonsteady flow*) occurs when at any point in a flow field the magnitude or direction of the flow velocity changes with time.

Figure 2.17(a) shows a steady-state groundwater flow pattern (dashed equipotentials, solid flowline) through a permeable alluvial deposit beneath a concrete dam. Along the line  $AB$ , the hydraulic head  $h_{AB} = 1000$  m. It is equal to the elevation of the surface of the reservoir above  $AB$ . Similarly,  $h_{CD} = 900$  m (the elevation of the tailrace pond above  $CD$ ). The hydraulic head drop  $\Delta h$  across the system is 100 m. If the water level in the reservoir above  $AB$  and the water level in the tailrace pond above  $CD$  do not change with time, the flow net beneath the dam will not change with time. The hydraulic head at point  $E$ , for example, will be  $h_E = 950$  m and will remain constant. Under such circumstances the velocity  $v = -K \partial h / \partial l$  will also remain constant through time. In a steady-state flow system the velocity may vary from point to point, but it will not vary with time at any given point.



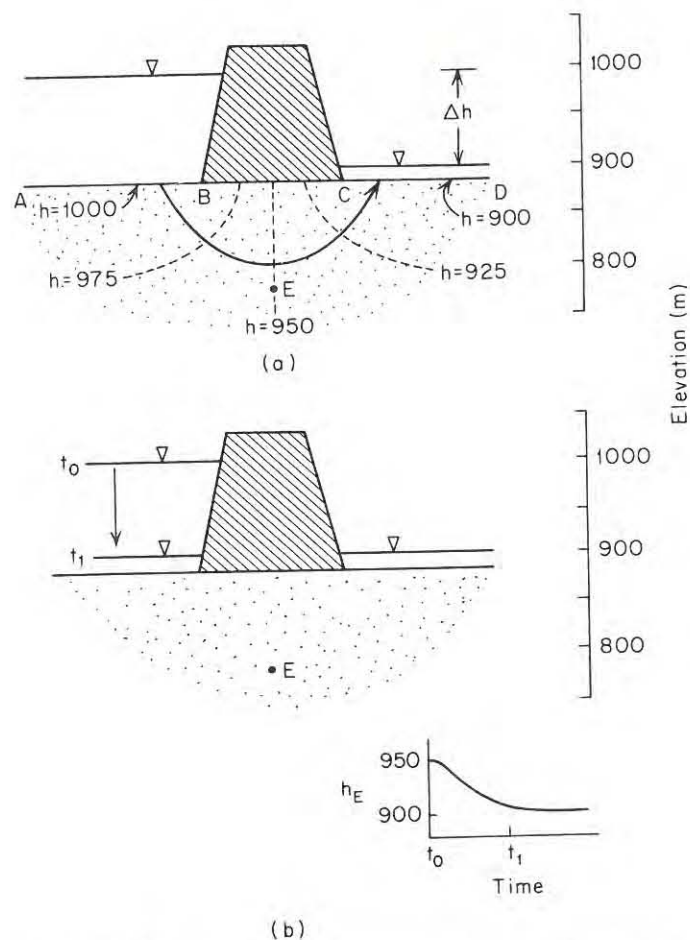


Figure 2.17 Steady-state and transient groundwater flow beneath a dam.

Let us now consider the transient flow problem schematically shown in Figure 2.17(b). At time  $t_0$  the flow net beneath the dam will be identical to that of Figure 2.17(a) and  $h_E$  will be 950 m. If the reservoir level is allowed to drop over the period  $t_0$  to  $t_1$ , until the water levels above and below the dam are identical at time  $t_1$ , the ultimate conditions under the dam will be static with no flow of water from the upstream to the downstream side. At point  $E$  the hydraulic head  $h_E$  will undergo a time-dependent decline from  $h_E = 950$  m at time  $t_0$  to its ultimate value of  $h_E = 900$  m. There may well be time lag in such a system so that  $h_E$  will not necessarily reach the value  $h_E = 900$  m until some time after  $t = t_1$ .

One important difference between steady and transient systems lies in the relation between their flowlines and pathlines. *Flowlines* indicate the instantaneous

directions of flow throughout a system (at all times in a steady system, or at a given instant in time in a transient system). They must be orthogonal to the equipotential lines throughout the region of flow at all times. *Pathlines* map the route that an individual particle of water follows through a region of flow during a steady or transient event. In a steady flow system a particle of water that enters the system at an inflow boundary will flow toward the outflow boundary along a pathline that coincides with a flowline such as that shown in Figure 2.17(a). In a transient flow system, on the other hand, pathlines and flowlines do not coincide. Although a flow net can be constructed to describe the flow conditions at any given instant in time in a transient system, the flowlines shown in such a snapshot represent only the directions of movement at that instant in time. In that the configuration of flowlines changes with time, the flowlines cannot describe, in themselves, the complete path of a particle of water as it traverses the system. The delineation of transient pathlines has obvious importance in the study of groundwater contamination.

A groundwater hydrologist must understand the techniques of analysis for both steady-state flow and transient flow. In the final sections of this chapter the equations of flow will be developed for each type of flow, under both saturated and unsaturated conditions. The practical methodology that is presented in later chapters is often based on the theoretical equations, but it is not usually necessary for the practicing hydrogeologist to have the mathematics at his or her fingertips. The primary application of steady-state techniques in groundwater hydrology is in the analysis of regional groundwater flow. An understanding of transient flow is required for the analysis of well hydraulics, groundwater recharge, and many of the geochemical and geotechnical applications.

## 2.9 Compressibility and Effective Stress

The analysis of transient groundwater flow requires the introduction of the concept of *compressibility*, a material property that describes the change in volume, or strain, induced in a material under an applied stress. In the classical approach to the strength of elastic materials, the *modulus of elasticity* is a more familiar material property. It is defined as the ratio of the change in stress  $d\sigma$  to the resulting change in strain  $d\epsilon$ . Compressibility is simply the inverse of the modulus of elasticity. It is defined as strain/stress,  $d\epsilon/d\sigma$ , rather than stress/strain,  $d\sigma/d\epsilon$ . The term is utilized for both elastic materials and nonelastic materials. For the flow of water through porous media, it is necessary to define two compressibility terms, one for the water and one for the porous media.

### Compressibility of Water

Stress is imparted to a fluid through the fluid pressure  $p$ . An increase in pressure  $dp$  leads to a decrease in the volume  $V_w$  of a given mass of water. The *compressibility*



of water  $\beta$  is therefore defined as

$$\beta = \frac{-dV_w/V_w}{dp} \quad (2.44)$$

The negative sign is necessary if we wish  $\beta$  to be a positive number.

Equation (2.44) implies a linear elastic relationship between the volumetric strain  $dV_w/V_w$  and the stress induced in the fluid by the change in fluid pressure  $dp$ . The compressibility  $\beta$  is thus the slope of the line relating strain to stress for water, and this slope does not change over the range of fluid pressures encountered in groundwater hydrology (including those less than atmospheric that are encountered in the unsaturated zone). For the range of groundwater temperatures that are usually encountered, temperature has a small influence on  $\beta$ , so that for most practical purposes we can consider  $\beta$  a constant. The dimensions of  $\beta$  are the inverse of those for pressure or stress. Its value can be taken as  $4.4 \times 10^{-10} \text{ m}^2/\text{N}$  (or  $\text{Pa}^{-1}$ ).

For a given mass of water it is possible to rewrite Eq. (2.44) in the form

$$\beta = \frac{-d\rho/\rho}{dp} \quad (2.45)$$

where  $\rho$  is the fluid density. Integration of Eq. (2.45) yields the *equation of state* for water:

$$\rho = \rho_0 \exp [\beta(p - p_0)] \quad (2.46)$$

where  $\rho_0$  is the fluid density at the datum pressure  $p_0$ . For  $p_0$  atmospheric, Eq. (2.46) can be written in terms of gage pressures as

$$\rho = \rho_0 e^{\beta p} \quad (2.47)$$

An incompressible fluid is one for which  $\beta = 0$  and  $\rho = \rho_0 = \text{constant}$ .

### Effective Stress

Let us now consider the compressibility of the porous medium. Assume that a stress is applied to a unit mass of saturated sand. There are three mechanisms by which a reduction in volume can be achieved: (1) by compression of the water in the pores, (2) by compression of the individual sand grains, and (3) by a rearrangement of the sand grains into a more closely packed configuration. The first of these mechanisms is controlled by the fluid compressibility  $\beta$ . Let us assume that the second mechanism is negligible, that is, that the individual soil grains are incompressible. Our task is to define a compressibility term that will reflect the third mechanism.

To do so, we will have to invoke the principle of effective stress. This concept was first proposed by Terzaghi (1925), and has been analyzed in detail by Skempton

(1961). Most soil mechanics texts, such as those by Terzaghi and Peck (1967) and Scott (1963), provide a full discussion.

For our purposes, consider the stress equilibrium on an arbitrary plane through a saturated geological formation at depth (Figure 2.18).  $\sigma_T$  is the total stress acting downward on the plane. It is due to the weight of overlying rock and water. This stress is borne in part by the granular skeleton of the porous medium and in part by the fluid pressure  $p$  of the water in the pores. The portion of the total stress that is not borne by the fluid is called the *effective stress*  $\sigma_e$ . It is this stress that is actually applied to the grains of the porous medium. Rearrangement of the soil grains and the resulting compression of the granular skeleton is caused by changes in the effective stress, not by changes in the total stress. The two are related by the simple equation

$$\sigma_T = \sigma_e + p \quad (2.48)$$

or, in terms of the changes,

$$d\sigma_T = d\sigma_e + dp \quad (2.49)$$

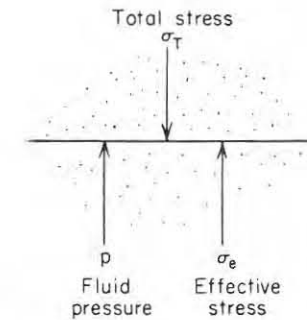


Figure 2.18 Total stress, effective stress, and fluid pressure on an arbitrary plane through a saturated porous medium.

Many of the transient subsurface flow problems that must be analyzed do not involve changes in the total stress. The weight of rock and water overlying each point in the system often remains essentially constant through time. In such cases,  $d\sigma_T = 0$  and

$$d\sigma_e = -dp \quad (2.50)$$

Under these circumstances, if the fluid pressure increases, the effective stress decreases by an equal amount; and if the fluid pressure decreases, the effective stress increases by an equal amount. For cases where the total stress does not change with time, the effective stress at any point in the system, and the resulting volumetric deformations there, are controlled by the fluid pressures at that point. Since  $p = \rho g \psi$  and  $\psi = h - z$  ( $z$  being constant at the point in question), changes



in the effective stress at a point are in effect governed by changes in the hydraulic head at that point:

$$d\sigma_e = -\rho g d\psi = -\rho g dh \quad (2.51)$$

### Compressibility of a Porous Medium

The compressibility of a porous medium is defined as

$$\alpha = \frac{-dV_T/V_T}{d\sigma_e} \quad (2.52)$$

where  $V_T$  is the total volume of a soil mass and  $d\sigma_e$  the change in effective stress.

Recall that  $V_T = V_s + V_v$ , where  $V_s$  is the volume of the solids and  $V_v$  is the volume of the water-saturated voids. An increase in effective stress  $d\sigma_e$  produces a reduction  $dV_T$  in the total volume of the soil mass. In granular materials this reduction occurs almost entirely as a result of grain rearrangements. It is true that individual grains may themselves be compressible, but the effect is usually considered to be negligible. In general,  $dV_T = dV_s + dV_v$ ; but for our purposes we will assume that  $dV_s = 0$  and  $dV_T = dV_v$ .

Consider a sample of saturated soil that has been placed in a laboratory loading cell such as the one shown in Figure 2.19(a). A total stress  $\sigma_T = L/A$  can be applied to the sample through the pistons. The sample is laterally confined by the cell walls, and entrapped water is allowed to escape through vents in the pistons to an external pool held at a constant known fluid pressure. The volumetric reduction in the size of the soil sample is measured at several values of  $L$  as  $L$  is increased in a stepwise fashion. At each step, the increased total stress  $d\sigma_T$  is initially borne by the water under increased fluid pressures, but drainage of the water from the sample to the external pool slowly transfers the stress from the water to the granular skeleton. This transient process is known as *consolidation*, and the time required for the consolidation process to reach hydraulic equilibrium at each  $L$  can be considerable. Once attained, however, it is known that  $dp = 0$  within the sample, and from Eq. (2.49),  $d\sigma_e = d\sigma_T = dL/A$ . If the soil sample has an original void ratio  $e_0$  (where  $e = V_v/V_s$ ) and an original height  $b$  [Figure 2.19(a)], and assuming that  $dV_T = dV_v$ , Eq. (2.52) can be written as

$$\alpha = \frac{-db/b}{d\sigma_e} = \frac{-de/(1 + e_0)}{d\sigma_e} \quad (2.53)$$

The compressibility  $\alpha$  is usually determined from the slope of a strain-stress plot in the form of  $e$  versus  $\sigma_e$ . The curve  $AB$  in Figure 2.19(b) is for loading (increasing  $\sigma_e$ ),  $BC$  is for unloading (decreasing  $\sigma_e$ ). In general, the strain-stress relation is neither linear nor elastic. In fact, for repeated loadings and unloadings, many fine-grained soils show hysteretic properties [Figure 2.19(c)]. The soil compressibility  $\alpha$ , unlike the fluid compressibility  $\beta$ , is not a constant; it is a function of the applied stress and it is dependent on the previous loading history.

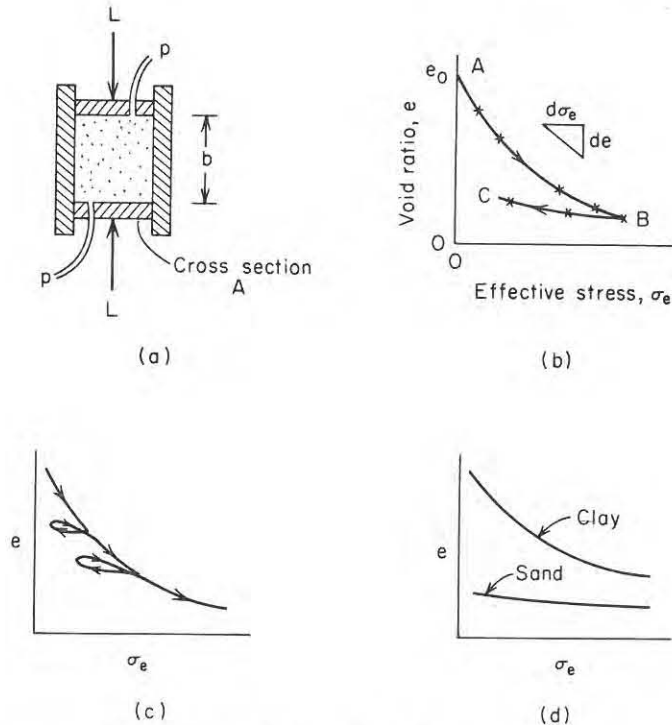


Figure 2.19 (a) Laboratory loading cell for the determination of soil compressibility; (b), (c), and (d) schematic curves of void ratio versus effective stress.

Figure 2.19(d) provides a schematic comparison of the  $e$ - $\sigma_e$  curves for clay and sand. The lesser slope for the sand curve implies a smaller  $\alpha$ , and its linearity implies an  $\alpha$  value that stays constant over a wide range of  $\sigma_e$ . In groundwater systems, the time-dependent fluctuations in  $\sigma_e$  are often quite small, so that even for clays, a constant  $\alpha$  can have some meaning. Table 2.5 is a table of compress-

Table 2.5 Range of Values of Compressibility\*

	Compressibility, $\alpha$ ( $\text{m}^2/\text{N}$ or $\text{Pa}^{-1}$ )
Clay	$10^{-6}$ – $10^{-8}$
Sand	$10^{-7}$ – $10^{-9}$
Gravel	$10^{-8}$ – $10^{-10}$
Jointed rock	$10^{-8}$ – $10^{-10}$
Sound rock	$10^{-9}$ – $10^{-11}$
Water ( $\beta$ )	$4.4 \times 10^{-10}$

\*See Table A1.3, Appendix I, for conversion factors.



sibility values that indicates the ranges of values that have been measured for various types of geologic materials. Original sources of compressibility data include Domenico and Mifflin (1965) and Johnson et al. (1968). The dimensions of  $\alpha$ , like  $\beta$ , are the inverse of those for stress. Values are expressed in SI units of  $\text{m}^2/\text{N}$  or  $\text{Pa}^{-1}$ . Note that the compressibility of water is of the same order of magnitude as the compressibility of the less-compressible geologic materials.

As noted in Figure 2.19(b) and (c), the compressibility of some soils in expansion (expansibility?) is much less than in compression. For clays, the ratio of the two  $\alpha$ 's is usually on the order of 10:1; for uniform sands, it approaches 1:1. For soils that have compressibility values that are significantly less in expansion than compression, volumetric deformations that occur in response to increasing effective stress [perhaps due to decreasing hydraulic heads as suggested by Eq. (2.51)] are largely irreversible. They are not recovered when effective stresses subsequently decrease. In a clay-sand aquifer-aquitard system, the large compactions that can occur in the clay aquitards (due to the large  $\alpha$  values) are largely irrecoverable; whereas the small deformations that occur in the sand aquifers (due to the small  $\alpha$  values) are largely elastic.

#### Aquifer Compressibility

The concept of compressibility inherent in Eq. (2.53) and in Figures 2.18 and 2.19 is one-dimensional. In the field, at depth, a one-dimensional concept has meaning if it is assumed that the soils and rocks are stressed only in the vertical direction. The total vertical stress  $\sigma_T$  at any point is due to the weight of overlying rock and water; the neighboring materials provide the horizontal confinement. The effective vertical stress  $\sigma_e$  is equal to  $\sigma_T - p$ . Under these conditions the *aquifer compressibility*  $\alpha$  is defined by the first equality of Eq. (2.53), where  $b$  is now the aquifer thickness rather than a sample height. The parameter  $\alpha$  is a vertical compressibility. If it is to be determined with a laboratory apparatus like that of Figure 2.19(a), the soil cores must be oriented vertically and loading must be applied at right angles to any horizontal bedding. Within an aquifer,  $\alpha$  may vary with horizontal position; that is,  $\alpha$  may be heterogeneous with  $\alpha = \alpha(x, y)$ .

In the most general analysis, it must be recognized that the stress field existing at depth is not one-dimensional but three-dimensional. In that case, aquifer compressibility must be considered as an anisotropic parameter. The vertical compressibility  $\alpha$  is then invoked by changes in the vertical component of effective stress, and the horizontal compressibilities are invoked by changes in the horizontal components of effective stress. Application of the concepts of three-dimensional stress analysis in the consideration of fluid flow through porous media is an advanced topic that cannot be pursued here. Fortunately, for many practical cases, changes in the horizontal stress field are very small, and in most analyses it can be assumed that they are negligible. It is sufficient for our purposes to think of the aquifer compressibility  $\alpha$  as a single isotropic parameter, but it should be kept in mind that it is actually the compressibility in the vertical direction, and that this is the only direction in which large changes in effective stress are anticipated.

To illustrate the nature of the deformations that can occur in compressible aquifers, consider the aquifer of thickness  $b$  shown in Figure 2.20. If the weight of overlying material remains constant and the hydraulic head in the aquifer is decreased by an amount  $-dh$ , the increase in effective stress  $d\sigma_e$  is given by Eq. (2.51) as  $\rho g dh$ , and the aquifer compaction, from Eq. (2.53) is

$$db = -\alpha b d\sigma_e = -\alpha b \rho g dh \quad (2.54)$$

The minus sign indicates that the decrease in head produces a reduction in the thickness  $b$ .

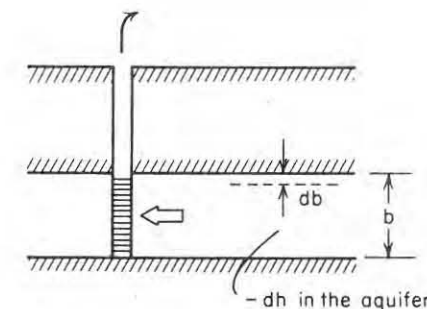


Figure 2.20 Aquifer compaction caused by groundwater pumping.

One way that the hydraulic head might be lowered in an aquifer is by pumping from a well. Pumping induces horizontal hydraulic gradients toward the well in the aquifer, and as a result the hydraulic head is decreased at each point near the well. In response, effective stresses are increased at these points, and aquifer compaction results. Conversely, pumping water into an aquifer increases hydraulic heads, decreases effective stresses, and causes aquifer expansion. If the compaction of an aquifer-aquitard system due to groundwater pumping is propagated to the ground surface, the result is *land subsidence*. In Section 8.12 this phenomenon is considered in detail.

#### Effective Stress in the Unsaturated Zone

The first equality in Eq. (2.51) indicates that the relationship between the effective stress  $\sigma_e$  and the pressure head  $\psi$  should be linear. This relation, and the concept of Figure 2.18 on which it is based, holds in the saturated zone, but there is now abundant evidence to suggest that it does not hold in the unsaturated zone (Narasimhan, 1975). For unsaturated flow, Bishop and Blight (1963) suggest that Eq. (2.51) should be modified to read

$$d\sigma_e = -\rho g \chi d\psi \quad (2.55)$$

where the parameter  $\chi$  depends on the degree of saturation, the soil structure, and the wetting-drying history of the soil. Curve ABC in Figure 2.21 shows such a



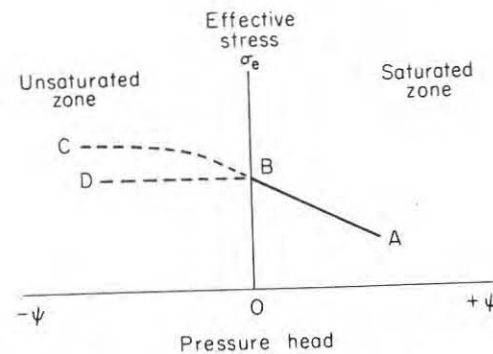


Figure 2.21 Relationship between effective stress and pressure head in the saturated and unsaturated zones (after Narasimhan, 1975).

relationship schematically. For  $\psi > 0$ ,  $\chi = 1$ ; for  $\psi < 0$ ,  $\chi \leq 1$ ; and for  $\psi \ll 0$ ,  $\chi = 0$ .

The  $\chi$  approach is an empirical one and its use reflects the fact that the capacity of fluid pressures less than atmospheric to support a part of the total stress in an unsaturated flow field is not yet fully understood. As a first approximation, it is not unreasonable to suppose that they have no such capacity, as suggested by curve ABD in Figure 2.21. Under this assumption, for  $\psi < 0$ ,  $\chi = 0$ ,  $d\sigma_e = d\sigma_T$ , and changes in pressure head (or moisture content) in the unsaturated zone do not lead to changes in effective stress.

The definition of the compressibility of a porous medium in the unsaturated zone is still given by Eq. (2.52) just as it is in the saturated zone, but the influence of the fluid pressure on the effective stress is now considered to be muted or non-existent.

## 2.10 Transmissivity and Storativity

There are six basic physical properties of fluid and porous media that must be known in order to describe the hydraulic aspects of saturated groundwater flow. These six have all now been introduced. They are, for the water, *density*  $\rho$ , *viscosity*  $\mu$ , and *compressibility*  $\beta$ ; and for the media, *porosity*  $n$  (or void ratio  $e$ ), *permeability*  $k$ , and *compressibility*  $\alpha$ . All the other parameters that are used to describe the hydrogeologic properties of geologic formations can be derived from these six. For example, we have seen from Eq. (2.28) that the saturated hydraulic conductivity  $K$  is a combination of  $k$ ,  $\rho$ , and  $\mu$ . In this section, we will consider the concepts of specific storage  $S_s$ , storativity  $S$ , and transmissivity  $T$ .

### Specific Storage

The *specific storage*  $S_s$  of a saturated aquifer is defined as the volume of water that a unit volume of aquifer releases from storage under a unit decline in hydraulic head. From Section 2.9 we now know that a decrease in hydraulic head  $h$  infers a

decrease in fluid pressure  $p$  and an increase in effective stress  $\sigma_e$ . The water that is released from storage under conditions of decreasing  $h$  is produced by two mechanisms: (1) the compactions of the aquifer caused by increasing  $\sigma_e$ , and (2) the expansion of the water caused by decreasing  $p$ . The first of these mechanisms is controlled by the aquifer compressibility  $\alpha$  and the second by the fluid compressibility  $\beta$ .

Let us first consider the water produced by the compaction of the aquifer. The volume of water expelled from the unit volume of aquifer during compaction will be equal to the reduction in volume of the unit volume of aquifer. The volumetric reduction  $dV_T$  will be negative, but the amount of water produced  $dV_w$  will be positive, so that, from Eq. (2.52),

$$dV_w = -dV_T = \alpha V_T d\sigma_e \quad (2.56)$$

For a unit volume,  $V_T = 1$ , and from Eq. (2.51),  $d\sigma_e = -\rho g dh$ . For a unit decline in hydraulic head,  $dh = -1$ , and we have

$$dV_w = \alpha \rho g \quad (2.57)$$

Now consider the volume of water produced by the expansion of the water. From Eq. (2.44),

$$dV_w = -\beta V_w dp \quad (2.58)$$

The volume of water  $V_w$  in the total unit volume  $V_T$  is  $nV_T$ , where  $n$  is the porosity. With  $V_T = 1$  and  $dp = \rho g d\psi = \rho g d(h - z) = \rho g dh$ , Eq. (2.58) becomes, for  $dh = -1$ ,

$$dV_w = \beta n \rho g \quad (2.59)$$

The specific storage  $S_s$  is the sum of the two terms given by Eqs. (2.57) and (2.59):

$$S_s = \rho g (\alpha + n\beta) \quad (2.60)$$

A dimensional inspection of this equation shows  $S_s$  to have the peculiar dimensions of  $[L]^{-1}$ . This also follows from the definition of  $S_s$  as a volume per volume per unit decline in head.

### Transmissivity and Storativity of a Confined Aquifer

For a confined aquifer of thickness  $b$ , the *transmissivity* (or *transmissibility*)  $T$  is defined as

$$T = Kb \quad (2.61)$$

and the *storativity* (or *storage coefficient*)  $S$  is defined as

$$S = S_s b \quad (2.62)$$



If we substitute Eq. (2.60) in Eq. (2.62), the expanded definition of  $S$  is seen to be

$$S = \rho g b(\alpha + n\beta) \quad (2.63)$$

The storativity of a saturated confined aquifer of thickness  $b$  can be defined in words as the volume of water that an aquifer releases from storage per unit surface area of aquifer per unit decline in the component of hydraulic head normal to that surface. The hydraulic head for a confined aquifer is usually displayed in the form of a potentiometric surface, and Figure 2.22(a) illustrates the concept of storativity in this light.

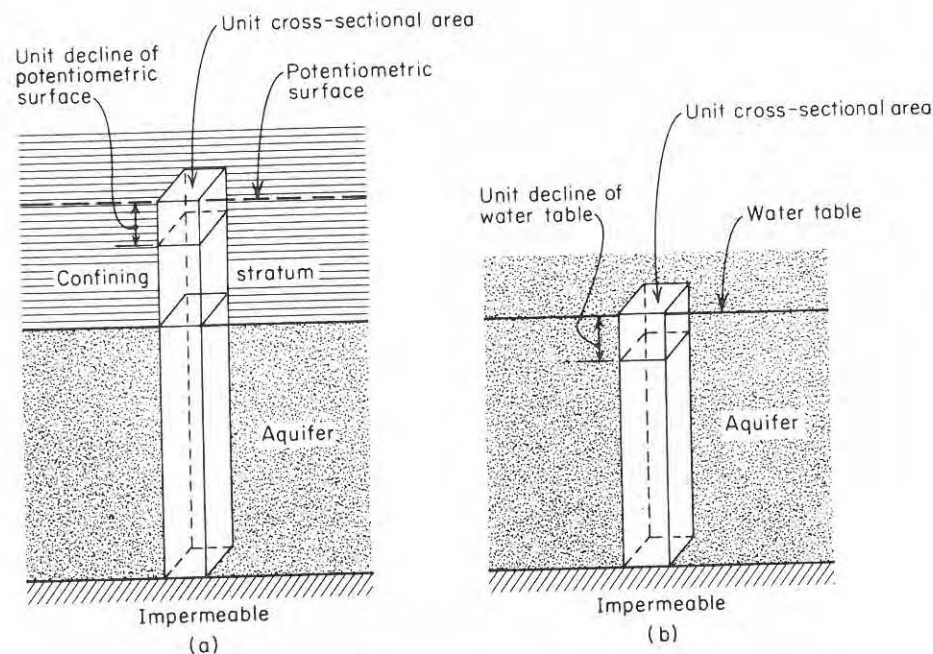


Figure 2.22 Schematic representation of the storativity in (a) confined and (b) unconfined aquifers (after Ferris et al., 1962).

In that the hydraulic conductivity  $K$  has dimensions  $[L/T]$ , it is clear from Eq. (2.61) that the transmissivity  $T$  has dimensions  $[L^2/T]$ . The SI metric unit is  $m^2/s$ .  $T$  and  $S$  are widely used terms in the North American water well industry and are often expressed in FPS engineering units. If  $K$  is expressed in gal/day/ft<sup>2</sup>, then  $T$  has units of gal/day/ft. The range of values of  $T$  can be calculated by multiplying the pertinent  $K$  values from Table 2.2 by the range of reasonable aquifer thicknesses, say 5–100 m. Transmissivities greater than 0.015 m<sup>2</sup>/s (or 0.16 ft<sup>2</sup>/s or 100,000 gal/day/ft) represent good aquifers for water well exploitation. Storativities are dimensionless. In confined aquifers, they range in value from 0.005 to 0.00005. Reference to the definition of  $S$ , coupled with a realization of its range of values,

makes it clear that large head changes over extensive areas are required to produce substantial water yields from confined aquifers.

Transmissivities and storativities can be specified for aquitards as well as aquifers. However, in most applications, the vertical hydraulic conductivity of an aquitard has more significance than its transmissivity. It might also be noted that in clay aquitards,  $\alpha \gg \beta$ , and the  $n\beta$  term in the definition of storativity [Eq. (2.63)] and specific storage [Eq. (2.60)] becomes negligible.

It is possible to define a single formation parameter that couples the transmission properties  $T$  or  $K$ , and the storage properties  $S$  or  $S_s$ . The *hydraulic diffusivity*  $D$  is defined as

$$D = \frac{T}{S} = \frac{K}{S_s} \quad (2.64)$$

The term is not widely used in practice.

The concepts of *transmissivity*  $T$  and *storativity*  $S$  were developed primarily for the analysis of well hydraulics in confined aquifers. For two-dimensional, horizontal flow toward a well in a confined aquifer of thickness  $b$ , the terms are well defined; but they lose their meaning in many other groundwater applications. If a groundwater problem has three-dimensional overtones, it is best to revert to the use of *hydraulic conductivity*  $K$  and *specific storage*  $S_s$ ; or perhaps even better, to the fundamental parameters *permeability*  $k$ , *porosity*  $n$ , and *compressibility*  $\alpha$ .

#### Transmissivity and Specific Yield in Unconfined Aquifers

In an unconfined aquifer, the transmissivity is not as well defined as in a confined aquifer, but it can be used. It is defined by the same equation [Eq. (2.61)], but  $b$  is now the saturated thickness of the aquifer or the height of the water table above the top of the underlying aquitard that bounds the aquifer.

The storage term for unconfined aquifers is known as the *specific yield*  $S_y$ . It is defined as the volume of water that an unconfined aquifer releases from storage per unit surface area of aquifer per unit decline in the water table. It is sometimes called the unconfined storativity. Figure 2.22(b) illustrates the concept schematically.

The idea of specific yield is best visualized with reference to the saturated-unsaturated interaction it represents. Figure 2.23 shows the water-table position and the vertical profile of moisture content vs. depth in the unsaturated zone at two times,  $t_1$  and  $t_2$ . The crosshatched area represents the volume of water released from storage in a column of unit cross section. If the water-table drop represents a unit decline, the crosshatched area represents the specific yield.

The specific yields of unconfined aquifers are much higher than the storativities of confined aquifers. The usual range of  $S_y$  is 0.01–0.30. The higher values reflect the fact that releases from storage in unconfined aquifers represent an actual dewatering of the soil pores, whereas releases from storage in confined aquifers represent only the secondary effects of water expansion and aquifer compaction



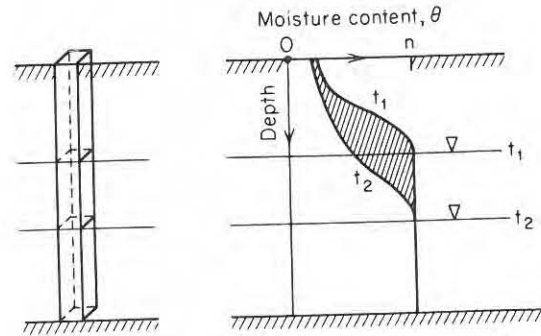


Figure 2.23 Concept of specific yield viewed in terms of the unsaturated moisture profiles above the water table.

caused by changes in the fluid pressure. The favorable storage properties of unconfined aquifers make them more efficient for exploitation by wells. When compared to confined aquifers, the same yield can be realized with smaller head changes over less extensive areas.

#### Storage in the Unsaturated Zone

In an unsaturated soil, changes in moisture content  $\theta$ , such as those shown in Figure 2.23, are accompanied by changes in the pressure head  $\psi$ , through the  $\theta(\psi)$  relationship displayed on the characteristic curve of Figure 2.13(a). The slope of this characteristic curve represents the unsaturated storage property of a soil. It is called the *specific moisture capacity*  $C$  and is defined as

$$C = \frac{d\theta}{d\psi} \quad (2.65a)$$

An increase of  $d\psi$  in the pressure head (say, from  $-200$  cm to  $-100$  cm on Figure 2.13) must be accompanied by an increase of  $d\theta$  in the moisture stored in the unsaturated soil. Since  $\theta(\psi)$  is nonlinear and hysteretic, so too, is  $C$ . It is not a constant; it is a function of the pressure head  $\psi$ :  $C = C(\psi)$ . In the saturated zone, in fact for all  $\psi > \psi_a$ , the moisture content  $\theta$  is equal to the porosity  $n$ , a constant, so that  $C = 0$ . A parallel formulation to Eq. (2.42) for  $C$  is

$$\begin{aligned} C &= C(\psi) & \psi < \psi_a \\ C &= 0 & \psi \geq \psi_a \end{aligned} \quad (2.65b)$$

The transmission and storage properties of an unsaturated soil are fully specified by the characteristic curve  $K(\psi)$  and one of the two curves  $\theta(\psi)$  or  $C(\psi)$ .

In an analogous manner to Eq. (2.64), the *soil-water diffusivity* can be defined as

$$D(\psi) = \frac{K(\psi)}{C(\psi)} \quad (2.66)$$

## 2.11 Equations of Groundwater Flow

In almost every field of science and engineering the techniques of analysis are based on an understanding of the physical processes, and in most cases it is possible to describe these processes mathematically. Groundwater flow is no exception. The basic law of flow is Darcy's law, and when it is put together with an equation of continuity that describes the conservation of fluid mass during flow through a porous medium, a partial differential equation of flow is the result. In this section, we will present brief developments of the equations of flow for (1) steady-state saturated flow, (2) transient saturated flow, and (3) transient unsaturated flow. All three of the equations of flow are well known to mathematicians, and mathematical techniques for their manipulation are widely available and in common use in science and engineering. Generally, the equation of flow appears as one component of a *boundary-value problem*, so in the last part of this section we will explore this concept.

In that so many of the standard techniques of analysis in groundwater hydrology are based on boundary-value problems that involve partial differential equations, it is useful to have a basic understanding of these equations as one proceeds to learn the various techniques. Fortunately, it is not an absolute requirement. In most cases, the techniques can be explained and understood without returning at every step to the fundamental mathematics. The research hydrogeologist must work with the equations of flow on a daily basis; the practicing hydrogeologist can usually avoid the advanced mathematics if he or she so desires.

#### Steady-State Saturated Flow

Consider a unit volume of porous media such as that shown in Figure 2.24. Such an element is usually called an *elemental control volume*. The law of conservation of mass for steady-state flow through a saturated porous medium requires that the rate of fluid mass flow *into* any elemental control volume be equal to the rate of fluid mass flow *out* of any elemental control volume. The *equation of continuity* that

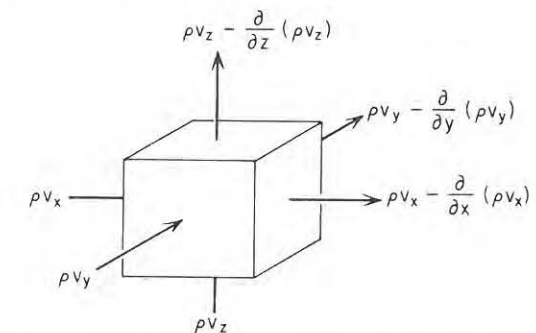


Figure 2.24 Elemental control volume for flow through porous media.



translates this law into mathematical form can be written, with reference to Figure 2.24, as

$$-\frac{\partial(\rho v_x)}{\partial x} - \frac{\partial(\rho v_y)}{\partial y} - \frac{\partial(\rho v_z)}{\partial z} = 0 \quad (2.67)$$

A quick dimensional analysis on the  $\rho v$  terms will show them to have the dimensions of a mass rate of flow across a unit cross-sectional area of the elemental control volume. If the fluid is incompressible,  $\rho(x, y, z) = \text{constant}$  and the  $\rho$ 's can be removed from Eq. (2.67). Even if the fluid is compressible and  $\rho(x, y, z) \neq \text{constant}$ , it can be shown that terms of the form  $\rho \partial v_x / \partial x$  are much greater than terms of the form  $v_x \partial \rho / \partial x$ , both of which arise when the chain rule is used to expand Eq. (2.67). In either case, Eq. (2.67) simplifies to

$$-\frac{\partial v_x}{\partial x} - \frac{\partial v_y}{\partial y} - \frac{\partial v_z}{\partial z} = 0 \quad (2.68)$$

Substitution of Darcy's law for  $v_x$ ,  $v_y$ , and  $v_z$  in Eq. (2.68) yields the equation of flow for steady-state flow through an anisotropic saturated porous medium:

$$\frac{\partial}{\partial x} \left( K_x \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left( K_y \frac{\partial h}{\partial y} \right) + \frac{\partial}{\partial z} \left( K_z \frac{\partial h}{\partial z} \right) = 0 \quad (2.69)$$

For an isotropic medium,  $K_x = K_y = K_z$ , and if the medium is also homogeneous, then  $K(x, y, z) = \text{constant}$ . Equation (2.69) then reduces to the equation of flow for steady-state flow through a homogeneous, isotropic medium:

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = 0 \quad (2.70)$$

Equation (2.70) is one of the most basic partial differential equations known to mathematicians. It is called *Laplace's equation*. The solution of the equation is a function  $h(x, y, z)$  that describes the value of the hydraulic head  $h$  at any point in a three-dimensional flow field. A solution to Eq. (2.70) allows us to produce a contoured equipotential map of  $h$ , and with the addition of flowlines, a flow net.

For steady-state, saturated flow in a two-dimensional flow field, say in the  $xz$  plane, the central term of Eq. (2.70) would drop out and the solution would be a function  $h(x, z)$ .

#### Transient Saturated Flow

The law of conservation of mass for transient flow in a saturated porous medium requires that the *net* rate of fluid mass flow into any elemental control volume be equal to the time rate of change of fluid mass storage within the element. With reference to Figure 2.24, the equation of continuity takes the form

$$-\frac{\partial(\rho v_x)}{\partial x} - \frac{\partial(\rho v_y)}{\partial y} - \frac{\partial(\rho v_z)}{\partial z} = \frac{\partial(\rho n)}{\partial t} \quad (2.71)$$

or, expanding the right-hand side,

$$-\frac{\partial(\rho v_x)}{\partial x} - \frac{\partial(\rho v_y)}{\partial y} - \frac{\partial(\rho v_z)}{\partial z} = n \frac{\partial \rho}{\partial t} + \rho \frac{\partial n}{\partial t} \quad (2.72)$$

The first term on the right-hand side of Eq. (2.72) is the mass rate of water produced by an expansion of the water under a change in its density  $\rho$ . The second term is the mass rate of water produced by the compaction of the porous medium as reflected by the change in its porosity  $n$ . The first term is controlled by the compressibility of the fluid  $\beta$  and the second term by the compressibility of the aquifer  $\alpha$ . We have already carried out the analysis (in Section 2.10) that is necessary to simplify the two terms on the right of Eq. (2.72). We know that the change in  $\rho$  and the change in  $n$  are both produced by a change in hydraulic head  $h$ , and that the *volume* of water produced by the two mechanisms for a unit decline in head is  $S_s$ , where  $S_s$  is the specific storage given by  $S_s = \rho g(\alpha + n\beta)$ . The mass rate of water produced (time rate of change of fluid mass storage) is  $\rho S_s \partial h / \partial t$ , and Eq. (2.72) becomes

$$-\frac{\partial(\rho v_x)}{\partial x} - \frac{\partial(\rho v_y)}{\partial y} - \frac{\partial(\rho v_z)}{\partial z} = \rho S_s \frac{\partial h}{\partial t} \quad (2.73)$$

Expanding the terms on the left-hand side by the chain rule and recognizing that terms of the form  $\rho \partial v_x / \partial x$  are much greater than terms of the form  $v_x \partial \rho / \partial x$  allows us to eliminate  $\rho$  from both sides of Eq. (2.73). Inserting Darcy's law, we obtain

$$\frac{\partial}{\partial x} \left( K_x \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left( K_y \frac{\partial h}{\partial y} \right) + \frac{\partial}{\partial z} \left( K_z \frac{\partial h}{\partial z} \right) = S_s \frac{\partial h}{\partial t} \quad (2.74)$$

This is the equation of flow for transient flow through a saturated anisotropic porous medium. If the medium is homogeneous and isotropic, Eq. (2.74) reduces to

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = \frac{S_s}{K} \frac{\partial h}{\partial t} \quad (2.75)$$

or expanding  $S_s$ ,

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = \frac{\rho g(\alpha + n\beta)}{K} \frac{\partial h}{\partial t} \quad (2.76)$$

Equation (2.76) is known as the *diffusion equation*. The solution  $h(x, y, z, t)$  describes the value of the hydraulic head at any point in a flow field at any time. A solution requires knowledge of the three basic hydrogeological parameters,  $K$ ,  $\alpha$ , and  $n$ , and the fluid parameters,  $\rho$  and  $\beta$ .

For the special case of a horizontal confined aquifer of thickness  $b$ ,  $S = S_s b$  and  $T = Kb$ , and the two-dimensional form of Eq. (2.75) becomes

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = \frac{S}{T} \frac{\partial h}{\partial t} \quad (2.77)$$



The solution  $h(x, y, t)$  describes the hydraulic head field at any point on a horizontal plane through the horizontal aquifer at any time. Solution requires knowledge of the aquifer parameters  $S$  and  $T$ .

The equation of flow for transient, saturated flow [in any of the forms given by Eqs. (2.74) through (2.77)] rests on the law of flow established by Darcy (1856), on the clarification of the hydraulic potential by Hubbert (1940), and on the recognition of the concepts of aquifer elasticity by Meinzer (1923), and effective stress by Terzaghi (1925). The classical development was first put forward by Jacob (1940) and can be found in its most complete form in Jacob (1950). The development presented in this section, together with the storage concepts of the earlier sections, is essentially that of Jacob.

In recent years there has been considerable reassessment of the classical development. Biot (1955) recognized that in compacting aquifers it is necessary to cast Darcy's law in terms of a relative velocity of fluid to grains, and Cooper (1966) pointed out the inconsistency of taking a fixed elemental control volume in a deforming medium. Cooper showed that Jacob's classical development is correct if one views the velocity as relative and the coordinate system as deforming. He also showed that De Wiest's (1966) attempt to address this problem (which also appears in Davis and De Wiest, 1966) is incorrect. Appendix II contains a more rigorous presentation of the Jacob-Cooper development than has been attempted here.

The classical development, through its use of the concept of vertical aquifer compressibility, assumes that stresses and deformations in a compacting aquifer occur only in the vertical direction. The approach couples a three-dimensional flow field and a one-dimensional stress field. The more general approach, which couples a three-dimensional flow field and a three-dimensional stress field, was first considered by Biot (1941, 1955). Verruijt (1969) provides an elegant summary of the approach.

For almost all practical purposes it is not necessary to consider relative velocities, deforming coordinates, or three-dimensional stress fields. The classical equations of flow presented in this section are sufficient.

### Transient Unsaturated Flow

Let us define the degree of saturation  $\theta'$  as  $\theta' = \theta/n$ , where  $\theta$  is the moisture content and  $n$  is the porosity. For flow in an elemental control volume that may be only partially saturated, the equation of continuity must reveal the time rate of change of moisture content as well as the time rate of change of storage due to water expansion and aquifer compaction. The  $\rho n$  term in Eq. (2.71) must become  $\rho n \theta'$ , and Eq. (2.72) becomes

$$-\frac{\partial(\rho v_x)}{\partial x} - \frac{\partial(\rho v_y)}{\partial y} - \frac{\partial(\rho v_z)}{\partial z} = n\theta' \frac{\partial \rho}{\partial t} + \rho \theta' \frac{\partial n}{\partial t} + n\rho \frac{\partial \theta'}{\partial t} \quad (2.78)$$

For unsaturated flow, the first two terms on the right-hand side of Eq. (2.78) are much smaller than the third term. Discarding these two terms, canceling the  $\rho$ 's

from both sides in the usual way, inserting the unsaturated form of Darcy's law [Eq. (2.41)], and recognizing that  $n d\theta' = d\theta$ , leads to

$$\frac{\partial}{\partial x} \left[ K(\psi) \frac{\partial h}{\partial x} \right] + \frac{\partial}{\partial y} \left[ K(\psi) \frac{\partial h}{\partial y} \right] + \frac{\partial}{\partial z} \left[ K(\psi) \frac{\partial h}{\partial z} \right] = \frac{\partial \theta}{\partial t} \quad (2.79)$$

It is usual to put Eq. (2.79) into a form where the independent variable is either  $\theta$  or  $\psi$ . For the latter case it is necessary to multiply the top and bottom of the right-hand side by  $\partial \psi$ . Then, recalling the definition of the specific moisture capacity  $C$  [Eq. (2.65)], and noting that  $h = \psi + z$ , we obtain

$$\frac{\partial}{\partial x} \left[ K(\psi) \frac{\partial \psi}{\partial x} \right] + \frac{\partial}{\partial y} \left[ K(\psi) \frac{\partial \psi}{\partial y} \right] + \frac{\partial}{\partial z} \left[ K(\psi) \left( \frac{\partial \psi}{\partial z} + 1 \right) \right] = C(\psi) \frac{\partial \psi}{\partial t} \quad (2.80)$$

Equation (2.80) is the  $\psi$ -based equation of flow for transient flow through an unsaturated porous medium. It is often called *Richards equation*, in honor of the soil physicist who first developed it (Richards, 1931). The solution  $\psi(x, y, z, t)$  describes the pressure head field at any point in a flow field at any time. It can easily be converted into an hydraulic head solution  $h(x, y, z, t)$  through the relation  $h = \psi + z$ . Solution requires knowledge of the characteristic curves  $K(\psi)$  and  $C(\psi)$  or  $\theta(\psi)$ .

The coupling of the unsaturated flow equation [Eq. (2.80)] and the saturated flow equation [Eq. (2.74)] has been attempted by Freeze (1971a) and by Narasimhan (1975). Improvements in the theory underlying saturated-unsaturated systems must await a better understanding of the principle of effective stress in the unsaturated zone.

### Boundary-Value Problems

A boundary-value problem is a *mathematical model*. The technique of analysis inferred by this latter term is a four-step process, involving (1) examination of the physical problem, (2) replacement of the physical problem by an equivalent mathematical problem, (3) solution of the mathematical problem with the accepted techniques of mathematics, and (4) interpretation of the mathematical results in terms of the physical problem. Mathematical models based on the physics of flow usually take the form of boundary-value problems of the type pioneered by the developers of potential field theory and as applied in physics to such problems as the conduction of heat in solids (Carslaw and Jaeger, 1959).

To fully define a transient boundary-value problem for subsurface flow, one needs to know (1) the size and shape of the region of flow, (2) the equation of flow within the region, (3) the boundary conditions around the boundaries of the region, (4) the initial conditions in the region, (5) the spatial distribution of the hydrogeologic parameters that control the flow, and (6) a mathematical method of solution. If the boundary-value problem is for a steady-state system, requirement (4) is removed.

Consider the simple groundwater flow problem illustrated in Figure 2.25(a). The region  $ABCD$  contains a homogeneous, isotropic porous medium of hydraulic



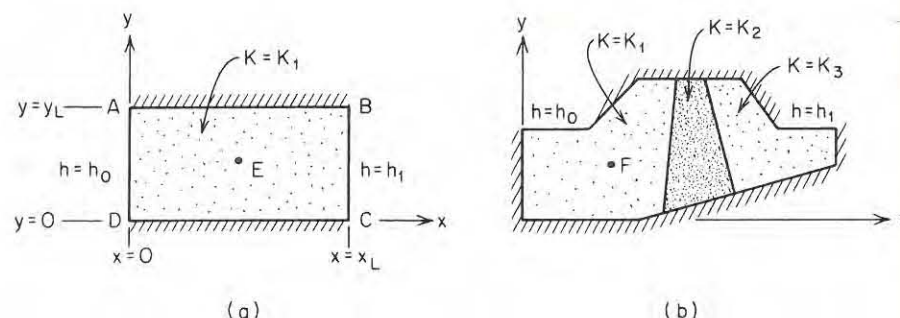


Figure 2.25 Two steady-state boundary-value problems in the  $xy$  plane.

conductivity  $K_1$ . The boundaries  $AB$  and  $CD$  are impermeable; the hydraulic heads on  $AD$  and  $BC$  are  $h_0$  and  $h_1$ , respectively. Assuming steady flow and setting  $h_0 = 100$  m and  $h_1 = 0$  m, we can see by inspection that the hydraulic head at point  $E$  will be 50 m. Apparently, we made implicit use of properties (1), (3), and (5) from the list above; our method of solution (6) was one of inspection. It is not clear that we needed to know the equation of flow within the region. If we move to a more difficult problem such as that shown in Figure 2.25(b) (an earthfill dam resting on a sloping base), the value of the hydraulic head at point  $F$  does not come so easily. Here we would have to invoke a mathematical method of solution, and it would require that we know the equation of flow.

The methods of solution can be categorized roughly into five approaches: (1) solution by inspection, (2) solution by graphical techniques, (3) solution by analog model, (4) solution by analytical mathematical techniques, and (5) solution by numerical mathematical techniques. We have just seen an example of solution by inspection. The methods of flow-net construction presented in Chapter 5 can be viewed as graphical solutions to boundary-value problems. Electrical analog models are discussed in Sections 5.2 and 8.9. Numerical solutions are the basis of modern computer simulation techniques as described in Sections 5.3 and 8.8.

The most straightforward approach to the solution of boundary-value problems is that of analytical solutions. Many of the standard groundwater techniques presented later in the text are based on analytical solutions, so it is pertinent to examine a simple example. Consider, once again, the boundary-value problem of Figure 2.25(a). The analytical solution is

$$h(x, y) = h_0 - (h_0 - h_1) \frac{x}{x_L} \quad (2.81)$$

This is the equation of a set of equipotential lines traversing the field  $ABCD$  parallel to the boundaries  $AD$  and  $BC$ . Since the equipotentials are parallel to the  $y$  axis,  $h$  is not a function of  $y$  and  $y$  does not appear on the right-hand side of Eq. (2.81). At point  $E$ ,  $x/x_L = 0.5$ , and if  $h_0 = 100$  m and  $h_1 = 0$  m as before, then

$h_E$  from Eq. (2.81) is 50 m, as expected. In Appendix III, the *separation-of-variables* technique is used to obtain the analytical solution Eq. (2.81), and it is shown that the solution satisfies the equation of flow and the boundary conditions.

## 2.12 Limitations of the Darcian Approach

Darcy's law provides an accurate description of the flow of groundwater in almost all hydrogeological environments. In general, Darcy's law holds (1) for saturated flow and for unsaturated flow, (2) for steady-state flow and for transient flow, (3) for flow in aquifers and for flow in aquitards, (4) for flow in homogeneous systems and for flow in heterogeneous systems, (5) for flow in isotropic media and for flow in anisotropic media, and (6) for flow in both rocks and granular media. In this text, we will assume that Darcy's law is a valid basis for our quantitative analyses.

Despite this soothing statement, or perhaps because of it, it is important that we examine the theoretical and practical limitations of the Darcian approach. It is necessary to look at the assumptions that underlie our definition of a continuum; examine the concepts of microscopic and macroscopic flow; investigate the upper and lower limits of Darcy's law; and consider the particular problems associated with flow in fractured rock.

### Darcian Continuum and Representative Elementary Volume

In Section 2.1, it was noted that the definition of Darcy's law requires the replacement of the actual ensemble of grains that make up a porous medium by a representative continuum. It was further stated that this continuum approach is carried out at a macroscopic rather than a microscopic scale. If Darcy's law is a macroscopic law, there must be a lower limit to the size of an element of porous media for which the law is valid. Hubbert (1940) has addressed this problem. He defined the term *macroscopic* with the aid of Figure 2.26. This diagram is a hypothetical plot of the porosity of a porous medium as it might be measured on samples of

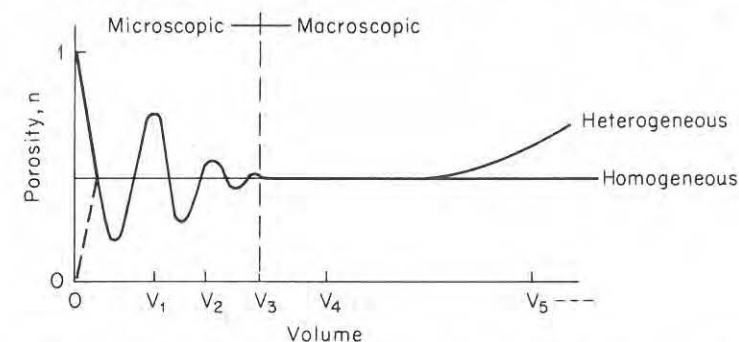


Figure 2.26 Microscopic and macroscopic domains and the representative elementary volume  $V_3$  (after Hubbert, 1956; Bear, 1972).



increasing volume  $V_1, V_2, \dots$ , taken at a point  $P$  within a porous medium. Bear (1972) defines the volume  $V_3$  in Figure 2.26 as the *representative elementary volume*. He notes that it is a volume that must be larger than a single pore. In fact, it must include a sufficient number of pores to permit the meaningful statistical average required in the continuum approach. Below this volume there is no single value that can represent the porosity at  $P$ . Throughout this text the values of porosity, hydraulic conductivity, and compressibility refer to measurements that could be carried out on a sample larger than the representative elementary volume. In a more practical sense, they refer to values that can be measured on the usual sizes of cored soil samples. Where the scale of analysis involves volumes, such as  $V_3$  in Figure 2.26, that may encompass more than one stratum in heterogeneous media, the scale is sometimes called *megascopic*.

The development of each of the equations of flow presented in Section 2.11 included the invocation of Darcy's law. It must be recognized, then, that the methods of analysis that are based on boundary-value problems involving these equations apply on a macroscopic scale, at the level of the Darcian continuum. There are some groundwater phenomena, such as the movement of a tracer through a porous medium, that cannot be analyzed on this scale. It is therefore necessary to examine the interrelationship that exists between the Darcy velocity (or specific discharge) defined for the macroscopic Darcian continuum and the microscopic velocities that exist in the liquid phase of the porous medium.

#### Specific Discharge, Macroscopic Velocity, and Microscopic Velocity

Our development will be more rigorous if we first differentiate, as Bear (1972) has done, between the *volumetric porosity*,  $n$ , which was defined in Section 2.5, and the *areal porosity*,  $n_A$ , which can be defined for any areal cross section through a unit volume, as  $n_A = A_v/A_T$ , where  $A_v$  is the area occupied by the voids and  $A_T$  is the total area. As suggested by Figure 2.27(a), various cross sections within a given unit volume may exhibit differing areal porosities  $n_{A1}, n_{A2}, \dots$ . The volumetric porosity,  $n$ , is an average of the various possible areal porosities,  $n_{A_i}$ .

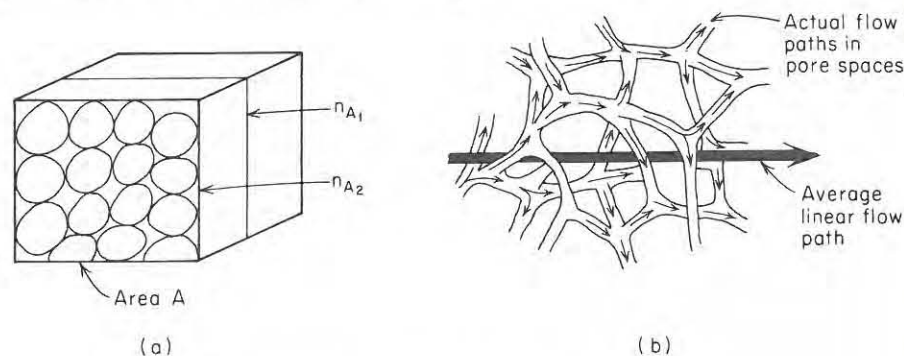


Figure 2.27 Concepts of (a) areal porosity and (b) average linear velocity.

For any cross section  $A$ , the *specific discharge*,  $v$ , is defined from Eq. (2.1) as

$$v = \frac{Q}{A}$$

In that the volumetric flux  $Q$  is divided by the full cross-sectional area (voids and solids alike), this velocity is identified as being pertinent to the macroscopic continuum approach. In actual fact, the flow passes through only that portion of the cross-sectional area occupied by voids. For cross section  $A_1$  we can define a velocity  $\bar{v}_1 = Q/n_{A1}A$  that represents the volumetric flux divided by the actual cross-sectional area through which flow occurs. For the various sections  $A_1, A_2, \dots$  we can define  $\bar{v}_1, \bar{v}_2, \dots$ . If we denote their average by  $\bar{v}$ , then

$$\bar{v} = \frac{Q}{nA} = \frac{v}{n} = \frac{-K}{n} \frac{\partial h}{\partial l} \quad (2.82)$$

The velocity  $\bar{v}$  is known under a variety of names. We will refer to it as the *average linear velocity*. In that  $Q$ ,  $n$ , and  $A$  are measurable macroscopic terms, so is  $\bar{v}$ . It should be emphasized that  $\bar{v}$  does not represent the average velocity of the water particles traveling through the pore spaces. These true, microscopic velocities are generally larger than  $\bar{v}$ , because the water particles must travel along irregular paths that are longer than the linearized path represented by  $\bar{v}$ . This is shown schematically in Figure 2.27(b). The true, microscopic velocities that exist in the pore channels are seldom of interest, which is indeed fortunate, for they are largely indeterminate. For all the situations that will be considered in this text, the Darcy velocity  $v$  and the average linear velocity  $\bar{v}$  will suffice.

As a basis for further explanation of  $\bar{v}$ , consider an experiment where a tracer is used to determine how much time is required for the bulk mass of groundwater to move a short but significant distance  $AB$  along a flow path.  $\bar{v}$  is then defined as the ratio of travel distance to travel time, where the travel distance is defined as the linear distance from  $A$  to  $B$  and the travel time is the time required for the tracer to travel from  $A$  to  $B$ . In light of this conceptualization of  $\bar{v}$ , Nelson (1968) has suggested a slightly different form of Eq. (2.82):

$$\bar{v} = \frac{Q}{\epsilon n A} = \frac{v}{\epsilon n} \quad (2.83)$$

where  $\epsilon$  is an empirical constant dependent on the characteristics of the porous medium. Data obtained in laboratory experiments by Ellis et al. (1968) using relatively uniform sands indicate values of  $\epsilon$  in the range 0.98–1.18. Values of  $\epsilon$  for nonuniform sands and for other materials do not exist at present. In studies of groundwater tracers and groundwater contamination the almost universal unstated assumption is that  $\epsilon = 1$ . For granular media this probably introduces little error. In fractured media the assumption may have less validity.



### Upper and Lower Limits of Darcy's Law

Even if we limit ourselves to the consideration of specific discharge on a macroscopic scale through the Darcian continuum, there may be limitations on the applicability of Darcy's law. Darcy's law is a linear law. If it were universally valid, a plot of the specific discharge  $v$  versus the hydraulic gradient  $dh/dl$  would reveal a straight-line relationship for all gradients between 0 and  $\infty$ . For flow through granular materials there are at least two situations where the validity of this linear relationship is in question. The first concerns flow through low-permeability sediments under very low gradients and the second concerns large flows through very high permeability sediments. In other words, there may be both a lower limit and an upper limit to the range of validity of Darcy's law. It has been suggested that a more general form of the porous media flow law might be

$$v = -K \left( \frac{dh}{dl} \right)^m \quad (2.84)$$

If  $m = 1$ , as it does in all the common situations, the flow law is linear and is called Darcy's law; if  $m \neq 1$ , the flow law is not linear and should not be called Darcy's law.

For fine-grained materials of low permeability, it has been suggested on the basis of laboratory evidence that there may be a threshold hydraulic gradient below which flow does not take place. Swartzendruber (1962) and Bolt and Groenevelt (1969) review the evidence and summarize the various hypotheses that have been put forward to explain the phenomenon. As yet, there is no agreement on the mechanism, and the experimental evidence is still open to some doubt. In any event, the phenomenon is of very little practical importance; at the gradients being considered as possible threshold gradients, flow rates will be exceedingly small in any case.

Of greater practical importance is the upper limit on the range of validity of Darcy's law. It has been recognized and accepted for many years (Rose, 1945; Hubbert, 1956) that at very high rates of flow, Darcy's law breaks down. The evidence is reviewed in detail by both Todd (1959) and Bear (1972). The upper limit is usually identified with the aid of the *Reynolds number*  $R_e$ , a dimensionless number that expresses the ratio of inertial to viscous forces during flow. It is widely used in fluid mechanics to distinguish between *laminar flow* at low velocities and *turbulent flow* at high velocities. The Reynolds number for flow through porous media is defined as

$$R_e = \frac{\rho v d}{\mu} \quad (2.85)$$

where  $\rho$  and  $\mu$  are the fluid density and viscosity,  $v$  the specific discharge, and  $d$  a representative length dimension for the porous medium, variously taken as a mean pore dimension, a mean grain diameter, or some function of the square root of the

permeability  $k$ . Bear (1972) summarizes the experimental evidence with the statement that "Darcy's law is valid as long as the Reynolds number based on average grain diameter does not exceed some value between 1 and 10" (p. 126). For this range of Reynolds numbers, all flow through granular media is laminar.

Flow rates that exceed the upper limit of Darcy's law are common in such important rock formations as karstic limestones and dolomites, and cavernous volcanics. Darcian flow rates are almost never exceeded in nonindurated rocks and granular materials. Fractured rocks (and we will use this term to refer to rocks rendered more permeable by joints, fissures, cracks, or partings of any genetic origin) constitute a special case that deserves separate attention.

### Flow in Fractured Rocks

The analysis of flow in fractured rocks can be carried out either with the *continuum* approach that has been emphasized thus far in this text or with a *noncontinuum* approach based on the hydraulics of flow in individual fractures. As with granular porous media, the continuum approach involves the replacement of the fractured media by a representative continuum in which spatially defined values of hydraulic conductivity, porosity, and compressibility can be assigned. This approach is valid as long as the fracture spacing is sufficiently dense that the fractured media acts in a hydraulically similar fashion to granular porous media. The conceptualization is the same, although the representative elementary volume is considerably larger for fractured media than for granular media. If the fracture spacings are irregular in a given direction, the media will exhibit trending heterogeneity. If the fracture spacings are different in one direction than they are in another, the media will exhibit anisotropy. Snow (1968, 1969) has shown that many fracture-flow problems can be solved using standard porous-media techniques utilizing Darcy's law and an anisotropic conductivity tensor.

If the fracture density is extremely low, it may be necessary to analyze flow in individual fissures. This approach has been used in geotechnical applications where rock-mechanics analyses indicate that slopes or openings in rock may fail on the basis of fluid pressures that build up on individual critical fractures. The methods of analysis are based on the usual fluid mechanics principles embodied in the Navier-Stokes equations. These methods will not be discussed here. Wittke (1973) provides an introductory review.

Even if we limit ourselves to the continuum approach there are two further problems that must be addressed in the analysis of flow through fractured rock. The first is the question of non-Darcy flow in rock fractures of wide aperture. Sharp and Maini (1972) present laboratory data that support a nonlinear flow law for fractured rock. Wittke (1973) suggests that separate flow laws be specified for the linear-laminar range (Darcy range), a nonlinear laminar range, and a turbulent range. Figure 2.28 puts these concepts into the context of a schematic curve of specific discharge vs. hydraulic gradient. In wide rock fractures, the specific discharges and Reynolds numbers are high, the hydraulic gradients are usually less



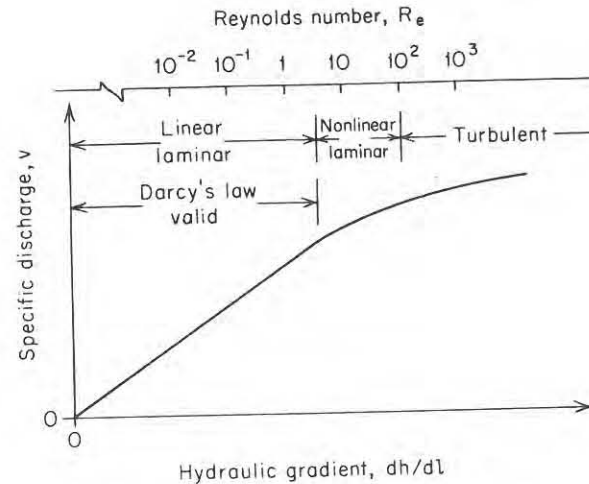


Figure 2.28 Range of validity of Darcy's law.

than 1, and the exponent  $m$  in Eq. (2.84) is greater than 1. These conditions lead to a downward deflection in the curve in Figure 2.28.

The second problem concerns the interaction of the three-dimensional stress field and the three-dimensional fluid flow field in rock. The general theoretical requirement for the coupling of these two fields was briefly discussed in Section 2.11, and reference was made there to the classic work of Biot (1941, 1955) for flow through porous media. For fractured rock, however, there is a further complication. Because the porosity of fractured rock is so low, the expansions and contractions of the fracture apertures that occur under the influence of changes in stress affect the values of hydraulic conductivity,  $K$ . The interaction between the fluid pressure  $p(x, y, z, t)$ , or the hydraulic head  $h(x, y, z, t)$ , and the effective stress  $\sigma_e(x, y, z, t)$  is thus complicated by the fact that  $K$  must be represented by a function,  $K(\sigma_e)$ . The analysis of such systems, and the experimental determination of the nature of the  $K(\sigma_e)$  function, is a continuing subject of research in the fields of rock mechanics and groundwater hydrology.

Many researchers involved in the application of groundwater theory in rock mechanics have proposed formulas that relate the fracture porosity  $n_f$  and the hydraulic conductivity  $K$  of jointed rocks to the joint geometry. Snow (1968) notes that for a parallel array of planar joints of aperture,  $b$ , with  $N$  joints per unit distance across the rock face,  $n_f = Nb$ , and

$$K = \left( \frac{\rho g}{\mu} \right) \left( \frac{Nb^3}{12} \right) \quad (2.86)$$

or

$$k = \frac{Nb^3}{12} \quad (2.87)$$

where  $k$  is the permeability of the rock.  $N$  and  $b$  have dimensions  $1/L$  and  $L$ , respectively, so that  $k$  comes out in units of  $L^2$ , as it should. Equation (2.86) is based on the hydrodynamics of flow in a set of planar joints. It holds in the linear-laminar range where Darcy's law is valid. It must be applied to a block of rock of sufficient size that the block acts as a Darcian continuum. A permeability  $k$ , calculated with Eq. (2.87), can be considered as the permeability of an equivalent porous medium; one that acts hydraulically like the fractured rock.

Snow (1968) states that a cubic system of like fractures creates an isotropic system with a porosity  $n_f = 3Nb$  and a permeability twice the permeability that any one of its sets would contribute; that is,  $k = Nb^3/6$ . Snow (1969) also provides predictive interrelationships between porosity and the anisotropic permeability tensor for three-dimensional joint geometries in which fracture spacings or apertures differ with direction. Sharp and Maini (1972) provide further discussion of the hydraulic properties of anisotropic jointed rock.

### 2.13 Hydrodynamic Dispersion

It is becoming increasingly common in the investigation of groundwater flow systems to view the flow regime in terms of its ability to transport dissolved substances known as solutes. These solutes may be natural constituents, artificial tracers, or contaminants. The process by which solutes are transported by the bulk motion of the flowing groundwater is known as *advection*. Owing to advection, nonreactive solutes are carried at an average rate equal to the average linear velocity,  $\bar{v}$ , of the water. There is a tendency, however, for the solute to spread out from the path that it would be expected to follow according to the advective hydraulics of the flow system. This spreading phenomenon is called *hydrodynamic dispersion*. It causes dilution of the solute. It occurs because of mechanical mixing during fluid advection and because of molecular diffusion due to the thermal-kinetic energy of the solute particles. Diffusion, which is a dispersion process of importance only at low velocities, is described in Section 3.4. Emphasis in the present discussion is on dispersion that is caused entirely by the motion of the fluid. This is known as *mechanical dispersion* (or *hydraulic dispersion*). Figure 2.29 shows a schematic example of the results of this dispersive process in a homogeneous, granular medium.

Mechanical dispersion is most easily viewed as a microscopic process. On the microscopic scale, dispersion is caused by three mechanisms (Figure 2.30). The first occurs in individual pore channels because molecules travel at different velocities at different points across the channel due to the drag exerted on the fluid by the roughness of the pore surfaces. The second process is caused by the difference in pore sizes along the flow paths followed by the water molecules. Because of differences in surface area and roughness relative to the volume of water in individual pore channels, different pore channels have different bulk fluid velocities.



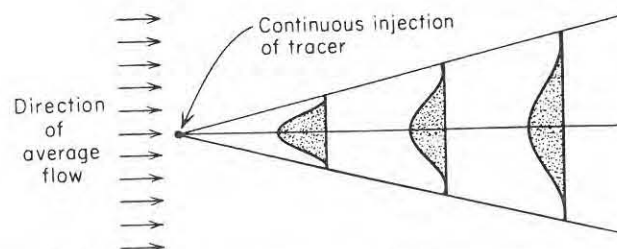


Figure 2.29 Schematic representation of the dilution process caused by mechanical dispersion in granular porous media.

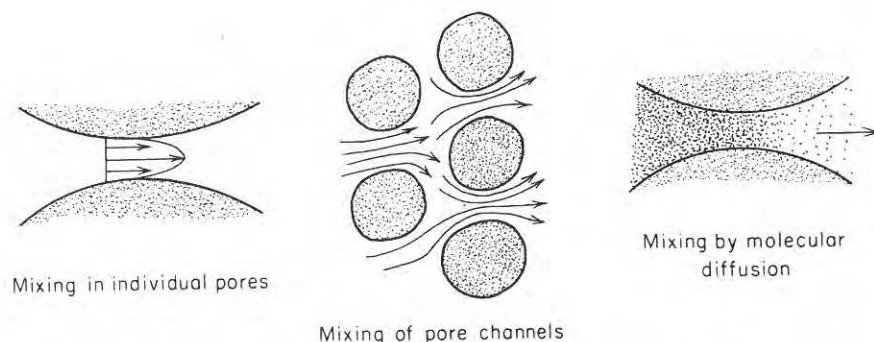


Figure 2.30 Processes of dispersion on a microscopic scale.

The third dispersive process is related to the tortuosity, branching, and inter-fingering of pore channels. The spreading of the solute in the direction of bulk flow is known as *longitudinal dispersion*. Spreading in directions perpendicular to the flow is called *transverse dispersion*. Longitudinal dispersion is normally much stronger than lateral dispersion.

Dispersion is a mixing process. Qualitatively, it has a similar effect to turbulence in surface-water regimes. For porous media, the concepts of average linear velocity and longitudinal dispersion are closely related. Longitudinal dispersion is the process whereby some of the water molecules and solute molecules travel more rapidly than the average linear velocity and some travel more slowly. The solute therefore spreads out in the direction of flow and declines in concentration.

When a tracer experiment is set up in the laboratory, the only dispersion that can be measured is that which is observable at the macroscopic scale. It is assumed that this macroscopic result has been produced by the microscopic processes described above. Some investigators believe that heterogeneities on the macroscopic scale can cause additional dispersion to that caused by the microscopic processes. The concept of macroscopic dispersion is still not well understood. Dispersive processes are pursued further in Chapter 9.

## Suggested Readings

- BEAR, J. 1972. *Dynamics of Fluids in Porous Media*. American Elsevier, New York, pp. 15–24, 52–56, 85–90, 122–129, 136–148.
- HUBBERT, M. K. 1940. The theory of groundwater motion. *J. Geol.*, 48, pp. 785–822.
- JACOB, C. E. 1940. On the flow of water in an elastic artesian aquifer. *Trans. Amer. Geophys. Union*, 2, pp. 574–586.
- MAASLAND, M. 1957. Soil anisotropy and land drainage. *Drainage of Agricultural Lands* ed. J. N. Luthin. American Society of Agronomy, Madison, Wisc., pp. 216–246.
- SKEMPTON, A. W. 1961. Effective stress in soils, concrete and rocks. *Conference on Pore Pressures and Suction in Soils*. Butterworth, London, pp. 4–16.
- STALLMAN, R. W. 1964. Multiphase fluids in porous media—a review of theories pertinent to hydrologic studies. *U.S. Geol. Surv. Prof. Paper 411E*.
- VERRUIT, A. 1969. Elastic storage of aquifers. *Flow Through Porous Media*, ed. R. J. M. De Wiest. Academic Press, New York, pp. 331–376.

## Problems

- The following field notes were taken at a nest of piezometers installed side by side at a single site:

Piezometer	a	b	c
Elevation at surface (m a.s.l.)	450	450	450
Depth of piezometer (m)	150	100	50
Depth to water (m)	27	47	36

Let  $A$ ,  $B$ , and  $C$  refer to the points of measurement of piezometers  $a$ ,  $b$ , and  $c$ . Calculate:

- The hydraulic head at  $A$ ,  $B$ , and  $C$  (m).
  - The pressure head at  $A$ ,  $B$ , and  $C$  (m).
  - The elevation head at  $A$ ,  $B$ , and  $C$  (m).
  - The fluid pressure at  $B$  ( $\text{N/m}^2$ ).
  - The hydraulic gradients between  $A$  and  $B$  and between  $B$  and  $C$ . Can you conceive of a hydrogeological situation that would lead to the directions of flow indicated by these data?
- Draw diagrams of two realistic field situations in which three piezometers installed side by side, but bottoming at different depths, would have the same water-level elevation.
  - Three piezometers located 1000 m apart bottom in the same horizontal aquifer. Piezometer  $A$  is due south of piezometer  $B$  and piezometer  $C$  is to the east of



- the line  $AB$ . The surface elevations of  $A$ ,  $B$ , and  $C$  are 95, 110, and 135 m, respectively. The depth to water in  $A$  is 5 m, in  $B$  is 30 m, and in  $C$  is 35 m. Determine the direction of groundwater flow through the triangle  $ABC$  and calculate the hydraulic gradient.
4. Show that the fluid potential  $\Phi$  is an energy term, by carrying out a dimensional analysis on the equation  $\Phi = gz + p/\rho$ . Do so for both the SI system of units and the FPS system of units.
  5. Three formations, each 25 m thick, overlie one another. If a constant-velocity vertical flow field is set up across the set of formations with  $h = 120$  m at the top and  $h = 100$  m at the bottom, calculate  $h$  at the two internal boundaries. The hydraulic conductivity of the top formation is 0.0001 m/s, the middle formation 0.0005 m/s, and the bottom formation 0.0010 m/s.
  6. A geologic formation has a permeability of 0.1 darcy (as determined by a petroleum company for the flow of oil). What is the hydraulic conductivity of the formation for the flow of water? Give your answer in m/s and in gal/day/ft<sup>2</sup>. What kind of rock would this likely be?
  7. (a) Four horizontal, homogeneous, isotropic geologic formations, each 5 m thick, overlie one another. If the hydraulic conductivities are  $10^{-4}$ ,  $10^{-6}$ ,  $10^{-4}$ , and  $10^{-6}$  m/s, respectively, calculate the horizontal and vertical components of hydraulic conductivity for the equivalent homogeneous-but-anisotropic formation.  
(b) Repeat for hydraulic conductivities of  $10^{-4}$ ,  $10^{-8}$ ,  $10^{-4}$ , and  $10^{-8}$  m/s, and for hydraulic conductivities of  $10^{-4}$ ,  $10^{-10}$ ,  $10^{-4}$ , and  $10^{-10}$  m/s. Put the results of the three sets of calculations in a table relating orders of magnitude of layered heterogeneity to resulting equivalent anisotropy.
  8. (a) From the volumetric definitions of porosity and void ratio, develop the relationships given in Eq. (2.40).  
(b) Is the porosity ever greater than the void ratio when both are measured on the same soil sample?
  9. The elevation of the ground surface at a soil-moisture measurement site is 300 cm. The soil is a sand and its unsaturated properties are represented by the drying curves of Figure 2.13. Draw a quantitatively accurate set of vertical profiles of moisture content, pressure head, and hydraulic head versus depth (as in Figure 2.12) for a 200-cm depth under the following conditions:  
(a) The moisture content is 20% throughout the profile.  
(b) The pressure head is  $-50$  cm throughout the profile.  
(c) The hydraulic head is 150 cm throughout the profile (static case).  
For cases (a) and (b), calculate the hydraulic gradients and the rates of flow through the profile. For case (c), determine the depth to the water table.
  10. Given a potentiometric surface with a regional slope of 7 m/km, calculate the natural rate of groundwater discharge through a confined aquifer with transmissivity,  $T = 0.002$  m<sup>2</sup>/s.

11. Show by dimensional analysis on the equation  $S = \rho gb(\alpha + n\beta)$  that the storativity is dimensionless.
12. (a) A horizontal aquifer is overlain by 50 ft of saturated clay. The specific weight (or unit dry weight) of the clay is 120 lb/ft<sup>3</sup>. The specific weight of water is 62.4 lb/ft<sup>3</sup>. Calculate the total stress acting on the top of the aquifer.  
(b) If the pressure head in the aquifer is 100 ft, calculate the effective stress in the aquifer.  
(c) If the aquifer is pumped and the hydraulic head at some point is reduced 10 ft, what will be the resulting changes in the pressure head, the fluid pressure, the effective stress, and the total stress?  
(d) If the compressibility of the aquifer is  $10^{-6}$  ft<sup>2</sup>/lb and its thickness is 25 ft, how much compaction will the aquifer undergo during the head reduction in part (c)?  
(e) If the porosity and hydraulic conductivity of the aquifer are 0.30 and 10 gal/day/ft<sup>2</sup>, calculate the transmissivity and storativity for the aquifer. The compressibility of water is  $2.1 \times 10^{-8}$  ft<sup>2</sup>/lb.
13. Review the problems that arise in the definition or use of the following classical groundwater terms: potentiometric surface, permeability, and groundwater flow velocity.